Smoothness constraint for cortical dipolar sources estimation

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Abstract

In the last decade, a wide range of approaches have been proposed to estimate the activity of physiological sources from multi-channel electroencephalographic (EEG) data. Two utterly different directions can be distinguished: brain source imaging (BSI) and blind source separation (BSS). While the first approach is based on the inversion of a given forward model, the latter blindly decomposes the EEG mixing by optimization of a contrast function excluding any physiological *priors* on the problem. All these methods have proven their ability in reconstructing efficiently the source activities in some well adapted situations. Nevertheless, the synthesis of a reliable lead field model for BSI is computationally demanding, and the criterion to be optimized in BSS are often inadequate with regards to the physiology of the problem. In this paper, a compromise between these two methodological trends is introduced. A BSS method is described taking account of physiological knowledge on the projection of the sources on the scalp map in conjunction with strong *priors* on the localization of the recorded sources. This estimation method is demonstrated to lead to a generalization of the classical Hjorth's laplacian montage, and provides satisfactory simulation results when the appropriate configurations on the sources are met.

Index Terms

EEG, Source Separation, Smoothness Constraint, Laplacian Montage

I. INTRODUCTION

Brain activity recorded by the mean of EEG are commonly considered to be those of radially-arrayed cortical pyramid cells [1]. When a sufficient amount of cells within a small region (or 'patch') produces near-synchronous field activities, the resulting cortical field is recorded by the EEG scalp electrodes. These recorded signals can thus be seen as the linear sum of near-instantaneously volume conducted activities of equivalent current dipoles placed in the middle of compact cortical areas of activated cells.

Two main trends of approaches are to be distinguished when dealing with brain sources estimation. A first class of methods assume that a current dipole exists at each mesh of the discrete cortical surface, with known projection on the EEG electrodes [2], [3]. These Brain Source Imaging (BSI) methods, however being popular and widely used in clinical context, are based on patient-dependent propagation model yielding high computational cost, and are prone to errors due to inherent modelling imprecision. A second class of methods estimate the sources from the EEG measurements, at the exclusion of any *priors* on the mixing model. These Blind Source Separation (BSS) approaches are based on the definition of a contrast function to be optimized, generally a second or higher order independence measure between the sources [4]. These approaches are known to be efficient in separating sources of artefacts from the physiological mixing [**?**], and it has been recently demonstrated [5] that some of these methods indeed produce plausible sources, *i.e.* sources whose projections on the scalp are almost dipolar. However, the independence criterion on which these BSS methods are based makes them irrelevant when considering correlated brain sources, as it is often the case either in physiological or pathological configurations [6], [7], [8].

These considerations motivate a source estimation method less model-dependent and resource demanding than the BSI approaches, also more physiologically relevant than the BSS approaches. With this objective in mind, we conjecture that replacing the independence criterion in the blind separation procedure by a biologically plausible one will lead to more *physiological* source estimates. The assumptions made in this paper are the following: according to previous physical studies on the EEG recording setup [9], the activities of radially-arrayed cortical pyramid cells can be modelled by an equivalent dipole placed in the middle of this cortex area. If the size of the activated 'patch' is sufficient, this activity is recorded by the EEG sensor situated on the scalp area covering this 'patch'. As a first step toward our objective, we will assume that the cortical surface beneath each EEG sensor can be modelled as a radial dipole, mainly contributing to the corresponding EEG channel while having attenuated impact on the neighbouring electrodes. Given simple electromagnetic approximations, this attenuation can be estimated, resulting in a smoothness constraint on the matrix mixing the sources on the sensors.

The paper is organized as follows: section II details the assumptions made on EEG recordings and describes how the smoothness constraint is applied on the scalp map projection of the sources. Section III explains how the sources are estimated and shows how this method generalizes the classical Hjorth's laplacian montage [10]. Results on simulated data are provided in section IV, demonstrating the ability of the method in reconstructing cortical dipolar sources.

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II. Smoothness constraint

A. EEG setup and modeling

The EEG setup consists in electrodes placed at regular interval on the scalp, covering about half of the head volume, modelled as a sphere in this paper. Each sensor has 4 neighbours (3 if it is located on the edge). The number of electrodes depends on the clinical application and can vary from 8 up to 128 in the case of High Resolution EEG (EEG-HR). The neuronal generators are widely modelled as equivalent current dipoles whose activities $\mathbf{S}$ propagate instantaneously and linearly through the anatomical structures with given propagation coefficients $\mathbf{A}$, resulting in a EEG multichannel potential mixture $\mathbf{X}$:

$$\mathbf{X} = \mathbf{A}\mathbf{S} \tag{1}$$

Considering BSS methods, both $\mathbf{A}$ and $\mathbf{S}$ have to be blindly estimated from $\mathbf{X}$, requiring strong assumptions about source statistical second or higher order independence. In this paper we will take benefit on neurophysiological *priors* on the configuration of the sources and their projections structure on the scalp EEG: the activities of N equivalent current dipoles placed beneath and radially pointing toward each EEG sensor will be estimated, as illustrated on fig.1. A such particular configuration allows to impose a particular $N \times N$ mixing system $\mathbf{A}$, which should provide dipolar (or at least smooth) scalp map projections for each source.

