EEG/SEEG signal modelling using frequency and fractal analysis

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***Abstract*—EEG (Electroencephalography) is used to measure the electrical activity of a human brain. It is widely used to analyse both normal and pathological data, because of its very high temporal resolution. Different algorithms were proposed in the literature for EEG signal processing, but a difficult issue is their validation on real signals. An important goal is thus to realistically simulate EEG data. The starting point of this research was the model proposed by Rankine et al. for the surface newborn EEG signal generation [1]. The model, based on both statistical, fractal and classical frequency modelling, has parameters estimated from the real data. A first objective is to validate and parametrize this model on adult surface EEG. A second and more important goal is to parametrize it and to apply it to depth EEG measurements (SEEG). The first results presented in this communication show that the proposed model can be applied in both cases (surface and depth adult EEG), although the parameters are slightly different. As expected [1], seizures cannot be modelled using this approach.**

***Index Terms*—EEG, simulation, fractal dimension**

I. Introduction

Electroencephalography (EEG) is the most widely used method to record electrical activity of the human brain. This data can be used to analyse the behaviour of the normal brain, as well as to diagnose different pathologies, as for example epilepsy. Since our knowledge about the generators of the electrical activity in brain is still on a fairly basic level, most of the signal processing algorithms developed for EEG signals can be validated only by medical expertise. In order to have reliable results, we need to use large datasets for testing. Since EEG recording is time consuming and problematic (because of the high variability of the signals), consistent large data sets are quite difficult to obtain. Simulated realistic datasets would help to build more consistent algorithms and test them more properly.

Depth EEG (called further as SEEG – Stereoelectroencephalography) uses the same principle of electrical activity recording like EEG, but electrodes are surgically inserted into the brain. As expected, because of the invasiveness of the technique, SEEG data is even less frequent than EEG data. Because of their acquisition method, the SEEG signals supposedly directly record brain sources, while the surface EEG is a mixture of source signals. Simulated signal can be useful both for SEEG dedicated studies and for forward/inverse problem applications: with a realistic source modelling, one can expect more realistic scalp EEG modelling. Moreover, in an inverse problem setup, simulated SEEG can be compared to the one obtained by the source estimation algorithms and thus used to validate them.

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II. EEG Models

There are several different approaches to model and simulate EEG signals, depending on the purpose of their applications. The most popular of them are:

- Source modelling from EEG signals (inverse problem, source separation)[2]
- Biological modelling (neurocomputing)[3], [4]
- EEG/SEEG modelling mimicking real signals [1]

Following [1], [5], we focus in this paper on third approach. **Signal imitation** is made using real signal characteristics. Datasets of real EEGs are analysed, in order to obtain these characteristics. Depending on the model, different supplementary assumptions are made, and validation is performed against large real datasets.

Rankine et al. separate two models having different characteristics: **seizure model** and **background model**, aiming to characterize different new-born real EEGs (Figs. 1 and 2).

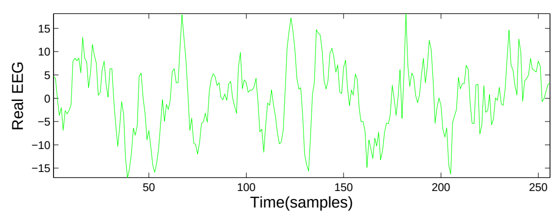


Fig. 1. Background EEG signal.

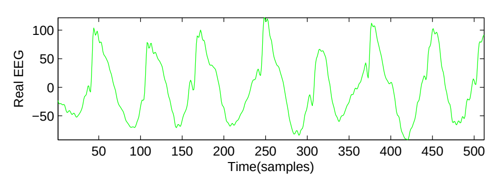


Fig. 2. Seizure EEG signal.

We focus here on the [1] background EEG model, our aim being to find if it can be applied on adult surface and depth data. We will asses its validity for both background and seizure signals. The different steps of the cited model and employed methodological tools, are described in more detail in the next section.

