# Reference estimation in EEG recordings

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**This version contains a slight improvement of the paper presented at EMBC'10, concerning the computing of the scaling factor (equation 15 in this version)**

**Abstract**

This work aims to analyze the reference (montage) problem in electroencephalographic (EEG) recordings. It is well accepted that EEG signals are a mixture of cerebral and extra-cerebral sources, and the solution to the reference problem depends on the hypothesized mixing model. We focus here on an acquisition model using a distant reference electrode and propose a method for determining and eliminating the reference signal which develops and improves Hu *et al.* work from [1]. The obtained solution, based on a constrained blind source separation (BSS) algorithm, outperforms the cited method on simulated noisy EEG signals for all noise levels.

## I. Introduction

An important issue for all for bio-potentials recording devices (EEG in particular) is to find a reference region in the human body with an electrical activity as neutral as possible. Indeed, the electrical activity at the reference affects measurements at all other active electrode sites [2], [3], [1]. As pointed out by all cited authors, it is impossible to find such a zero-potential reference, and all recording devices use the so-called Common Reference (CR) montage: *measuring electrodes* are referenced to a particular chosen *reference electrode*. Nevertheless, to ease the interpretation and to permit the use of some signal processing techniques like synchronicity measures (coherence and derived methods), several multiple combinations of these CR measures have been derived by some simple manipulations. The most common of these montages are the Average Reference (AR) and the Bipolar-Longitudinal (BL) montages. The AR montage is obtained in two steps: (a) compute an average signal from all measuring electrodes recorded signals and (b) subtract it from all these channels, while the BL montage is obtained by making the difference between two neighboring measuring electrodes (placed longitudinally on the skull according to some pre-defined system, 10-20 being the most well known).

Regardless of the employed montage, all measured EEG signals can be seen as a result of an unknown mixture of several unknown cortical sources, extra-cortical artefacts and noise. Blind Source Separation (BSS) can be used to separate these mixed measured signals in "independent" sources, which can be further-on used either for artefact elimination or for normal or pathological brain activity evaluation [4], [5], [6], [7]. Another application of BSS was proposed by Hu, Stead and Worrel [1], which introduce two methods for reference identification and removal based on blind source separation of the BL montage. Their methods are based on a strong hypothesis on the electrical activity propagation, which allow to write a constrained model for the EEG recordings. We briefly present in the second section of this paper the hypothesis and the methods introduced in the cited paper [1].

The main contribution of this letter is presented in the third section. Hypothesizing a mixture model similar to the one proposed by [1], we develop a more accurate BSS based method of reference identification and removal, which can be applied (with slight modifications) regardless of the employed montage (CR, AR of BL). The fourth section presents comparative results on simulated and real signals and discusses the limits of all BSS based approaches (ours included) and their possible further applications.

## II. Reference identification by BSS

The classical instantaneous linear mixing model writes:

$$\boldsymbol{x}(k) = \boldsymbol{A}\boldsymbol{s}(k) \tag{1}$$

where $\boldsymbol{x}$ is a vector of $M$ observed signals (EEG measuring electrodes), $\boldsymbol{A}$ is the unknown full-column rank mixing matrix ($M \times N$) and $\boldsymbol{s}$ is the vector of $N$ independent unknown sources (in the classical approach $M = N$, that is, we have the same number of sensors and sources).

In order to estimate the original sources, a reverse linear transformation $\boldsymbol{B}$ must be obtained such as:

$$\boldsymbol{y} = \boldsymbol{B}\boldsymbol{x} \tag{2}$$

with $\boldsymbol{y}$ a vector of $N$ estimated sources and $\boldsymbol{B}$ the $N \times M$ linear transformation that allows separating the mixed signals in their independent components. Theoretically, such transformation $\boldsymbol{B}$ should be the (left-) inverse of the mixing matrix $\boldsymbol{A}$. However, obtaining the exact inverse of the mixing matrix $\boldsymbol{A}$ is impossible (column permutation or multiplication by a constant is equivalent to source permutation or multiplication) [8]. Thus, source separation algorithms try to find a matrix $\boldsymbol{B}$ such as $\boldsymbol{G} = \boldsymbol{B}\boldsymbol{A}$ be a permuted and scaled diagonal matrix (one non-null value by line and column), which implies that the sources are recovered, except for their order and their amplitude.