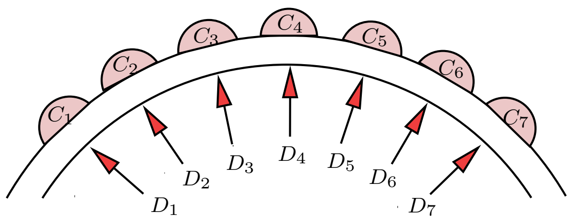

Fig. 1. Placement of dipoles with respect to the sensor locations. All dipoles are beneath the sensors at a given depth, and radial to the surface.

B. Definition of the smoothness criterion

Our main motivation is to develop a source estimation method that provides biologically plausible results, meaning that the projections of each estimated source on the scalp should have a dipolar pattern. This paper is a first effort in this direction, and the dipolar objective is here simplified to a smoothness constraint.

Like in the case of most BSS algorithms, and in particular in the case of Independent Component Analysis (ICA) the proposed method can be separated in two steps: an initialization step (*e.g.* decorrelation for ICA algorithms) followed by the application of a second criterion (independence for ICA, smoothness in our case). The role of the initialization step is to pre-condition the data by introducing priors in concordance with the final objective.

We will derive first the smoothness criterion, which constitutes the main contribution of this work. Considering the assumptions made in section II-A and following simple electromagnetic propagation laws, a projection rule of the dipolar sources on the sensors placed on the scalp is defined. This leads to a particular type of mixing matrix $\mathbf{A}$ and thus to a particular separation matrix imposing the smoothness constraint, defined as described further.

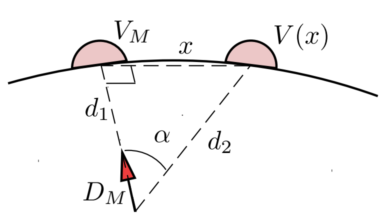

Fig. 2. Geometrical approximation of the dipole-electrodes configuration. V_M is the potential recorded by the electrode pointed by the (radially-oriented) dipole D_M, while $V(x)$ is the potential at a distance x from this maximum potential point.

Our basic assumption is that the observed potential V_M generated by a dipole D_M on its relative sensor decreases with the square of the distance between them:

$$V_M = \frac{j}{K \cdot d_1^2} \tag{2}$$

where j is the amplitude of the current dipole D_M, K is a propagation coefficient and d_1 the depth of the source (dipole). As the sources are assumed to be dipolar and pointing toward to each of the sensors, and considering rather small distances between neighbouring sensors, the geometrical approximation of the fig. 2 is proposed: the neighbouring sensors with respect to V_M are assumed to be distributed on a plane orthogonal to the direction of the dipole. This allows to estimate the distance d_2 between the dipole and a neighbouring sensor. Indeed, given the depth d_1 of this source (*i.e.* the distance to its toward-pointing sensor) and the known sensor-interval distance x, we can write $d_2^2 = x^2 + d_1^2$. Then equation (2) can be expanded to compute any potential value produced by D_M on this plane:

$$V(x) = V_M \frac{d_1^2 \cos\alpha}{d_2^2} \tag{3}$$

Noticing that $\cos\alpha = \frac{d_1}{d_2}$, equation (3) can be written as:

$$V(x) = \frac{V_M}{\left(\frac{x^2}{d_1^2}+1\right)^{\frac{3}{2}}} \tag{4}$$

The values of $V(x)$ computed at the positions of the electrodes constitute the columns of the target mixing matrix. In this way, we insure that the scalp potentials generated by a single dipolar source follows the spatial pattern given by the equation (4).

An important parameter that determines the shape of $V(x)$, and thus the projection (mixing) matrix, is the source depth parameter d_1 (see fig. 3). If this depth is assumed to be known and fixed, the proposed source separation approach turns into a *semi-blind* or *informed* source separation method.

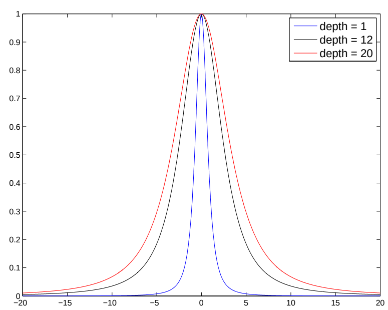

Fig. 3. Potential variation on the scalp as a function of the distance x to the electrode pointed by the dipole and for three different dipole depths d_1.