A. Background EEG modelling

The background surface EEG model [1] uses the fact that the EEG power spectrum approximately follows a power law:

$$S(f) \approx \frac{c}{|f|^{\gamma}} \tag{1}$$

where c is constant, f is frequency and γ is the power law exponent[1]. If one wants to generate a simulated EEG signal $x(t)$, the first step is to express $S(f)$ as $X(f)X^*(f)$, with $X(f)$ being the amplitude spectrum of the EEG signal $x(t)$, obtained by the Fourier transform:

$$X(f) = \frac{\sqrt{c}}{|f|^{\frac{\gamma}{2}}} e^{j\theta(f)}, \quad (2)$$

where $\theta(f)$ is the phase of the Fourier transform. In order to obtain a more realistic signal, [1] proposes to generate several $X_i(f)$ using different phase vectors $\theta_i(f)$. Several $x_i(t)$ can be obtained by inverse Fourier transform from $X_i(f)$, and the final simulated background EEG signal is generated as

$$x(t) = \sum_i \mathcal{F}^{-1}(X_i(f)) \quad (3)$$

As it can be seen, this model needs three parameters: c, γ and $\theta(f)$. The amplitude c is of secondary importance, so we will focus only on the last two parameters. In order to use realistic values, they must be extracted from real data.

1) Parameter estimation: The method used in [1] to estimate the power law exponent γ exploits the linear relationship between γ and the fractal dimension FD of a signal [6], expressed by:

$$FD = \frac{5-\gamma}{2} \quad (4)$$

This step is useful because the FD can be estimated from the real EEGs using one of the fractal dimension estimation methods. Different fractal dimension estimators (such as Box-counting, Information and Correlation dimensions) can be used, with quite similar results on classical fractals. Higuchi's FD estimation[7] is a particular example of fractal dimension derived from box-counting. This algorithm works directly in the time domain (analysing the geometrical form of signal), so it can be used for relatively short time intervals.

As said previously, in order to simulate realistic signals, the needed parameters (FD and $\theta(f)$) must respect real signals characteristics. As in [1], we have estimated them using the following procedure, applied to a database of real adult background EEG/SEEG signals:

- estimate the FD and compute the phase for each signal
- assume that, over the database, FD follows a beta distribution and estimate the distribution parameters (method of moments [8]). Probability density function of a beta distribution with two parameters, α and β can be expressed as

  $$pdf(x,\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad x \in [0,1]$$

  where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\mathrm{d}t$ is the Γ function.
- assume that the phase θ follows a uniform distribution in $[-\pi, \pi]$
- test (Kolmogorov-Smirnov) the empirical distributions against theoretical distributions generated using the previously estimated parameters.

[1]Since real EEGs are non-stationary, γ is considered constant for every epoch of 4 seconds (assuming a quasi-stationary signal during one epoch).

2) Signal simulation: Assuming that estimated realistic probability distributions have been obtained for both the fractal dimensions FD and for the phases $\theta(f)$, a realistic simulated background EEG can be generated by randomly choosing a value for FD and a phase vector $\theta(f)$ and introducing them in (4),(2) and (3).

In order to validate the approach, [1] suggests to extract FD and $\theta(f)$ from a real EEG measurements and to use the described method to generate a synthetic signal: if the method is correct, than the original signal and the simulated one should be similar (correlated). The correlation index, noted further on as ρ, can be computed in time domain (ρ_t), as well in frequency (after computing the Welch periodogramm, ρ_f) and in time-frequency (spectrograms after short-time Fourier transforms, ρ_{tf}).

III. Results

The described model was applied to different classes of EEG signals: surface and depth, background and seizure. The database contained 400 signal fragments from 3 different patients, 4 seconds each. Seizure periods were pointed out by neurologists beforehand. Surface EEG signal was filtered with cut-off frequencies at 0.5 and 30Hz whereas source SEEG signal was filtered with low-pass filter at 128Hz (no assumptions on SEEG signal spectral behaviour was made). Consequently, surface EEG signals contained 256 samples and source SEEG signals contained 1024 samples for every 4 seconds window.

A. Adult surface EEG

At first, the simulation method was applied to adult surface EEG data. For generality, we tested the model both to background and seizure EEG, downsampled to 64Hz (as in [1]).

The power spectral density (PSD) was computed for several time windows of 4s length, to find out if it exhibits $1/f$ process behaviour (figure 3). Under the $1/f$ hypothesis, the fractal dimension FD can be estimated using (4).

