Reference estimation in EEG: analysis of equivalent approaches

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Abstract

This paper demonstrates the theoretical equivalence between the different solutions proposed in the literature for the reference-estimation problem in electroencephalographic (EEG) recordings. By reference, we understand an unknown, non-null, time-varying potential, measured at the reference electrode situated sufficiently distant from the measuring electrodes. Despite the theoretical equivalence of the various approaches, they do not yield identical results in practice. This discrepancy is primarily due to the practical implementation of the underlying approach. We show in this context that the most reliable solution avoids blind source separation and montage transformation in addition to making full use of available a priori knowledge.

Index Terms

EEG, reference estimation, blind source separation

I. Introduction

An important issue for bio-potentials recording devices (EEG in particular) is to find a reference region on the human body with as low to neutral electrical activity as possible. Indeed, the electrical activity at the reference affects measurements at all other active electrode sites [3–6]. As pointed out by all cited authors, it is impossible to find such a zero-potential reference, and all recording devices use the so-called *common reference* (CR) montage: *measuring electrodes* are referenced to a particular chosen *reference electrode*.

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In depth EEG recordings [5, 6], the signals are acquired from intracranial contacts, placed along a nail electrode inserted in the brain (see Fig. 1). In this setup, the reference electrode is placed on the surface of the head, and the signal it records is assumed to be uncontaminated by (in signal processing terminology: independent of, or atleast uncorrelated with) the electrical activity recorded by the measuring contacts[1]. However, although the reference potential is supposed independent of the depth sources, it is not necessarily null: the surface reference electrode records also muscle and eye artefacts and might also be contaminated by random noise. These extra-cerebral artefacts and noise appear on all measured signals, as they are obtained as a potential difference electrical activity recorded by the measuring contacts and the reference electrode.

To eliminate the influence of the reference electrode, and consequently to ease the interpretation and the use of different signal processing techniques like synchrony measures (coherence and similar methods), several montages have been derived from the CR recordings by simple manipulations. In depth-EEG signal analysis, the most commonly employed montage is the Bipolar Montage (BM), obtained by taking the difference between two neighboring measuring contacts of a nail electrode. All depth signals are interpreted by clinicians using this montage as images of the local neural activity, implicitly reference free.

Still, both in scalp- and in depth-EEGs, direct measures obtained by the CR montage can be useful for the interpretation, as they offer a global view complementary to the local view furnished by the BM montage. Unfortunately, they are contaminated by the electrical activity recorded by the reference contact.

Hu, Stead and Worrel [5] proposed a first attempt to reduce this influence using an ICA-based blind source separation (BSS) approach. A modification of this approach, replacing the ICA operator by a principal component analysis (PCA) was later proposed by the same authors as a faster, more robust solution [6].

The approach of [5] served as the motivation for [7], wherein a simpler, more elegant and robust solution was proposed by directly exploiting the particular structure of the mixing matrix. Moreover (see [8]), this last semi-blind source separation solution (sBSS) was shown to be equivalent to the minimum power distortionless response filter (MPDR), well known in array signal processing, and which also maximises the signal-to-noise ratio (SNR).

[1]Although in practice this independence assumption is difficult to verify, current assumptions in EEG measurements state that surface potentials are mainly generated by pyramidal neurons situated in the neo-cortex [1], and that the electrical activity of the profound neural structures do not reach the surface of the head [2].

Our communication here furthers the analysis presented in [8]. Specifically we demonstrate that the BSS-based approaches of [5, 6] are *identical* to the sBSS/MPDR solution and that the ICA (respectively, PCA) operations suggested in [5, 6] are not only *unnecessary* but might potentially *degrade* the solution. This explains the discrepancy between the results obtained by the sBSS/MPDR approach and the approaches of [5, 6], despite their theoretical equivalence.

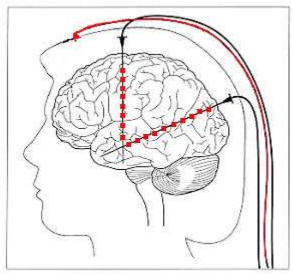

Fig. 1: Depth EEG implantation scheme: the nail electrodes, having 10 to 15 measuring points, are inserted in the brain, while the reference electrode is placed on the surface of the head

This paper is organized as follows: we first briefly present the signal model for the reference estimation, followed by the approach of [5, 6]. Section IV then presents the sBSS/MPDR solution from [7, 8]. The proof of the theoretical equivalence of all the approaches, which forms the main contribution of the paper, is presented in Section V. This is followed by a discussion on the implementation issues which explains the significant performance differences between the algorithms.

