On the observability of the flying capacitor converter

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***Abstract*—This paper address the continuous state estimation of a class of switched systems having modes in which the state is not or partially observable. Using a relevant LaSalle principle for switched systems, a characterization of invariant sets and associated control laws for which the state cannot be estimated, is given. This result is applied on a flying capacitor converter to prove convergency of the estimation error under suitable control laws. This work is also used to explain why rate of the estimate error decreases with high switching frequency scheme.**

***Index Terms*—Observer design, invariance principle, power converter**

I. Introduction

In the case of industrial applications with power of a few megawatts, the switching components voltage becomes very high (several kilovolts). Therefore, the switching frequency must be maintained to a low value and bulky filters are needed for obtaining an appropriated output [1]. To palliate this drawback, a new class of power electronic converters has appeared, called multicell converters. These structures consist of a series connection of switching devices with passive storage elements, which are used to generate intermediate voltage levels [2]. The control law for these structures needs to maintain the intermediate voltage levels at some constant values and to regulate the voltage or the load current. The main advantages of the multilevel converter is that the spectral quality of the output signal is improved by a high switching frequency between the intermediate voltage levels [3]. The downside is that, excepting simpler DC-DC converters, the control of multilevel converters is more complex [4], [5].

Emerging control techniques such as stabilizing control [6], predictive control [7] [8], flat control [9] are based on observer technics. Moreover, the robustness of the proposed schemes is also improved and ensured with parameters estimations. Thus, there are a demanding for state or parameter estimations to improve algorithms and to reduce the number of sensors. The recursive filter such as Extended or Unscented Kalman Filter are efficients and the most frequently used techniques [10], [11]. But proof of the convergency are generally omitted or intractable due to nonlinearity of the control system and the dependency of the observability property w.r.t. the input [12]. Promising approaches for hybrid systems may be linking to algebraic observability [13].

In this paper, for the class of affine switched systems, a switched observer is proposed as state estimator and applied on a flying capacitor converter. The originality of the proposed approach yields to a complete characterization of invariant sets and associated control laws for which the estimator doesn't converge. This characterization do not depend on of the chosen switched observer and is intrinsically linked to the structure of the system.

In the next section, the problem statement is given and it is shown that that the state estimation of the flying capacitor relies in the stability of a linear switched systems having non positive definite matrices. The section III is devoted to the characterization of invariant set and associated control laws. A discuss is introduced at the beginning of section IV on how to determine the invariant set. Then an algorithm is proposed to solve this problem. Switched observer design with pole placement is illustrated in section V. Then the proposed characterization is used to explain why the rate of convergency of the observer is decreasing with the switching frequency.

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II. Problem statement

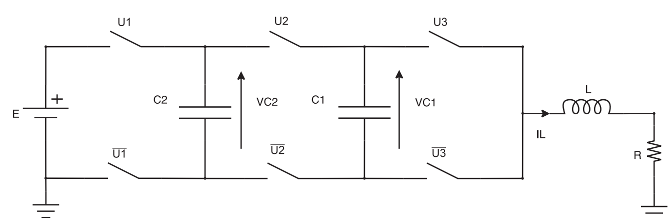

Fig. 1. Multilevel converter. The state variables are the capacitor voltages and the current load. The control is the switch position of U_1, U_2 and U_3.

The state equations of the converter have an affine form given by

$$\dot{x}(t) = A_0 x(t) + \textstyle\sum_{i=1}^{3} u_i(t)(A_i x(t) + B_i) \tag{1}$$

with a three dimensional state $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$, a three dimensional boolean control vector $u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \in \{0, 1\}^3$, the matrices A_i, $i = 0, 1, 2, 3$, defined by

$$A_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}, \; A_1 = \begin{bmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & 0 \\ \frac{1}{L} & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & -\frac{1}{C_2} \\ -\frac{1}{L} & \frac{1}{L} & 0 \end{bmatrix}, \; A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix},$$

and $B_1 = B_2 = 0$, $B_3 = [0, 0, E/L]^T$. The states x_1, x_2 are the voltages in each capacitor and x_3 is the load current.

The control u refers to the switches position and we count 2^3 modes following the values of u.

The aim is to estimate the voltage of the capacitors in the case where only the current in the load is measured. The output y is defined by the equation $y = Cx$ with $C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$.