This classical BSS model supposes ideally measured signals, *i.e.*, zero-referenced. As real electrophysiological recordings have a non-null common reference, Hu *et al.* [1] proposed a modified mixing model including the common reference signal, which can be written as:

$$\boldsymbol{x}_c = \begin{bmatrix} & -1 \\ \boldsymbol{A} & \vdots \\ & -1 \end{bmatrix} \begin{bmatrix} \boldsymbol{s} \\ r \end{bmatrix} = \boldsymbol{Q} \begin{bmatrix} \boldsymbol{s} \\ r \end{bmatrix}, \tag{3}$$

with $r$ being the common reference signal and $\boldsymbol{Q}$ the new mixing matrix $M \times N + 1$. Implicitly, although not clearly stated nor proven, this approach is based on the hypothesis that the non-zero reference signal is independent from all other sources and thus from ideally zero-referenced $\boldsymbol{x}$. In the recording setup from [1], the signals are acquired from intracranial electrodes (depth or subdural grids) referenced to a metallic plot placed on the head surface (vertex). The authors suppose that the intracranial electrical activity is independent from the one on the surface (that is, the signal $r$ can be considered as an independent source)[1].

From equation (3) it can be easily seen that the common referenced measures can be written as:

$$\boldsymbol{x}_c = \boldsymbol{x} - r[1 \dots 1]^T \tag{4}$$

In [1], the CR and the BL montages are used to develop two methods for reference signal identification. The bipolar montage BL can be obtained by making differences between pairs of signals from $\boldsymbol{x}_c$, respectively by subtracting the corresponding lines of the matrix $\boldsymbol{A}$. This subtraction obviously eliminates the influence of the reference signal $r$ from the BL montage.

We briefly present here the second more elaborated method. The basic idea is that by separating the reference-free BL montage, one will obtain the $\boldsymbol{y}_b$ sources as a linear mixture of all subjacent independent zero-referenced sources $\boldsymbol{s}$, although not well separated. Therefore, ideal zero-referenced signals $\boldsymbol{x}$ can be obtained by mixing $\boldsymbol{y}_b$ and thus the CR measures $\boldsymbol{x}_c$ also:

$$\boldsymbol{x}_c = \begin{bmatrix} & -1 \\ \overline{\boldsymbol{A}} & \vdots \\ & -1 \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_b \\ r \end{bmatrix} = \overline{\boldsymbol{Q}} \begin{bmatrix} \boldsymbol{y}_b \\ r \end{bmatrix}, \tag{5}$$

with $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{Q}}$ the mixing matrices allowing to obtain the measured signals from the sources $\boldsymbol{y}_b$ estimated from the BL montage separation (themselves a linear combination of the real sources $\boldsymbol{s}$) and the reference $r$. After some manipulations (see [1] for details) one can find the elements of the $\overline{\boldsymbol{A}}$ matrix:

$$\overline{a}_b(i,j) = \frac{\mathbb{E}[x_{c,i} y_{b,j}]}{\mathbb{E}[y_{b,j}^2]}, \quad i = 1 \dots N, j = 1 \dots P$$

with $x_{c,i}$ being the $i^{th}$ signal of the CR montage, $y_{b,j}$ the $j^{th}$ estimated source obtained after separating the BL montage and $\mathbb{E}[.]$ the mathematical expectation. Next, $N$ estimated versions of the reference signal $\widehat{r}_2$ are obtained as:

$$\widehat{r}_{2,i} = x_{c,i} - \sum_{l=1}^{P} \frac{\mathbb{E}[x_{c,i} y_{b,l}]}{\mathbb{E}[y_{b,l}^2]} y_{b,l}, i = 1 \dots N \tag{6}$$

or, in matrix form, $\boldsymbol{r}_2 = \boldsymbol{x}_c - \overline{\boldsymbol{A}}\boldsymbol{s}$. The final estimated reference as an averaged version of (6):

$$\widehat{r}_2 = \frac{1}{N} \sum_{i=1}^{N} \widehat{r}_{2,i} \tag{7}$$

## III. Proposed algorithm for reference identification

The previously described approach leads to convincing results, but it has two main drawbacks: it needs averaging of the different obtained references and, mainly, it depends on the employed BSS algorithm. We propose in this section a novel approach that addresses both problems.

[1] A different model, based on the assumption that the reference electrode itself records a mixture of sources, which could be more realistic for cephalic references in both intracranial and scalp recordings, is proposed in [9].

### A. Unscaled reference estimation

Assume the mixing model (3) with a full rank mixing matrix $\boldsymbol{Q}$. Classical BSS algorithms consist in two successive linear transforms: (1) a second order whitening and (2) an orthogonal transform (see for example [8] for details).