II. Signal model

Regardless of the employed montage, the measured EEG signals at each of the M sensors can be considered as a mixture of several unknown cortical sources, extra-cortical artefacts and noise. This is compactly represented by the following signal model:

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) \tag{1}$$

where $\mathbf{x}(n) \in \mathbb{R}^{(M\times 1)}$ is the vector of observations at time instant n (measured EEG signals after sampling and quantization) and $\mathbf{s}(n) \in \mathbb{R}^{(Q\times 1)}$ is the corresponding vector of source realisations (underlying brain activity) at the same instant. The different sources s_q are supposed to be at least uncorrelated ($\mathrm{E}\{s_q s_{q'}\} = 0,\ \forall q \neq q'$) if not statistically independent. $\mathbf{A} \in \mathbb{R}^{(M\times Q)} = \left(\mathbf{a}_1, \ldots, \mathbf{a}_Q\right)$ represents the linear combination of the sources to yield the observation vector $\mathbf{x}$, where $\mathbf{a}_q \in \mathbb{R}^{(M\times 1)}$. This model, also known as the instantaneous mixing model, is widely accepted in the EEG processing field [9]. For ease of exposition, we will subsequently drop the time index n.

Blind source separation (BSS) separates these mixed measured signals into "independent" sources, which can be used either for artefact elimination or for (normal or pathological) brain activity evaluation (see, for example, Sanei and Chambers [9] for a review).

The application of BSS that interests us in this paper is that of reference identification and removal in depth-EEG recordings. Prior work in this area was performed by Hu *et al.* [5, 6]. They make the basic assumption that the non-zero reference signal r can be seen as a source, independent from the other brain sources in the observed mixture, and present several BSS-based approaches to extract this source.

The signal model they consider is that of a common-reference EEG recording, which is obtained by modifying (1) as:

$$\mathbf{x}_c = \begin{bmatrix} & -1 \\ \mathbf{A} & \vdots \\ & -1 \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ r \end{bmatrix}, \tag{2}$$

where r is the non-zero reference signal and $\mathbf{x}_c$ now denotes the vector of measured common-reference EEG signals. This formulation is a straightforward result of referencing the potentials at the measuring electrodes to the common reference electrode.

III. Reference estimation via bipolar montage transformation

In [5, 6], the reference signal is estimated in two stages. In the first stage the CR montage of (2) is transformed into the bipolar montage (BM) by computing pair-wise differences of the signals in $\mathbf{x}_c$. In matrix form, this transform can be written as a left multiplication by a *blocking* matrix $\mathbf{B}$ of dimensions $(M-1)\times M$:

$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \cdots & 1 & -1 & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}. \tag{3}$$

The corresponding $M-1$ dimensional BM signal vector is then:

$$\mathbf{x}_b = \mathbf{B}\mathbf{x}_c\,. \tag{4}$$

In the second stage, these BM signals are decomposed into $P \leq M-1$ independent ([5]) or principal ([6]) components:

$$\overline{\mathbf{s}}_{\text{ica}} = \mathbf{G}_{\text{ica}}\mathbf{x}_b = \mathbf{G}_{\text{ica}}\mathbf{B}\mathbf{x}_c \tag{5}$$

$$\overline{\mathbf{s}}_{\text{pca}} = \mathbf{G}_{\text{pca}}\mathbf{x}_b = \mathbf{G}_{\text{pca}}\mathbf{B}\mathbf{x}_c \tag{6}$$

These reference-free signals are then utilized to obtain an estimate of r as:

$$\widehat{r} = -\frac{1}{M}\sum_{m=1}^{M}\left(x_{c,m} - \sum_{l=1}^{P}\frac{\mathrm{E}\{x_{c,m}\overline{s}_l\}}{\mathrm{E}\{\overline{s}_l^2\}}\overline{s}_l\right) \quad , \tag{7}$$

where the $x_{c,m}$ are the elements of $\mathbf{x}_c$, and $\overline{s}_l$ is the lth source recovered from either (5) or (6) such that:

$$\mathrm{E}\{\overline{s}_l\overline{s}_{l'}\} = 0, \quad \forall l \neq l' \, , \tag{8}$$

where the condition in (8) is guaranteed due to the properties of the ICA and PCA transforms.

IV. sBSS/MPDR REFERENCE ESTIMATION

The independent reference estimation problem was approached differently by [7] and [8]. Starting from the same model of equation (2), and assuming that the reference can be found as a linear combination of the measured signals, the authors observed that applying BSS to estimate a full separation matrix is not necessary, as in fact only one source (namely r) needs to be estimated, and it has a known mixing column. For this, one row of the full separation matrix suffices.