Notice that the state components are only partially observable for every fixed configuration of the switches. For example, if the switch value is $u = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, then the voltage in the capacitor C_1 cannot be estimated since it is disconnected or if the switch value is $u = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ then only the sum of the voltages in capacitors C_1 and C_2 can be estimated. Thus, it is not possible to consider an observer for arbitrary switching law. A question of interest is then how to characterize the switching law for which it is impossible to observe the state?

Here such characterization will be illustrated in the case of a Luenberger switched observer. The purpose is to study the behavior of a switched observer of the form:

$$\dot{\hat{x}} = A_0\hat{x} + L_0(y - \hat{y}) + \textstyle\sum_{i=1}^{3} u_i(A_i\hat{x} + B_i + L_i(y - \hat{y})).$$

Defining the estimation error $e = x - \hat{x}$, its dynamics are given by:

$$\dot{\hat{e}} = \tilde{A}_0 e + \textstyle\sum_{i=1}^{3} u_i \tilde{A}_i e, \tag{2}$$

with $\tilde{A}_i = A_i - L_i C$, $i = 0, 1, 2, 3$.

In view of equation (2), the problem statement consists in studying the stability of a linear switched system

$$\dot{x} = f(t, x) = A_{\sigma(t)} x, \ \ \sigma(t) \in \{1, \ldots, I\}, \tag{3}$$

where the state $x(t) \in \mathbb{R}^n$, A_i, $i \in \{1, \ldots, I\}$ are non positive definite matrices ($A_i \leq 0$) and the switching law $\sigma(\cdot)$ is a measurable function of time t defined on $[0, +\infty)$.

III. Characterization of invariant sets

As it is well known in the continuous time case [14], a sufficient condition guaranteeing the global asymptotic stability of the continuous-time switched systems (3) is the existence of a common Lyapunov function. When we consider *weak* common Lyapunov functions, which differ from standard Lyapunov ones in having just *nonpositive* derivative along system trajectories, global asymptotic stability properties can still be guaranteed using additional conditions such as, for instance, a dwell time assumption on the switchings [15], [16], [17], [18]. In this paper, no such assumptions are used and it is only supposed that the switching law is a Lebesgue measurable function. An interesting question is to characterize the ω-limit sets and its associated controls in presence of a *weak* common Lyapunov functions.

Assume that there exists a symmetric positive definite matrix P ($P > 0, P^T = P$) such that for all $x \in \mathbb{R}^n$,

$$x^T P x > 0$$
$$x^T (A_i^T P + P A_i) x \leq 0, \forall i \in \{1, \ldots, I\}.$$

The function

$$V(x) = x^T P x$$

is a weak common Lyapunov function. What could be said about the asymptotic behavior of the trajectories? An answer is given in the following theorem.

Theorem 1: Let $x_0 \in \mathbb{R}^n$ and $\sigma(\cdot)$ be a switching law defined on $[0, +\infty)$. Let x be the solution of the switched system corresponding to $\sigma(\cdot)$ and with initial condition x_0. Then the ω-limit set $\Omega(x)$ is included in a level set of V and for all $y_0 \in \Omega(x)$, there exists an absolutely continuous function y which satisfies

$$\begin{aligned} \dot{y} &= A_\alpha y \quad \text{for a.e. } t \geq 0 \\ y(0) &= y_0 \\ y(t) &\in \Omega(x) \quad \text{for every } t \geq 0, \end{aligned}$$

and where the time variant matrix $A_\alpha = \sum_{i=1}^{I} \alpha_i A_i x$ evolves following a control law $\alpha(t) = (\alpha_1(t), \ldots, \alpha_I(t))$ taking its values in the set

$$\Delta = \left\{ \alpha = (\alpha_1, \ldots, \alpha_I) \mid \textstyle\sum_{i=1}^{I} \alpha_i = 1, \alpha_i \geq 0, i = 1, \ldots, I \right\}.$$

In particular,

$$\dot{V}(y(t)) = 0$$

for every $t \geq 0$, where, with a standard abuse of notations, $\dot{V}(y(t))$ denotes the derivative of $t \mapsto V(y(t))$.