The first step of our algorithm also consists in whitening. The measured signals are multiplied by a matrix $\boldsymbol{W}$ $(M \times M)$ obtained from the eigen-factorization of the covariance matrix $\boldsymbol{R}_c = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^T$ of $\boldsymbol{x}_c$:

$$\boldsymbol{W} = \boldsymbol{D}^{-1/2}\boldsymbol{V}^T \tag{8}$$

to obtain a set of decorrelated signals of unit variance $\boldsymbol{z}$:

$$\boldsymbol{z} = \boldsymbol{W}\boldsymbol{x}_c = \boldsymbol{W}\boldsymbol{Q}[\boldsymbol{s} \;\; r]^T = \boldsymbol{T}[\boldsymbol{s} \;\; r]^T \tag{9}$$

In theory, source estimates could be found by inverting $\boldsymbol{T}$, but this is impossible, as it depends on the unknown $\boldsymbol{Q}$. In classical BSS, $M = N + 1$ so $\boldsymbol{T}$ is unitary, and the next step consists in finding source estimates $\boldsymbol{y} = \boldsymbol{U}\boldsymbol{z}$ by multiplying the whitened signals $\boldsymbol{z}$ by another unitary matrix $\boldsymbol{U}$ $(M \times M)$, computed to optimize some independence criterion.

When $M < N + 1$, the separation problem is underdetermined. Still, in theory, the minimum squared error solution would be given by the pseudo-inverse of $\boldsymbol{T}$, again impossible to compute because it depends on $\boldsymbol{Q}$. But as $\boldsymbol{T} = \boldsymbol{W}\boldsymbol{Q}$ is orthogonal ($M \times N + 1$, orthogonal rows), its pseudo-inverse is its transpose $\boldsymbol{T}^T$. Let $\boldsymbol{U} = \boldsymbol{T}^T$. Minimum squared error source estimates could be found as:

$$[\hat{\boldsymbol{s}} \;\; \hat{r}]^T = \boldsymbol{U}\boldsymbol{z} \tag{10}$$

As said previously, $\boldsymbol{U}$ cannot be found by transposing the unknown $\boldsymbol{T}$, but only the last row of $\boldsymbol{U}$ (noted $\boldsymbol{u}_{N+1}$) is needed to find an estimate of $r$. But $\boldsymbol{u}_{N+1}^T$ is the last column of $\boldsymbol{T}$:

$$\begin{aligned} \boldsymbol{T} = \boldsymbol{W}\boldsymbol{Q} &= \begin{bmatrix} & -1 \\ \boldsymbol{A} & \vdots \\ & -1 \end{bmatrix} \\ &= \begin{bmatrix} & -\sum_{j=1}^{M} w(1,j) \\ \boldsymbol{W}\boldsymbol{A} & \vdots \\ & -\sum_{j=1}^{M} w(M,j) \end{bmatrix} \end{aligned} \tag{11}$$

Consequently, the last of $\boldsymbol{U}$ can be obtained by summing the columns of the whitening matrix $\boldsymbol{W}$:

$$\boldsymbol{u}_{N+1} = [-1 \cdots -1]\boldsymbol{W}^T \tag{12}$$

so the reference estimation $\hat{r}$ can be found as:

$$\hat{r}_n^* = \boldsymbol{u}_{N+1}\boldsymbol{W}\boldsymbol{x}_c = [-1 \cdots -1]\boldsymbol{R}_c^{-1}\boldsymbol{x}_c \tag{13}$$

### B. Reference scaling

The previous development starts from the hypothesis that the sources $\boldsymbol{s}$, reference $r$ included, and the whitened signals $\boldsymbol{z}$ have equal unit variance. In general setups this is not necessarily the case, so the amplitude of the reference $\hat{r_n^*}$ (13) is not well estimated. A last step is then necessary to find the scaled reference $\hat{r}_n = \alpha\hat{r}_n^*$ and thus the right amplitude $\alpha$. The solution is given by the minimization of the mean squared error: $\min_{\alpha} \mathbb{E}[(\alpha\hat{r}_n^* - r)^2]$.