This observation was exploited in [7], wherein a so-called semi-blind source separation method was proposed. This approach was shown, in [8], to be equivalent to a constrained power minimisation approach, well known in array literature as the minimum power distortionless response (MPDR) approach. It was further shown in [8] that this solution also maximises the signal-to-noise ratio and thus is the optimal solution in terms of least squared error.

This solution yields the reference estimate as:

$$\widehat{r} = \mathbf{w}^T\mathbf{x}_c \, , \quad \text{with} \quad \mathbf{w} = -\frac{\mathbf{\Phi}_{\mathbf{x}_c\mathbf{x}_c}^{-1}}{\mathbf{1}^T\mathbf{\Phi}_{\mathbf{x}_c\mathbf{x}_c}^{-1}\mathbf{1}}\mathbf{1} \tag{9}$$

where $\mathbf{\Phi}_{\mathbf{x}_c\mathbf{x}_c} = \mathrm{E}\{\mathbf{x}_c\mathbf{x}_c^T\}$ is the correlation matrix of the measured signals $\mathbf{x}_c$ and $\mathbf{1}$ is a vector of ones.

V. Proof of equivalence and implementation issues

Numerical simulations presented in [7] show that the sBSS/MPDR approach outperforms the bipolar/FastICA method proposed by [5], especially for noisy signals. Similar simulations (not presented here) show that by changing the BSS algorithm from FastICA to SOBI-RO also leads to different performances. Finally, Hu *et al.* [6] also confirmed that the use of PCA instead of FastICA leads to better results. These observations tend to suggest that the methods are essentially different. As it will be shown next, this is not the case.

Indeed, equation (7) may be expressed in a more compact manner as follows:

$$\widehat{r} = -\frac{1}{M}\mathbf{1}^T\left(\mathbf{x}_c - \mathrm{E}\left\{\mathbf{x}_c\overline{\mathbf{s}}^T\right\}\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}}^{-1}\overline{\mathbf{s}}\right), \tag{10}$$

where $\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}}$ is a *diagonal* matrix such that $[\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}}]_{l,l} = \mathrm{E}\left\{\overline{s}_l^2\right\}$ (which is an identity matrix if BSS is used as in [5]). We may further simplify (10) by substituting for $\overline{\mathbf{s}}$ from (5) (resp. (6)) as:

$$\widehat{r} = -\frac{1}{M}\mathbf{1}^T\left(\mathbf{x}_c - \mathrm{E}\left\{\mathbf{x}_c\mathbf{x}_c^T\right\}\mathbf{B}^T\mathbf{G}^T\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}}^{-1}\mathbf{G}\mathbf{B}\mathbf{x}_c\right) \tag{11}$$

Recognizing that $\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}} = \mathbf{G}\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T$, we finally obtain: Also, recognizing that $\boldsymbol{\Phi}_{\overline{\mathbf{s}}\overline{\mathbf{s}}} = \mathbf{G}\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T$, where $\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c} = \mathrm{E}\left\{\mathbf{x}_c\mathbf{x}_c^T\right\}$ is the correlation matrix of $\mathbf{x}_c$, we finally obtain:

$$\widehat{r} = -\frac{1}{M}\mathbf{1}^T\left(\mathbf{x}_c - \boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T\left(\mathbf{G}\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T\right)^{-1}\mathbf{G}\mathbf{B}\mathbf{x}_c\right) \tag{12}$$

On the other hand, the solution of (9) can be factorized as:

$$\begin{aligned}\widehat{r} &= -\frac{\mathbf{1}^T\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}^{-1}}{\mathbf{1}^T\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}^{-1}\mathbf{1}}\mathbf{x}_c \\ &= -\frac{1}{M}\mathbf{1}^T\left(\mathbf{x}_c - \boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\left(\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{x}_c\right),\end{aligned} \tag{13}$$

where $\mathbf{B}$ is the bipolar transform from (4). This factorisation is demonstrated in [10].

Note, firstly, the similarities between (12) and (13). In the case that the ICA and PCA transforms preserve the rank (i.e., no dimension reduction is applied and the $\mathbf{G}_{\mathrm{ica}}$ and $\mathbf{G}_{\mathrm{pca}}$ are fully ranked, square matrices of dimensions $(M-1)$), the matrix $(\mathbf{G}\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T)^{-1}$ may be factorized as:

$$(\mathbf{G}\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T\mathbf{G}^T)^{-1} = \mathbf{G}^{-T}(\mathbf{B}\boldsymbol{\Phi}_{\mathbf{x}_c\mathbf{x}_c}\mathbf{B}^T)^{-1}\mathbf{G}^{-1} \tag{14}$$

Substituting this value in (12) it is easy to see that the solution of [5, 6] is *identical* to the MPDR solution of [7, 8]. The PCA (resp. ICA) transforms are *redundant*.

