A predictive control approach for DC-DC power converters and cyclic switched systems.

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***Abstract*—Classical control laws for power converters are based on the average model. Usually, they have a good performance in the transient. Nevertheless, the steady state behavior is not well-controlled (waveform, subharmonics, etc.). This article shows a predictive approach that reaches an optimal periodic cycle from a set-point of the average model. The method uses the sensitivity functions and a Newton algorithm which allows to track an optimal trajectory based on a cost function.**

***Index terms*— Cycle, sensitivity function, Gauss-Newton algorithm, non-linear programming.**

I. INTRODUCTION

Power converters are devices that ensure a supply to the electric machines by the conversion of an electrical signal (voltage, current, frecuency). This is obtained by switches operating (open or closed) at high frequencies. They allow "to trim" a current or a voltage. Thus, it is necessary to design a control law for deciding the position of the switches. Usually, classical analysis and control laws lie on average models approaches [1], [2]. However, they are low frequency approaches and do not take into account the discontinuous aspect of the switches. Therefore, resulting waveform can create subharmonics or inter-harmonics of the cutting frequency. There are also approaches based on sampling linear models. From the non-linear sampling model, a linearized model around the desired operation point is used [3]. The non-linear recurrence is not easily obtained and oscillations between switching instants appear. Other emerging strategies consist in directly controlling the different switches without using an average model. There exist already some of these control laws that have been tested by industrial companies. They show an improvement of 50% in response time compared to the techniques traditionally used [4].

In this work, we are concerned with the problem of controlling systems, with a cyclic behaviour in steady state and whose inputs can be described with binary values (0 or 1). This situation gives us a good source of interesting applications, not only for power converters, but also for other systems. There are already some other strategies applied to these systems based on sliding modes [5], predictive control [6] and passivity control [7].All these methods do not take into account the cyclic behavior of the system.

The model predictive control has already been applied to power systems. However, the different approaches are focused in particular examples. See [8] for a full bridge DC/DC converter application or [9] for an active front-end rectifier. The approach presented in this article is formulated from a switched system point of view. This allows to establish a general method which does not depend on the application

The method presented in this paper provides the theoretical background to a new predictive control approach based on sensitivity function and a Newton algorithm. First, this approach establishes an optimal limit cycle which is a reference cycle. Second, a predictive control is created in order to reach a reference cycle in steady state. Some simulation results can be seen in [10] without providing a theoretical basis.

Since the method solves several optimization problems, the on-line implementation cannot be ensured for complex converters. However, the optimal solutions can be interpolated by the off-line training of a neural network (NN). This network interpolates the trajectories and allows to satisfy time contraints in real time between two sampled time.

The present article is organized as follows: In Section 2, we present the general problem. Section 3 is devoted to the open loop analysis and the limit cycle research. Section 4 shows how to obtain optimal switching times from a Newton algorithm and sensitivity functions. Section 5 is concerned by the construction of the control law in closed loop by predictive control. Section 6 shows simulation and experimental results in a buck-boost and a multilevel converter. In section 7, some conclusions and future work are presented.

II. PROBLEM FORMULATION

Since the power converters are switched systems, they can be described by a piecewise differencial equation:

$$\dot{x}(t) = A_{\sigma_i} x(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, σ_i is the active mode of the system It is determined by the switches positions. $\sigma_i \in \{1, \ldots, 2^r\}$ where r is the number of switches in the power converter. $A_{\sigma_i} \in \mathbb{R}^{n\times n}$. It has been proven in [11] that an equivalent model for the power converters is a control affine model written as:

$$\dot{x}(t) = f\left(x(t)\right) + g\left(x(t)\right) u(t) \tag{2}$$

where $u(t)$ is a boolean vector. $f(x(t)) \in \mathbb{R}^n$, $g(x(t)) = [g_1(x(t)), \ldots, g_r(x(t))] \in \mathbb{R}^{n\times r}$, $g_k(t) \in \mathbb{R}^r$, $k \in \overrightarrow{r} \triangleq \{1, \ldots, r\}$. Indeed, each mode σ_i produce different values of the boolean vector $u(t)$. The control problem is to find a switching law such that the system in closed loop is stable.

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x must be regulated around an average reference value $\bar{x}^\infty$. Let us assume that all the states of (2) are measurable.

Definition 1: A switching sequence is a finite sequence represented by $(\mathcal{T}, \mathcal{I}, s)$ where:

- s is the (finite) length of the sequence.
- $\mathcal{T} = \{t_i\}_{i=0}^{s}$ is a strictly increasing time sequence composed by the instant values when a mode is switched on.
- $\Omega = \{\sigma_i\}_{i=0}^{s-1}$ is a map set called mode sequence. A mode σ_i is switched on at time t_i, $i = 0, \ldots, s-1$.

A switching sequence is shown in Fig. 1.