As the ideally zero-referenced mixture $\boldsymbol{x}$ does not depend on $\alpha$, the expression to be minimized can be rewritten as a function of any signal $x_{c,i},\ (i = 1 \ldots M)$ from the CR montage:

$$\begin{aligned} \mathbb{E}[(\alpha_i\hat{r}_n^* - r)^2] &= \mathbb{E}[(x_i - r + \alpha_i\hat{r}_n^*)^2] = \mathbb{E}[(x_{c,i} + \alpha_i\hat{r}_n^*)^2] \\ &= \mathbb{E}[x_{c,i}^2] + \alpha_i^2\mathbb{E}[\hat{r}_n^{*2}] + 2\alpha_i\mathbb{E}[x_{c,i}\hat{r}_n^*] \end{aligned} \tag{14}$$

|  | Noise power $\sigma_n^2$ | | |
|---|---|---|---|
|  | $0.5\sigma_r^2$ | $\sigma_r^2$ | $2\sigma_r^2$ |
| $SNR_{\hat{r}_2}$ | 7.19dB | 5.42dB | 3.06dB |
| $SNR_{\hat{r}_n}$ | 7.72dB | 5.92dB | 3.39dB |

TABLE I: Mean signal to error ratios for the two estimated references and three noise powers, averaged over 2500 simulations (50 mixing matrices $\times$ 50 noise sequences)

The minimization of (14) with respect to $\alpha_i$ leads for each $x_{c,i}$ to:

$$\begin{aligned}\alpha_i &= -\frac{\mathbb{E}[x_{c,i}\hat{r}_n^*]}{\mathbb{E}[\hat{r}_n^{*2}]} = -\frac{\mathbb{E}[x_{c,i}[-1\cdots-1]\boldsymbol{R}_c^{-1}\boldsymbol{x}_c]}{\mathbb{E}[\hat{r}_n^{*2}]} \\ &= \frac{[1\ldots1]\boldsymbol{R}_c^{-1}\mathbb{E}[x_{c,i}\boldsymbol{x}_c]}{\mathbb{E}[\hat{r}_n^{*2}]} = \frac{[1\ldots1]\boldsymbol{R}_c^{-1}\boldsymbol{R}_c(.,i)}{\mathbb{E}[\hat{r}_n^{*2}]} \\ &= \frac{[1\ldots1]\begin{bmatrix}0\\ \vdots\\ 1\\ \vdots\\ 0\end{bmatrix}}{\mathbb{E}[\hat{r}_n^{*2}]} = \frac{1}{\mathbb{E}[\hat{r}_n^{*2}]}\end{aligned} \quad (15)$$

independent of the channel number $i$.

To summarize, under the initial hypothesis that the reference $r$ is independent from all other sources $\boldsymbol{s}$ constituting the measured CR mixture $\boldsymbol{x}_c$ (eq. 3), an estimate of the reference can be found by the following procedure:

1) Whiten the measured CR montage $\boldsymbol{x}_c$
2) Compute $\boldsymbol{u}_{N+1}$ according to (12)
3) Compute the unscaled reference $\hat{r}_n^*$ from (13)
4) Scale $\hat{r}_n^*$ by (15) to find the final reference estimate $\hat{r}_n = \alpha\hat{r}_n^*$

## IV. Results and discussion

This section presents some simulation results comparing the newly proposed method to the second method from [1] (comparisons with the first method from [1] are not presented here, as the second method performs better, according to the authors). The sources $\boldsymbol{s}$ were created to simulate frequencies in real EEGs (physiological brain rhythms) and an eye-blinking artefact, the simulated reference $r$ being an ECG type signal (as for a scalp EEG recorded with a neck reference for example) (figure 1a). The ideal zero-reference mixture $\boldsymbol{x}$ was obtained using a random matrix (uniform distribution in [-1 1]). Starting from $\boldsymbol{x}$ and $r$, we obtained the common reference montage $\boldsymbol{x}_c$. Both $\boldsymbol{x}$ and $\boldsymbol{x}_c$ are presented figures 1b and 1c. As it can be seen, the mixture is underdetermined (7 sources, reference included, and 6 measures).

In order to test the robustness of our method to noisy inputs, we added zero-mean Gaussian to the CR mixture $\boldsymbol{x}_c$. Different noise variances were used: $\sigma_n^2 = 0.5\sigma_r^2, \sigma_r^2, 2\sigma_r^2$ (with $\sigma_r^2$ being the reference variance).