VI. Implementation issues

Still, despite this formal proof of equivalence, the numerical results are worse when using Hu's method with FastICA [5] than when using sBSS/MPDR approaches [7, 8] or PCA based method from [6]. Clearly, some assumptions made while proving the theoretical equivalnce of the approaches are not respected when using FastICA. Indeed, FastICA (and also other BSS algorithms, especially the iterative ones), sometimes fails to converge toa full ranked $\mathbf{G}_{\mathrm{ica}}$ and to retrieve $M-1$ independent sources. This indicates that the ICA approach has discarded some sources, which then end up corrupting the reference estimate in (12).

In general, where the ICA and PCA approaches incorrectly perform *dimension reduction* $(\mathrm{rank}(\mathbf{G}) < (M-1))$ the approaches of [5, 6] would yield worse estimates of r as compared to the sBSS/MPDR approach. As shown in [7], this is especially true when the signals are noisy: in this case, a noise subspace will be incorrectly separated from the signal subspace, resulting in a dimension reduction.

Another issue appears when using SOBI-RO and similar BSS algorithms. In this case, to be robust to noise, a so-called robust whitening is introduced instead of the simple whitening. This leads to an approximate orthogonalization of $\overline{\mathbf{s}}$. Thereby (8) is violated, and the result of (12) is degraded.

Neither of these problems appear when using PCA without dimension reduction (equivalent to simple whitening) or when using sBSS/MPDR approaches.

With respect to computational complexity: as the sBSS/MPDR approach does not require any PCA, ICA or BM transform, it has the least computational complexity. This point is anyway moot because, as already demonstrated, the PCA/ICA transforms are unnecessary for the existing reference estimation framework.

As a last point: the sBSS/MPDR approach is general and can be easily applied for estimating any source if knowledge of its mixing coefficients are available.

VII. Conclusion

The goal of this short paper was to show that different solutions proposed in the literature for reference estimation in depth EEG signals are theoretically *strictly equivalent*, with the two-stage ICA-/PCA-based approaches incorporating unnecessary transforms, which increase the computational complexity. Despite this proof, the implementations are not numerically equivalent, because of the lack of robustness in the two-stage ICA(resp. PCA)-based approaches. The sBSS/MPDR solution, on the other hand, makes full use of the available *a priori* knowledge and, in additional to lower computational burden is also numerically more robust.

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References

[1] D. Cosandier-Rimélé, J.-M. Badier, P. Chauvel, and F. Wendling, "A physiologically plausible spatio-temporal model for EEG signals recorded with intracerebral electrodes in human partial epilepsy," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 3, pp. 380–388, 2007.

[2] H. P. Zaveri, R. B. Duckrow, and S. S. Spencer, "Concerning the observation of an electrical potential at a distance from an intracranial electrode contact," *Clinical Neurophysiology*, Letter to the editor, vol. 120, no. 10, pp. 1873–1875, 2009.

[3] J. Dien, "Issues in the application of the average reference: review, critiques and recommendations," *Behavior Research Methods, Instruments, & Computers*, vol. 30, no. 1, pp. 34–43, 1998.

[4] D. Yao, "A method to standardize a reference of scalp EEG recordings to a point at infinity," *Physiological Measurement*, vol. 22, pp. 693–711, 2001.

[5] S. Hu, M. Stead, and G. Worrel, "Automatic identification and removal of scalp reference signal for intracranial EEGs based on Independent Component Analysis," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 9, pp. 1560–1572, 2007.

[6] S. Hu, M. Stead, and G. Worrell, "Removal of scalp reference signal and line noise for intracranial EEGs," in *Networking, Sensing and Control, IEEE International Conference on*, Apr. 2008, pp. 1486–1491.

[7] R. Ranta, R. Salido-Ruiz, and V. Louis-Dorr, "Reference estimation in EEG recordings," in *32nd Ann. Int. Conf. of the IEEE-EMBS*, Buenos Aires, Argentina, Aug. 2010.

[8] N. Madhu and R. Ranta, "A unified treatment of the reference estimation problem," *submitted*, 2011.

[9] S. Sanei and J. Chambers, *EEG Signal Processing*. John Wiley & Sons, 2007.

[10] B. Breed and J. Strauss, "A short proof of the equivalence of LCMV and GSC beamforming," *IEEE Signal Process. Lett.*, vol. 9, no. 6, pp. 168–169, 2002.