Static Output Feedback Control for switched systems

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Abstract

In this paper we investigate the design of a static output feedback control for switching discrete time systems. Looking at the problem of stability analysis, we give two conditions allowing to check the existence of a particular class of Lyapunov functions. Although one can recover the first condition as a particular case from the second one, it is proved that the two conditions are equivalent for stability analysis. Then we show that when one is faced with a static output feedback control problem, the second condition leads to a sufficient solution to build a stabilizing static output feedback controller which is less conservative than the solution based on the first condition.

1 Introduction

We consider the class of switched hybrid systems given by

$$\begin{array}{rcl} x_{k+1} & = & A_{\alpha(k)}x_k + B_{\alpha(k)}u_k \\ y_k & = & C_{\alpha(k)}x_k \end{array} \qquad (1)$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^m$ is the control input and $y_k \in \mathbb{R}^p$ is the output vector. $\alpha(k)$ is a switching rule defined by $\alpha(k) : \mathbb{N} \to \mathcal{I}$ with $\mathcal{I} = \{1, ..., N\}$. Such systems are said to be switched and belong to the class of hybrid systems. We assume that the matrix $C_{\alpha(k)}$ is of full row-rank and that the switching rule $\alpha(k)$ in not known a priori but its instantaneous value is available in real time. Here, we are interested in the design of output feedback control

$$u_k = K_{\alpha(k)}y_k, \qquad (2)$$

ensuring stability of the closed loop switching system:

$$x_{k+1} = \big(A_{\alpha(k)} + B_{\alpha(k)}K_{\alpha(k)}C_{\alpha(k)}\big)x_k \qquad (3)$$

Due to space limitations, the proofs of all the proposed theorems have been dropped and can be found in [1].

2 Stability analysis

In this section, we investigate the stability of the origin of the autonomous switched system under arbitrary switching laws using Lyapunov theory. To this end, define the indicator functions $\xi_i(k)$, for all $i = 1, \dots, N$, as:

$$\xi_i(k) = \begin{cases} 1 & \text{when the switched system is described by the } i\text{-th mode } A_i \\ 0 & \text{otherwise} \end{cases}$$

$$\xi(k) = \begin{bmatrix} \xi_1(k), & ..., & \xi_N(k) \end{bmatrix}^T$$

Then the autonomous switched system can be written as

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k)A_i x_k \qquad (4)$$

Results in [2] use parameter dependent Lyapunov functions to check stability of polytopic time varying systems. The application to systems as (4) leads to Lyapunov function as

$$V(k, x_k) = x_k^T P(\xi(k))x_k = x_k^T\Big(\sum_{i=1}^{N} \xi_i(k)P_i\Big)x_k \qquad (5)$$

with $P_1, ...P_N$ symmetric positive definite matrices. If such a positive definite Lyapunov function $V(k, x_k)$ exists and is such that $\Delta V(k, x_k) = V(k+1, x_{k+1}) - V(k, x_k)$ is negative definite along the solutions of (4), then the origin of the system given by (4) is globally asymptotically stable.

Theorem 1 *[1] The following statements are equivalent*

i) There exist a Lyapunov function of the form (5) whose difference is negative definite proving asymptotic stability of the system (4).

ii) There exist N symmetric matrices $P_1, ..., P_N$ satisfying

$$\begin{bmatrix} P_i & A_i^T P_j \\ P_j A_i & P_j \end{bmatrix} > \mathbf{0} \qquad \forall (i, j) \in \mathcal{I} \times \mathcal{I} \qquad (6)$$

The Lyapunov function is then given by: $V(k, x_k) = x_k^T\big(\sum_{i=1}^{N} \xi_i(k)P_i\big)x_k$.

iii) There exist N symmetric matrices $S_1, ..., S_N$ and N matrices $G_1, ..., G_N$ satisfying

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T A_i^T \\ A_i G_i & S_j \end{bmatrix} > \mathbf{0} \qquad \forall (i, j) \in \mathcal{I} \times \mathcal{I} \qquad (7)$$

The Lyapunov function is then given by: $V(k, x_k) = x_k^T\big(\sum_{i=1}^{N} \xi_i(k)P_i\big)x_k$ with $P_i = S_i^{-1}$.

Condition (6) has also been proposed in [3] to check stability of piecewise affine linear systems. Notice that we can recover condition ii) by imposing $G_i = S_i$ and letting $S_i^{-1} = P_i$ in condition iii). However, Theorem 1 shows that both conditions are equivalent and then present the same level of conservatism. As we will see in the next sections, the contribution of Theorem 1, in addition to prove the equivalence i)$\Longleftrightarrow$ ii)$\Longleftrightarrow$ iii), is to propose an alternative condition which will be less conservative when output feedback control synthesis problem is considered.