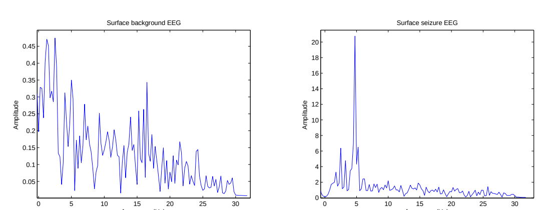
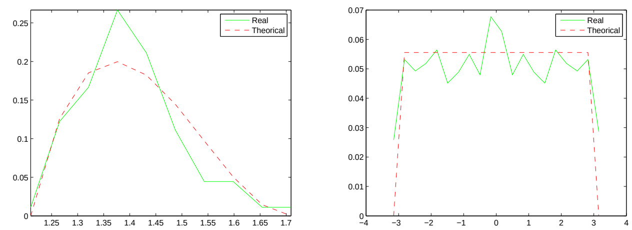


Fig. 3. PSD of an adult background (left) and seizure (right) EEG signal

1) Parameter extraction for background EEG data: Fractal dimension (thus γ) and phase spectrum were calculated for every time window from the database and empirical distributions were estimated as described previously. Results are shown in figure 4. γ was found to follow a beta distribution with $\alpha = 1.936$ and $\beta = 2.975$. θ was found to follow uniform distribution in $[-\pi, \pi]$. These hypothesis were confirmed using Kolmogorov-Smirnov test at a 5% significance level.



(a) γ (b) Phase

Fig. 4. Surface background EEG FD and phase spectrum distributions

TABLE I
CORRELATIONS (MEAN AND SD) FOR BACKGROUND EEG

ρ	new-born [1]	adult
ρ_t	0.795 (0.081)	0.675 (0.075)
ρ_f	0.716 (0.131)	0.803 (0.150)
ρ_{tf}	0.817 (0.113)	0.705 (0.075)

2) Parameter extraction for seizure EEG data: The same procedure could be applied also for seizure signals. Still, as seen in Fig. 3, the PSD does not display a $1/f$ process behaviour: because of rhythmic seizure activity, a peak in the seizure frequency band might be observed. Consequently, eq. (4) does not hold and other modelling techniques must be applied (see also [1]).

3) Validation: In order to validate the approach, the second procedure described previously was used: starting from a real signal, FD is estimated and thus γ. Its phase spectrum was computed (θ) as well as its power (used to estimate c). These parameters were used to generate a particular synthetic signal that was later compared with the real one using the validation procedure described previously (3 correlations ρ_t, ρ_f, ρ_{tf}).

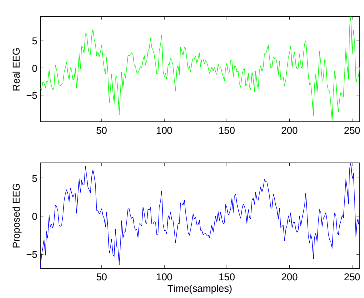


Fig. 5. Real and simulated background EEG signals

a) Background EEG: The obtained modelling results are rather similar between new-born and adult data. Adult modelled signals show a better correlation than new-borns in the frequency domain, but correlation in time and time-frequency domains are lower. Globally, it seems that adult background EEGs can be modelled using the described approach (validated in [1] for new-borns).

b) Seizure EEG: Same analysis was performed for seizure EEGs. Since Rankine et al. had proposed another model for seizure EEGs, it was decided to compare the correlations of our model with Rankine's et al. model.

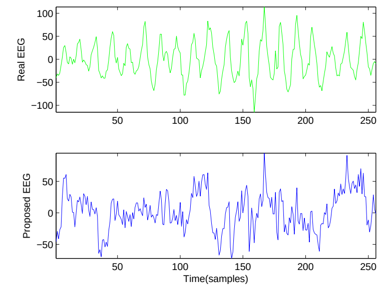


Fig. 6. Real and simulated seizure EEG signals

TABLE II
CORRELATIONS (MEAN AND SD) FOR SEIZURE EEG

ρ	new-born [1]	adult
ρ_t	0.345 (0.176)	0.661 (0.705)
ρ_f	0.799 (0.093)	0.494 (0.178)
ρ_{tf}	0.901 (0.056)	0.680 (0.090)

Here it can be seen that, unlike in the previous case, in the time domain this method give better results than [1]. On the contrary, in frequency domain correlations are very low. This might be related to the power spectrum density of surface seizure EEG that does not follow $1/f$ law. Still, due to the high result in the time domain, we think that after an appropriate power spectrum density estimation (i.e. different from $1/f$ process), this model could be used also to seizure EEGs.

B. Adult depth EEG (SEEG)

The main difference from the practical point of view between applying the same approach on EEG and SEEG data is that, since the frequencies contained in the SEEG might be higher, filtering and downsampling is not applied. Examples of power spectra are given Fig. 7.