III. Smooth source estimation

A. Initialization step and smoothing transform construction

Let consider $\mathbf{W}$ as a given initial projections matrix. This matrix could be for example a whitening/sphering matrix, if we aim to initialize the smoothing with decorrelated source estimates. Another possible choice would be a matrix containing some prior physiological information (for example coding a particular grouping of the sources on the cortex).

Given an initialization $\mathbf{W}$, the observations $\mathbf{X}$ are related to some sources $\mathbf{Z}$ by:

$$\begin{aligned} \mathbf{X} &= \mathbf{WZ} \\ \mathbf{Z} &= \mathbf{W}^{-1}\mathbf{X} \end{aligned} \tag{5}$$

where $\mathbf{Z}$ is an initial source estimation, not necessarily dipolar (*i.e.*, before applying the smoothness constraint). In other words, the projection on the scalp of a source $\mathbf{z}$ (*i.e.*, the corresponding column of $\mathbf{W}$) do not always corresponds to the smooth pattern of a dipolar source. From this initialization point, we need to derive a new matrix $\mathbf{L}_W$ which columns provides such smooth projections. The procedure is following: the absolute maximum value of each column $1 \le i \le N$ of $\mathbf{W}$ is taken as maximum potential value V_M, corresponding to the electrode e_M on which the dipolar source D_i is maximally projected. The distance x from each electrode e_j to e_M is known, and using equation (4) the relative potential with respect to our smoothness constraint can be computed, resulting in a target mixing matrix $\mathbf{L}_W$. We will then look for the transform $\mathbf{B}$ that maps each column i of $\mathbf{W}$ as $V(x)$ (*i.e.*, as a column of $\mathbf{L}_W$). In matrix form:

$$\mathbf{L}_W = \mathbf{WB} \tag{6}$$

Consequently, the choice of the initial matrix $\mathbf{W}$ is sensitive as it will affect the construction of the associated objective matrix $\mathbf{L}_W$.

B. *Smoothness constraint application*

Using equation (1) and the objective mixing matrix $\mathbf{L}_W$, the estimation of the source activities is straightforward. By imposing our target mixing matrix $\mathbf{L}_W$, we will transform the initial sources $\mathbf{Z}$ into dipolar sources $\mathbf{S}_d$ having each projection smooth on the scalp map. More precisely, $\mathbf{X}$ from (5) can be written using transform $\mathbf{B}$ (6) as:

$$\mathbf{X} = \mathbf{W}\mathbf{B}\mathbf{B}^{-1}\mathbf{Z} \tag{7}$$

The operator $\mathbf{B}$ emphasizes the effect of the smoothing operator on the sources $\mathbf{Z}$:

$$\mathbf{S}_d = \mathbf{B}^{-1}\mathbf{Z} \tag{8}$$

Compactly, $\mathbf{X}$ is then written as the smooth projection $\mathbf{L}_W$ of these estimated dipolar sources $\mathbf{S}_d$:

$$\mathbf{X} = \mathbf{L}_W\mathbf{S}_d \tag{9}$$

In the present work, we do not assume any *priors* on the sources to be retrieved. The method is then initialized using the identity matrix $\mathbf{W} = \mathbf{I}$, corresponding to our assumption that a source per sensor is to be estimated. Then the initial sources $\mathbf{Z}$ are taken as original measurements $\mathbf{X}$. After applying the smoothness constraint $\mathbf{L}_I$, we obtain the following estimation of the sources:

$$\mathbf{S}_I = \mathbf{L}_I^{-1}\mathbf{X} \tag{10}$$

Before presenting the results, we first discuss how our method relates to the Hjorth's laplacian montage [10] when using the identity as initialization matrix.

C. *Analogy with the Hjorth's laplacian montage*

The classical Hjorth's laplacian montage described in [10] consists in subtracting from each electrode potential the mean of its neighbouring electrode potentials. The founding assumption underlying this study is similar than those made in the present paper: each EEG sensor measurement is mostly impacted by the local neuronal activities on the cortex. Thus the motivation underlying this simple operation is to identify the corresponding activity underlying each sensor. The method decreases the correlation between the reconstructed measurements because of the elimination of common propagated signals.

By imposing a plausible source projections pattern on the scalp surface, our method acts very similarly to the Hjorth's laplacian montage. A close look at the inverse of the matrix $\mathbf{L}_I$ reveals that indeed from each channel $\mathbf{X}_i$ is subtracted a weighted mean of the neighbouring electrodes. The further an electrode is from the considered electrode e_i, the lower its weight in the mean to be subtracted. The proposed method can then be seen as a generalization of the Hjorth's laplacian montage: unlike for the classical laplacian, the weights are not fixed in our method. These weights depend on the source depth, which can be given by anatomical *priors*, *e.g.* if accurate medical imaging modalities are available.