To have a reliable simulation, 50 random mixing matrices were used to obtain $\boldsymbol{x}$ and $\boldsymbol{x}_c$, and for each of them we generated 50 noise simulations for each noise power. According to our simulations, the noise power influences both estimated references $\hat{r}_2$ (7) and $\hat{r}_n$ (13), but for all cases, the newly introduced $\hat{r}_n$ is closer to the real reference $r$ than $\hat{r}_2$. Mean signal to error ratios (averages over 50 mixing matrices $\times$ 50 noise sequences) were computed according to:

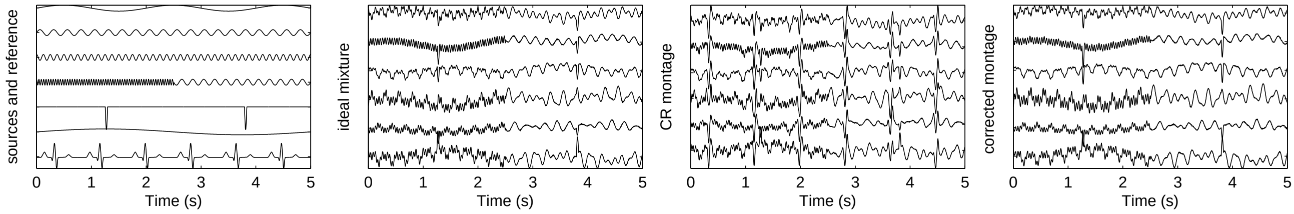
$$SNR_{\hat{r}} = \mathbb{E}[r^2]/\mathbb{E}[(r-\hat{r})^2]$$

both for $\hat{r}_2$ and $\hat{r}_n$, the results being given table I[2].

The main application of the estimated reference is the construction of a corrected montage, obtained as $\hat{\boldsymbol{x}} = \boldsymbol{x}_c + \hat{r}_n[1\ldots1]^T$. An example without noise, to ease the comparison, is shown figure 1d. This reconstruction can be used further on either for clinical interpretation or for automatic procedures based on synchrony measures among EEG channels (coherence and similar). Indeed, the presence of a common reference in the measures will increase this synchrony and may lead to erroneous conclusions [10], [11] (see figure 2 for an example).

It must be noted here that the above procedure (as the one described in [1]) starts from the hypothesis that the reference signal is independent from the other sources (*i.e.*, the reference electrode records only one source). This is generally the case for depth EEG recordings (first applications, nor presented here, are promising), but not for head referenced surface EEG,

[2]Moreover, other simulations performed with highly underdetermined mixtures ($M \ll N$, not shown here) indicate that our method significantly improves the reference estimation, compared to $\hat{r}_2$.




(a) Simulated sources $\boldsymbol{s}$ and reference $r$ (last row). (b) Zero-reference ideal montage $\boldsymbol{x}$ (c) CR montage $\boldsymbol{x}_c$ (d) Corrected montage $\hat{\boldsymbol{x}}$

Fig. 1: Simulation example

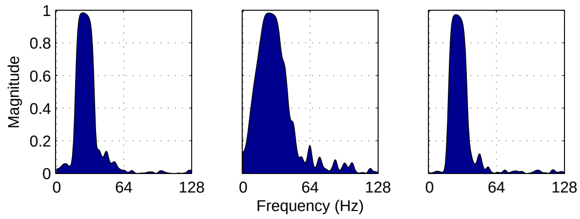



Fig. 2: Coherence between signals 1 and 5 for the ideal zero-referenced mixture $\boldsymbol{x}$ (left), CR montage $\boldsymbol{x}_c$ (center) and corrected montage $\hat{\boldsymbol{x}}$ (right).

when the reference is itself a mixture of sources (see [9] for details on this model). Applying these reference elimination methods when the initial hypothesis is not respected might lead to false results: if the real mixing matrix doesn't have a known column the estimated reference will be meaningless.

A first sight appealing application would be the improvement of cerebral source estimation by BSS: constraining the mixing matrix as in (1) leads to the estimation of one of the sources, namely $r$. Using $\hat{r}_n$ to correct the montage means the elimination of one of the sources from the mixture, so an easier separation. Unfortunately, this reasoning is false: the reference $r$ is itself estimated as a linear combination of the CR measures $\boldsymbol{x}_c$ (13) and, as long as the estimation is not *perfect*, the corrected montage $\hat{\boldsymbol{x}}$ will still be a linear mixture of all sources, $r$ included.

## V. Conclusion

Reference estimation and elimination in EEG recordings remains a challenging and useful problem because of the potential benefits in interpretation and automatic processing. The method introduced in this paper, derived from a BSS model (independent reference) improves previously proposed approaches by approximately 10% of SNR (in dB). The method is robust to noise and to mixing matrix characteristics.

Potential uses of the proposed reference elimination algorithm are mainly in synchrony analysis, but it can also be routinely employed to derive a new EEG montage, alternative to classical average reference or bipolar montages.

An interesting perspective is to estimate the error between the estimated reference and the real one. First results, not shown here, indicate that it depends (non-linearly) on the mixing characteristics (number of electrodes, mixing coefficients). A detailed analysis should indicate in which conditions the estimate is reliable and should be used.

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