Roughly speaking, the $\Omega(x)$ limit set is made with Fillipov solution of the switched systems included in a level set and given by the differential inclusion:

$$\dot{x} \in F(x) \tag{4}$$

where the set-valued map $x \mapsto F(x) = \sum_{i=1}^{I} \alpha_i A_i x \mid \alpha \in \Delta\} \subset \mathbb{R}^n$.

Define, for every $x \in \mathbb{R}^n$ and every $i = 1, \ldots, N$ the directional derivative of V in the direction $A_i x$ as $\dot{V}_i(x) = x^T(A_i^T P + P A_i)x \leq 0$. Therefore, for every $v \in F(x)$ the directional derivative of V in the direction v is given by

$$\begin{aligned} \dot{V}(x; v) &= x^T (A_\alpha^T P + P A_\alpha) x \\ &= \textstyle\sum_{i=1}^{I} \alpha_i (x^T (A_i^T P + P A_i) x) \\ &= \textstyle\sum_{i=1}^{I} \alpha_i \dot{V}_i(x) \leq 0 \end{aligned}$$

since the $\alpha_i' s$ are positive.

Then, the directional derivative of V along any direction in the set $F(x)$ is nonpositive. Then, the solutions of (4) included into a level set of V satisfy the following control problem:

Problem 2: Find the controls $\alpha^* \in L^\infty([0, +\infty), \Delta)$ and the associated trajectories x satisfying

$$\dot{x} = A_{\alpha^*} x \ a.e \tag{5}$$
$$\text{subject to } \textstyle\sum_{i=1}^{I} \alpha_i^* \dot{V}_i(x) = 0.$$

By homogeneity, solving Problem 2 amounts to finding the trajectories of (4) that are included in a given level set of V and the associated control functions. Notice that if α^* and x satisfy Problem 2, then

$$\alpha^* \in \arg\max_{\alpha \in \Delta} \alpha_i \dot{V}_i(x).$$

Denote by Inv the union of all supports of trajectories x corresponding to a solution of Problem 2 and by Ω_{Inv} the union of all theirs ω-limit sets. It follows from Theorem 1 that

Inv contains both Ω_{Inv} and the set Ω, defined as the union of all ω-limit sets associated to (3).

Therefore, Problem 2 may be useful to characterize the ω-limit set and the associated control for the switched system (3). An important question is: "What is the relative size of Ω with respect to Inv and Ω_{Inv}?" The answer can be obtained using the following density result.

Theorem 3: (See [19, Theorem 1].) The Carathéodory solutions of (3) are dense among the solutions of $\dot{x} \in F(x)$ in the following sense: Let x a global solution of (4) starting from x_0 and let $\varepsilon : [0, +\infty) \to (0, +\infty)$ be continuous. Then there exists a solution ξ of (3) starting from $\xi_0 \in B(x_0, \varepsilon(0))$ such that $\|\xi(t) - x(t)\| < \varepsilon(t)$ for all $t \in [0, +\infty)$.

In other words, all global solution of (4) could be approached arbitrarily close by global solutions of (3).

In view of this density theorem the ω-limit set of any trajectory of (4) is the ω-limit set of a trajectory of (3). Therefore, $\Omega_{\mathrm{Inv}} \subset \Omega \subset \mathrm{Inv}$.

Assume now that we get a trajectory solving Problem 2 which is not entirely an ω-limit set. Since this solution is included in a level set of V, it is clear, that its ω-limit set remains in the same level set.

IV. Algorithm to compute invariant set

The simplest situation occurs when on a time interval (a, b), $a < b$, there exists one index i such that

$$\dot{V}_i(x) = x^T C_i^T C_i x \equiv 0$$

and $\dot{V}_j(x) \neq 0$ for $j \neq i$.

The control is then given by $\alpha_j(t) = \delta_{ij}$ for all $t \in (a, b)$. In this situation, since $x^T C_i^T C_i x = 0$ vanishes identically, by successive differentiations we get the set of equations:

$$\begin{aligned} C_i x &= 0 \\ C_i A_i x &= 0 \\ &\vdots \\ C_i A_i^{n-1} x &= 0 \end{aligned}$$

and the trajectory x is therefore contained in the unobservable subspace associated to the pair (C_i, A_i), denoted in the following by $\mathrm{Ker}(\mathcal{O}(C_i, A_i))$.