IV. RESULTS

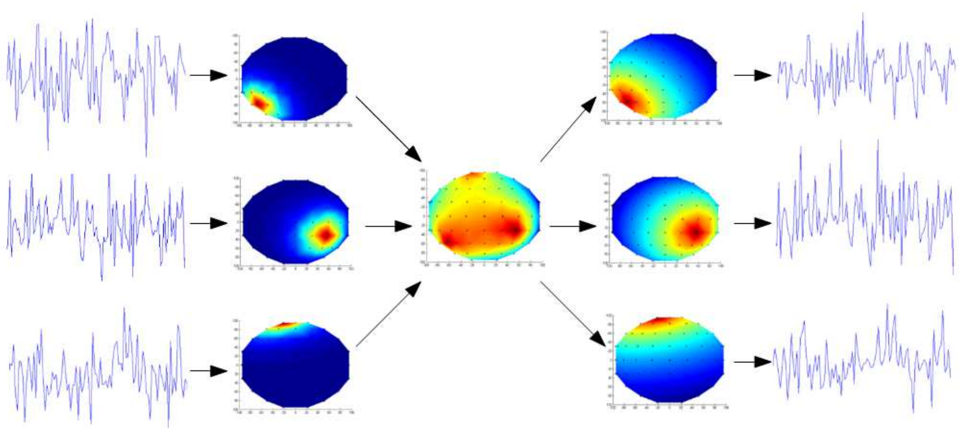

Fig. 4. Example of original sources, their obtained mixture and the estimated scalp projections and sources for a dipole depth of 12 mm (informed).

In order to validate the proposed source estimation approach we simulate correlated random sources (generalized Gaussian distribution), as well as plausible sources produced by a neural population model [6]. These simulated brain sources $\mathbf{S}$ are mixed (projected on the electrodes) using a 3 layers lead-field matrix $\mathbf{A}$ (Rush & Driscoll model [11]) generated for the given configuration of electrodes and dipoles (one dipole beneath each electrode). We assumed a 64 channels EEG. An example of sources, their individual scalp projections (*i.e.*, the columns of $\mathbf{A}$) and the resulting scalp map of the mixture $\mathbf{X}$ is given on the left side of the figure 4. The right side of the same figure displays the estimated sources and their projections using our method.

Table I presents mean correlation coefficients between the original sources $\mathbf{S}$ and the estimated ones $\hat{\mathbf{S}}$ (to avoid edge effects, the sources corresponding to the border electrodes were excluded from the comparison). For this particular simulation setup, corresponding both to the hypothesis of the Hjorth's laplacian montage and to its generalization proposed in this paper, both methods show similar performances when the dipoles are placed on the cortex surface. The advantage of our generalized method becomes compelling for more profound or superficial sources (see table I), provided that the depths of the sources are correctly informed.

TABLE I

MEAN CORRELATION COEFFICIENTS (1000 REALIZATIONS). THE DEPTH OF THE DIPOLES IS GIVEN WITH RESPECT TO THE STANDARD RADIUS OF THE INNER SPHERE IN THE RUSH & DRISCOLL MODEL (80 MM)

Dipole depth	Generalized laplacian	Hjorth's laplacian
-5 mm	0.93	0.87
0 mm	0.85	0.85
5 mm	0.80	0.76
10 mm	0.77	0.63
20 mm	0.69	0.35

V. CONCLUSIONS AND FUTURE RESEARCH

Our method shows consistent performance on simulated data corresponding to the working hypothesis. The results are superior to those obtained using the Hjorth's laplacian montage for superficial and profound cortical sources. The proposed method generalizes the laplacian montage because the decreasing of the scalp potential can be parametrized if the depth distance between the cortical surface and the scalp is known. Moreover, it may generalize further the laplacian montage as it provides the possibility to estimate the weights associated to each channel through the estimation of each source depth. This estimation can be either obtained from the data or by imaging modalities. It has to be emphasized that the method proposed in this paper stands as an introduction to physiologically plausible semi-blind source separation methods.

Possible short term developments will aim at studying the effect of other initializations, and in particular using a sphering procedure, as this pre-whitening step is known for its capacity to provide good estimation of cortical and radially oriented dipolar components [5]. Also, the influence of additive noise or perturbations (EMG, ECG) on the measurements will be evaluated.

Next, the *priors* on the source positions have to be relaxed (each dipole could be assumed to be localized anywhere on the cortical surface, not strictly beneath the electrodes). Furthermore, the simple smoothness constraint proposed in this work has also to be improved for a more plausible and accurate dipolar scalpmap. From an applicative point of view, we expect that such methods will provide accurate quantification of the correlations between the reconstructed cortical dipolar sources, with the objective to identify brain functional networks.

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