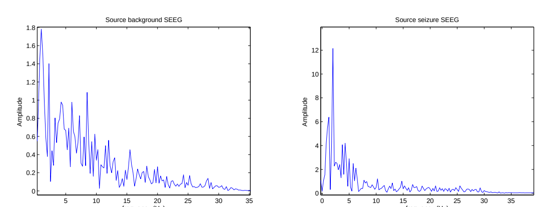
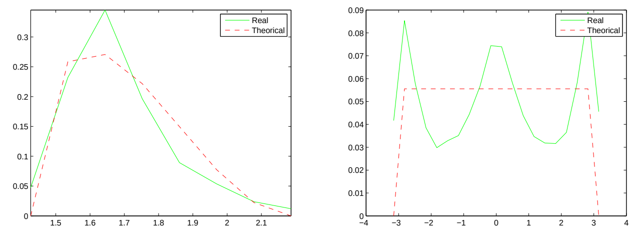


Fig. 7. PSD of an adult background (left) and seizure (right) SEEG signal

1) Parameter extraction for background SEEG data: Fractal dimension (and thus γ) and phase spectrum were estimated for every time window. Results are shown in Fig. 8.

According to power spectrum density (Fig. 8), we can see that SEEG could be considered as a $1/f$ process. γ distribution was found to follow beta distribution with $\alpha = 1.578$ and $\beta = 2.945$[2] This hypothesis was tested with Kolmogorov-Smirnov test and could not be rejected at the 5% significance level. Meanwhile θ distribution was found not to exhibit uniform distribution. Therefore, in order to improve this model, some other distributions should be tried as descriptors of the θ distribution.

[2]Note that the values are quite different from the surface EEG.



(a) γ (b) Phase

Fig. 8. Source background SEEG FD and phase spectrum distributions

2) Parameter extraction for seizure SEEG data: The same procedure has been applied also for seizure SEEGs. As expected, the PSD does not display a $1/f$ behaviour, and phase distribution as well is far from the uniform distribution: the described approach is not appropriate for a reasonable simulation of seizure SEEG data.

3) Validation: Similar procedure as on EEG has been done (except downsampling and filtering).

a) Background SEEG: As before, for every particular signal of background SEEG a synthetic signal was generated using the extracted parameters.

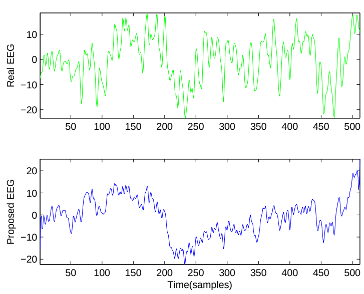


Fig. 9. Real and simulated background SEEG signals

TABLE III
CORRELATION (MEAN AND SD) FOR BACKGROUND SEEG

ρ	adult SEEG
ρ_t	0.587 (0.064)
ρ_f	0.582 (0.201)
ρ_{tf}	0.720 (0.049)

According to Table III, the simulated and real signals are moderately correlated (a higher value for the time-frequency correlation though). Still, as shown in Fig. 9, the modelling gives visually correct results when compared to real data.

b) Seizure SEEG: Same procedure was applied for seizure time windows, but the obtained signals show very low correlations results both in time and frequency domains. Again, this is probably due to the specific frequency content of epileptic seizures. An example is displayed in Fig. 10.

IV. CONCLUSION AND FUTURE RESEARCH

The goal of the research presented in this paper was to explore if an existing model of surface new-born background EEG [1] can be used for adult EEGs (background and seizure, surface and depth). According to our results, it seems that it is possible (although slightly less reliable) to generate an adult background EEG than a newborn EEG. Similarly, it is harder (but possible) to mimic background SEEG signals than surface EEGs. On the contrary, seizure EEG/SEEG signals cannot be reliably generated, probably due to the model assumption on the spectral behaviour ($1/f$).

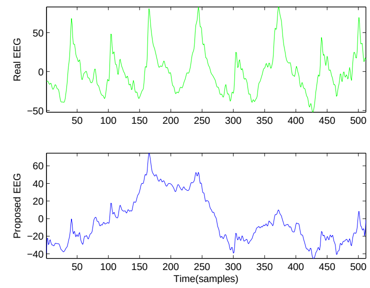


Fig. 10. Real and simulated seizure SEEG signals

A first immediate perspective is to confirm the presented findings on a larger database. It might be useful to introduce some categorisation in order to have more specific classes of EEG signals to work with (depending on the actual cerebral activity or on the recording site). Finally, it could be interesting to apply different models for the power spectrum estimation (besides $1/f$, clearly not appropriate for seizure data) and for the phase (not necessarily following a uniform distribution, as seen in the SEEG case).

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