3 Static output feedback design

The synthesis of a switched static output feedback can be reduced, using Theorem 1, to find P_i and K_i ($\forall i \in \mathcal{I}$) such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$

$$\begin{bmatrix} P_i & (A_i + B_i K_i C_i)^T P_j \\ P_j (A_i + B_i K_i C_i) & P_j \end{bmatrix} > \mathbf{0} \qquad (8)$$

or equivalently find S_i, G_i and K_i ($\forall i \in \mathcal{I}$) such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T (A_i + B_i K_i C_i)^T \\ (A_i + B_i K_i C_i) G_i & S_j \end{bmatrix} > \mathbf{0} \qquad (9)$$

The problem of solving numerically (8) or (9) for (P_i, K_i) or (S_i, G_i, K_i) respectively is non-convex in general. This makes the problem of output feedback a very difficult one. A sufficient condition is given in the following Theorems. These conditions have the advantage to be convex and then can be tested numerically with efficient LMI algorithms.

Theorem 2 *[1] If there exist symmetric matrices S_i, matrices U_i and V_i ($\forall i \in \mathcal{I}$) such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$*

$$\begin{pmatrix} S_i & (A_i S_i + B_i U_i C_i)^T \\ A_i S_i + B_i U_i C_i & S_j \end{pmatrix} > \mathbf{0} \qquad (10)$$

and

$$V_i C_i = C_i S_i \qquad \forall i \in \mathcal{I} \qquad (11)$$

then the output feedback given by (2) with

$$K_i = U_i V_i^{-1} \qquad \forall i \in \mathcal{I} \qquad (12)$$

stabilizes the system (1).

Theorem 3 *[1] If there exist symmetric matrices S_i, matrices G_i, U_i and V_i ($\forall i \in \mathcal{I}$) such that $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$*

$$\begin{pmatrix} G_i + G_i^T - S_i & (A_i G_i + B_i U_i C_i)^T \\ A_i G_i + B_i U_i C_i & S_j \end{pmatrix} > \mathbf{0} \qquad (13)$$

and

$$V_i C_i = C_i G_i \qquad \forall i \in \mathcal{I} \qquad (14)$$

then the output feedback control given by (2) with

$$K_i = U_i V_i^{-1} \qquad \forall i \in \mathcal{I} \qquad (15)$$

stabilizes the system (1).

The above Theorems deserve some comments. In Theorem 2, the Lyapunov matrices P_i ($\forall i \in \mathcal{I}$) are constrained to satisfy, in addition to stability condition, the equality constraint (11). In Theorem 3, the Lyapunov matrices $P_i = S_i^{-1}$ ($\forall i \in \mathcal{I}$) are let free and have to satisfy the stability condition only. The equality constraint (14) has to be satisfied by the slack variables G_i ($\forall i \in \mathcal{I}$). This means that the equivalence between conditions given in Theorem 2 and Theorem 3 fails. As it will be shown by the numerical evaluation given at the end of the paper, reporting the constraint (14) on the slack variables G_i makes the conditions of Theorem (3) less conservative than those given in Theorem 2.

4 Numerical evaluation

Recall that our switched system is characterized by: the number of modes (N), the system order (n), the number of inputs (m) and number of outputs (p). For fixed values of (N, n, m, p), we generate randomly 100 switched systems of the form (1). For each switched system we try to compute a stabilizing output feedback control using the constant Lyapunov Function (CLF) conditions, the conditions of Theorem 2 and the conditions of Theorem 3. Due to space limitations, we are not able to include all the performed tests in this paper. The full results are given in [1]. However, the following table gives some tests showing the improvement in the number of success in providing an output feedback stabilizing control allowed by the condition proposed here.

Dim.	Succ.	$N=2$	3	4	5	6
$n=3$	CLF	93	74	60	62	47
$m=2$	$Th2$	100	97	99	97	92
$p=2$	$Th3$	**100**	**98**	**100**	**98**	**95**
$n=3$	CLF	78	46	42	16	17
$m=1$	$Th2$	98	87	93	80	70
$p=2$	$Th3$	**100**	**95**	**98**	**94**	**88**
$n=3$	CLF	77	56	34	26	18
$m=2$	$Th2$	96	95	91	82	83
$p=1$	$Th3$	**96**	**96**	**93**	**90**	**90**
$n=4$	CLF	27	10	0	0	0
$m=1$	$Th2$	79	50	28	26	8
$p=2$	$Th3$	**97**	**83**	**64**	**48**	**34**
$n=4$	CLF	47	8	4	2	0
$m=2$	$Th2$	89	71	47	28	16
$p=1$	$Th3$	**95**	**78**	**65**	**46**	**32**
$n=5$	CLF	8	1	0	0	0
$m=2$	$Th2$	60	38	23	4	4
$p=2$	$Th3$	**85**	**69**	**42**	**26**	**19**

5 Conclusion

In this paper, static output feedback for switching systems has been investigated. Motivated by stability analysis results developed for polytopic time varying uncertain systems, a condition have been proposed to build a switching static output feedback control. As the proposed conditions are LMI based conditions, they can be easily extended to other control problems (H_2, H_∞ ...).

References

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