Assume now that there exists a subset of indices $I_0 \subseteq \{1, \ldots, I\}$ of cardinality $|I_0| > 1$ such that for all $i_0 \in I_0$,

$$C_{i_0} x \equiv 0 \tag{6}$$

on (a, b) and $C_j x(t) \neq 0$ if $j \notin I_0$ and $t \in (a, b)$. In particular x, restricted to the interval (a, b), evolves in $\bigcap_{i_0 \in I_0} \mathrm{Ker}(C_{i_0})$ and the associated control law (see Problem 2) takes its values in the subset

$$\Delta_{I_0} = \left\{ \alpha \in \Delta \mid \textstyle\sum_{i_0 \in I_0} \alpha_{i_0} = 1 \right\}. \tag{7}$$

By differentiating (6) and replacing $\dot{x}$ by $\sum_{i \in I_0} \alpha_i A_i x$, we get

$$\textstyle\sum_{i \in I_0} \alpha_i C_{i_0} A_i x = 0$$

for all $i_0 \in I_0$.

We see the former as a linear relation between control components, and we notice that either at least one coefficient $C_{i_0} A_i x$ is a non-vanishing function (giving an algebraic condition on α) or they all vanish, providing us with $|I_0|$ extra algebraic conditions on x that can again be differentiated. In the case in which some coefficients are vanishing and some other are not, we are given an algebraic condition on α and some algebraic conditions on x that can again be differentiated.

We obtain in this way a recurrent procedure. Notice that we do not need to require additional regularity assumptions on x in order to consider all these subsequent differentiations, since at each step only the differentiability of x is required.

We are therefore justified to define, for every $i_0 \in I_0$,

$$p_{i_0} = \min \left\{ k \mid \exists i \in I_0, \frac{\partial}{\partial \alpha_i} \frac{d^k}{dt^k} C_{i_0} x \not\equiv 0 \text{ on } (a, b) \right\},$$

that is, p_{i_0} is the minimal number of time derivatives of $C_{i_0} x$ guaranteeing the appearance of at least one component of the control with a nonzero coefficient.

By eventually restricting the interval (a, b), we will assume that all such non-vanishing coefficients are nonzero everywhere in (a, b).

For each $i_0 \in I_0$ the following conditions are fulfilled on (a, b),

$$\begin{cases} C_{i_0} x & = 0 \\ C_{i_0} A_{i_1} x & = 0, \ i_1 \in I_0 \\ & \cdots \\ C_{i_0} A_{i_1} A_{i_2} \cdots A_{i_{p_{i_0}-1}} x & = 0, \ \left(i_1, \cdots, i_{p_{i_0}-1}\right) \in I_0^{p_{i_0}-1} \end{cases} \tag{8}$$

and

$$\textstyle\sum_{i_k \in I_0} \alpha_{i_k} C_{i_0} A_{i_1} A_{i_2} \cdots A_{i_{p_{i_0}}} x = 0, \left(i_1, \cdots, i_{p_{i_0}}\right) \in I_0^{p_{i_0}}. \tag{9}$$

Even if the the last equation can be differentiated once again (this depends on the regularity of α), no additional information concerning the values of the control components can be obtained since their derivatives will appear.

The system of equations (7), (8), (9) provides us, at every instant of time, with a set of algebraic relations between the control α and the point x.

The set of solution of (8), denoted $S_{p_{i_0}}$ is monotone non increasing w.r.t. p_{i_0}. Then, assume that for every set I_0, there exists a finite maximum number $p_{i_0\,\max}$ beyond which (that is for all $p_{i_0} > p_{i_0\,\max}$) the solution set of (8) is constant.

As the number p_{i_0} is *a priori* unknown, the following algorithm may be used to try to determine a solution.

Algorithm 4: for all subset $I_0 = \{s_1, s_2, \ldots, s_{|I_0|}\}$ of $\{1, \ldots, I\}$
for each $i_0 \in I_0$
set $p_{i_0} = 1$ and $S_0 = \{\emptyset\}$
step 1: check if the solution set of (8) is constant $S_{p_{i_0}} = S_{p_{i_0}-1}$
if not set $p_{i_0} := p_{i_0} + 1$ and return to step 1
end if
set $p_{i_0\,\max} := p_{i_0} - 1$
end for

for all $p = (p_{s_1}, p_{s_2}, \ldots, p_{s_{|I_0|}})$ with $1 \leq p_{s_k} \leq p_{s_k \max}$ for $k = 1, 2, \ldots, |I_0|$

compute the solution set of (8) taking $i_0 = s_k$ for $k = 1, \ldots, |I_0|$ and check if the intersection of all such solution sets reduces to the singleton $\{0\}$

if not solve the algebraic system of equations (7), (8), (9)

end if

end for

end for

V. Application to the flying capacitor converter

It is easy to show that if a matrix $P > 0$ satisfies $\tilde{A}_i^T P + P\tilde{A}_i \leqslant 0$, for $i = 0, 1, 2, 3$, then P must be of the type

$$P = \begin{pmatrix} p_1 & p_4 & 0 \\ p_4 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}.$$

Moreover, in order to avoid a positive eigenvalue in $\tilde{A}_i^T P + P\tilde{A}_i$, the gains L_i and P must be such that

$$p_1\left(\frac{u_2 - u_1}{C_1} - L_1(u)\right) + p_4\left(\frac{u_3 - u_2}{C_1} - L_2(u)\right) + p_3\frac{u_1 - u_2}{L} = 0$$
$$p_4\left(\frac{u_2 - u_1}{C_1} - L_1(u)\right) + p_2\left(\frac{u_3 - u_2}{C_1} - L_2(u)\right) + p_3\frac{u_2 - u_3}{L} = 0$$

where $L_1(u)$ and $L_2(u)$ are respectively the first and the second component of $L(u) = L_0 + \sum_{i=1}^{3} u_i L_i$.

As mention above, for a given mode, the subsystem is not or partially observable. The dynamic of such a system is fast and the smallest eigenvalues is p=-3000. Choosing a pole placement at $pole = -10000$ for each observable subspace of $\tilde{A}_i = A_i - L_i C$, the matrix gain are given by

$$L_0 = 10^4[0, 0, 5.7]$$
$$L_1 = 10^6[8.975, 4.5, -0.0]$$
$$L_2 = 10^6[-4.475, 4.475, 0]$$
$$L_3 = 10^6[-4.5, -8.975, 0]$$

with the matrix P having the following coefficients

$$p_1 = p_2 = 90$$
$$p_4 = -45$$
$$p_3 = 6.075 \times 10^6.$$

A. Characterization of the invariant set and associated controls

A simple computation using algorithm 4 provides an invariant set and the associated generic singular control laws:

$$\mathrm{Inv} = \{e \mid e_3 = 0\} \qquad (10)$$

$$U_{\mathrm{Inv}} = \{u \in [0\ 1]^3 : e_2 u_3 - u_1 e_1 + u_2 e_1 - u_2 e_2 = 0\}, \qquad (11)$$

which do not depend on L.

For example, an error trajectory starting from a point $e \in \mathrm{Inv}$ and corresponding to a control u whose first two components are arbitrarily chosen and the third one is set to $u_3 = -\frac{-u_1 e_1 + u_2 e_1 - u_2 e_2}{e_2}$ remains in Inv. Notice that such choice of u_3 does not necessarily belong to $[0, 1]$, but it is not difficult to show that there exist values of (u_1, u_2, e_1, e_2) for which this is the case.

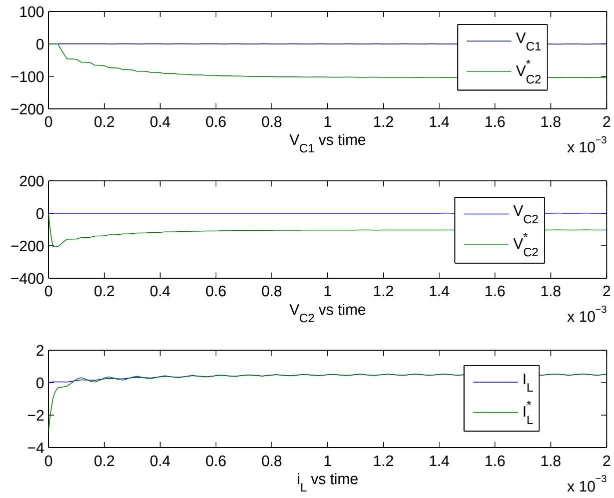

Fig. 2. Example of a singular control making unobservable the Flying Capacitor Converter

In the case where $e_2 = 0$, we get $u_1 = u_2$ and u_3 free.

For all control laws built in this way, system (2) is just stable and evolves on a level set of the matrix P. In the figure (2), for example, we have applied the singular control $u_3 = -\frac{-u_1 e_1 + u_2 e_1 - u_2 e_2}{e_2}$. It can be observed that once the error e_3 is in or near 0 (that is e in or near Inv) the errors in the estimation of the voltage values remain constant.

B. Observability of the operating point

Generally, the goal of the control applied on the flying capacitor converter is to regulate the load current and to maintain in average the voltage in each capacitor to a fixed value of $2E/3$ in capacitor C_2 and $E/3$ in capacitor C_1. The operating points of the flying capacitor converter may therefore be defined in average value by $x_{\mathrm{ref}} = \begin{bmatrix} 2E/3 & E/3 & i_{\mathrm{ref}} \end{bmatrix}^T$.

It is interesting to notice that the only control value that maintains the current in average around a nonzero value is a singular law. Indeed, for systems exhibiting a cyclic behavior, the desired operating point is the mean value of x on the cycle.

To find the admissible operating point set, one can study the evolution of the average state $\bar{x}$ given by the convolution product

$$\bar{x}(t) = \sqcap_{T_p} * x(t) = \frac{1}{T_p}\int_{t-T_p}^{t} x(\tau) d\tau \qquad (12)$$

where T_p is the cycle period and $\sqcap_{T_p}$ is a rectangular window function.

The dynamical model of $\bar{x}$ is obtained differentiating (12). However, the derivative is generally hard to use because of its nonlinear form.

A solution consists in defining the average state model

$$\dot{\tilde{x}} = A_0\tilde{x} + \sum_{i=1}^{3}\bar{u}_i(t)(A_i\tilde{x}(t) + B_i), \;\; \overline{u}_i \in [0\;1] \qquad (13)$$

which gives an approximation of the dynamics of $\bar{x}$. $\bar{u}$ is the average value of u on the cycle. It has been proven that $\tilde{x}$ is close to $\bar{x}$ and x when T_p is small with respect to the system dynamics (see [20]).

The operating points are then defined as the equilibria of the average state model, that is, the elements of the set

$$X_{\text{ref}} = \{x_{\text{ref}} \in \mathbb{R}^n \mid A_0x_{\text{ref}} + \sum_{i=1}^{3}u_{i,\text{ref}}(A_ix_{\text{ref}} + B_i) = 0, \\ u_{i,\text{ref}} \in [0,1]\}. \qquad (14)$$

Since no mode allows to hold the given reference x_{ref} as $u_{\text{ref}} \in (0,1)$, the only possibility for the switched system is to enter into a cyclic behavior around the reference x_{ref}.

Here, equation $A_0x + \sum_{i=1}^{3}u_i(A_ix + B_i) = 0$ has a unique solution $u_1 = u_2 = u_3$ leading to the equilibrium $i = \frac{E}{R}u_3 \neq 0$. This law corresponds to a singular law of the type seen at the beginning of the section, rendering the system unobservable.

Consequently, a better approximation of the singular law $u_1 = u_2 = u_3$ by the switching law leads to a smaller convergence rate of the observer. For example, if we consider two periodic switching laws, one obtained from the other by a uniform time-rescaling, which realize $u_1 = u_2 = u_3$ in average, we can see in Figures 3 and 4 that the convergence rate of system (2) decreases when the switching frequency increases. For this example the observer convergence is achieved at time $t = 2 \times 10^{-3}s$ for a switching frequency of $f_s = 10$ kHz while it only happens at time $t = 1.0 \times 10^{-2}s$ for a switching frequency of $f_s = 30$ kHz.

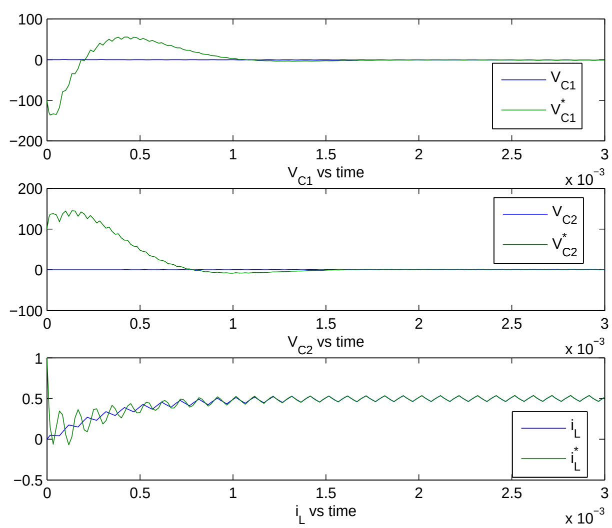

Fig. 3. Convergence rate vs the switching frequency ($\mathrm{f}_s = 10$ kHz)

The method presented in this article has been validated in simulation with the nominal parameter values $C_1 = C_2 = 40\mu F$, $L = 0.01H$, $R = 30\Omega$.

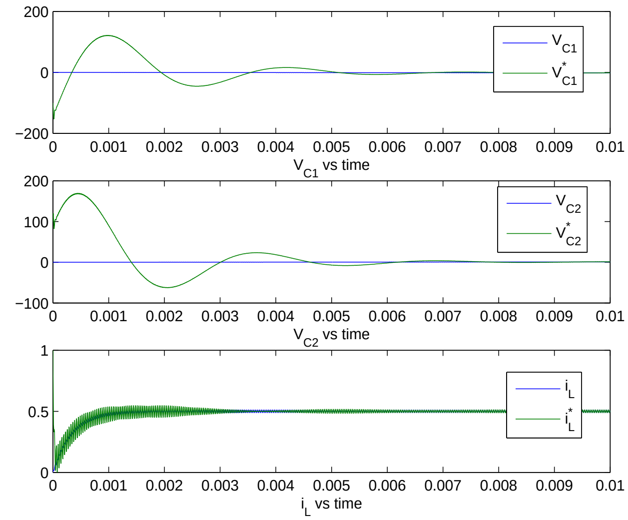

Fig. 4. Convergence rate vs the switching frequency ($\mathrm{f}_s = 30$ kHz)

C. Toward a separation principle

Known as a Principle of separation of estimation and control, it has been proved that if a stable observer and stable state feedback are designed for a linear time-invariant system, then the combined observer and feedback will be stable. The Separation principle does not hold in general for nonlinear systems and in particular for switched systems. Nevertheless a version for discrete time linear switched systems is proved in [21]. But its continuous version remains to write. On the figure 5, an optimal state feedback law [6] is used jointly with the proposed observer.

The defined operating point is $v_{C10} = \frac{1}{3}E$, $v_{C20} = \frac{2}{3}E$ and $i_L = 1A$ (here $R = 10$ ohm). We can see that the control target is clearly achieved.

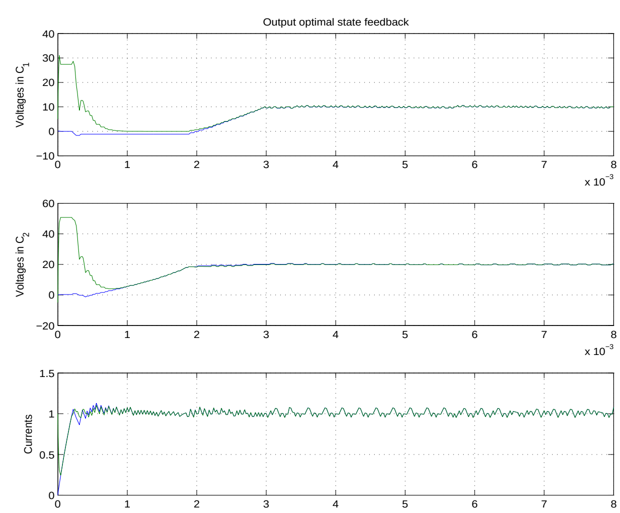

Fig. 5. Output feedback law

VI. Conclusion

In this paper, a pole placement design of a switched observer is proposed and applied on a flying capacitor converter. A complete characterization of invariant set and associated control laws for the estimate error dynamic systems explains in which case the system state cannot be estimated. A contrary, the result proves the asymptotic convergency of the observer under suitable control laws (i.e. for switching law not associated to an invariant set). An amazing consequence is that the operating points of the flying capacitor are associated to an invariant set making unobservable the system. Fortunately, this points are defined by mean values remaining observable the system in a neighborhood. In view of these results, it is clear that asymptotic stability property are not sufficient and an important question concerns the estimation of the observer rate convergency.

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