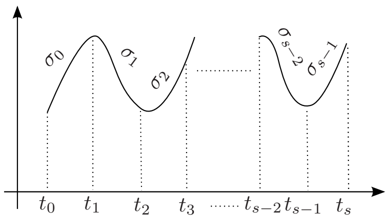

Fig. 1. The switching sequence $(\mathcal{T}, \mathcal{I}, s)$. Each mode σ_i is switched on at $t = t_i$

Usually, the power converters has a steady state depicted by a limit cycle around an equilibrium point of the average model. Several cycles with the same average value may exist. However with the method presented here, it is possible to select one of the cycles and use it as a reference for the steady state. This selection can be carried out with a cost function for reducing the quadratic norm, optimizing the harmonic content, etc. After finding an optimal limit cycle, a tracking problem appears. Techniques based on predictive methods can be used. Then, the design control can be divided in two parts:

1) Near the optimal limit cycle determined by the switching sequence $(\mathcal{T}^\infty, \Omega^\infty, s^\infty)$ where $\mathcal{T}^\infty = \{t_i^\infty\}_{i=0}^{s}$,$\Omega^\infty = \{\sigma_i^\infty\}_{i=0}^{s-1}$. In this step, the predictive control computes only the switching times $\mathcal{T}$. Ω and s are fixed to the values Ω^∞ and s^∞.
2) Far from the limit cycle: $\mathcal{T}$ and Ω are optimized and the length of the sequence is fixed to s^∞.

In the next section, we describe how to obtain the limit cycle and its switching sequence $(\mathcal{T}^\infty, \Omega^\infty, s^\infty)$.

III. Determination of a cyclic steady state

In this part, an optimal limit cycle is determined. The goal is to obtain the best time sequence $\mathcal{T}^\infty$, the modes sequence Ω^∞ and its length s^∞ $(1 < s^\infty < s_{max})$. These values must produce a specific waveform. In order to minimize the oscillations around a desired operation point, an integral quadratic criterion is chosen around the average reference value $\overline{x}^\infty$:

$$J(\mathcal{T}^\infty, \Omega^\infty, s^\infty) = \min_{\mathcal{T}, \Omega, s} \int_0^{T_p} [x(t) - \overline{x}^\infty]^T \mathcal{Q} [x(t) - \overline{x}^\infty] dt \tag{3}$$

where $\mathcal{Q} = \mathcal{Q}^T > 0$ and T_p is the period of the cycle. T_p is not *a-priori* known, but it is bounded by a constant $T_{p,max}$. It means:

$$T_p < T_{p,max} \tag{4}$$

The cyclic behavior is imposed by the constraint:

$$x(0) = x(T_p) \tag{5}$$

A duration between switches has to be verified. In DC/DC converters, the switches have a minimum time t_{min} between two states (dwell time - closed to open or open to closed). Thus, the following additional constraints must be considered:

$$\begin{aligned} \delta_k(t_i) \geq t_{min} |u_k(t_i) - u_k(t_{i-1})| \qquad & \forall i = 1, \ldots, s \\ \dot{\delta}_k(t_i) = 1 \qquad & \forall k = 1, \ldots, r \\ \delta_k(t_{i+1}) = 0 \qquad if \qquad & |u_k(t_i) - u_k(t_{i-1})| \neq 0 \end{aligned} \tag{6}$$

Eq. (6) is a condition on minimal duration of the mode σ_j where t_{min} is a constant. δ_k is the elapsed time from the last activation of the switch.

Remark 2: Other criteria can be also considered. e.g., in order to minimize the harmonics, the function (3) uses a filtered state variable instead of the state variable [6]. .

Solution of (3) subject to constraints (4), (5) and (6) give the reference trajectory x^∞. More exactly, fixing a modes sequence Ω and a value of s, the problem is reduced to determine the switching instants. The following algorithm can be used:

Algorithm 3: The values of $\mathcal{T}$, Ω and s can be obtained through three steps:

1) For s and Ω fixed, the cost function $J(\mathcal{T}, \Omega, s)$ described by equation (3) is optimized.
2) The point 1 is taken again with a new couple Ω and s until all possible sequences are analysed.
3) The modes sequence producing the minimum value of $J(\mathcal{T}, \Omega, s)$ is chosen.

The algorithm finishes because Ω is finite and s is bounded Usually, analytic solution of the optimization problem (3) with constraints (4), (5), and (6) is hardly obtained. That is why optimization methods must be applied. The solution defines an execution $(\mathcal{T}^\infty, \Omega^\infty, \mathcal{X}^\infty)$ and the modes number s^∞.

Remark 4: The method can be numerically very long because the optimization problem must be solved $\sum_{s=1}^{s_{max}} r^s$ times. This optimization is carried out off-line.

From this section and section II, it is observed that the optimization of switching time is needed. The next section is devoted to this topic.

IV. Switching time optimization

Through this section, a method for optimizing the switching times is presented. The modes sequence is supposed to be fixed. The method is used for any of the considered strategy (near or far from the limit cycle). The research of $\mathcal{T}$ is formalized as another optimization problem with a quadratic error between the trajectory x and the reference steady state x^∞. However, the interesting values are only located at the switching instant. Therefore, switching time control problem can be written as:

$$\begin{aligned} \min \quad & J(\mathcal{T}) = \sum_{i=1}^{s} (x(t_i) - x_i^*)^T \mathcal{Q} (x(t_i) - x_i^*) \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ & x(0) = x_0 \qquad \text{(Given initial condition)} \\ & t_s = T_p^\infty \qquad \text{(Condition on the period).} \end{aligned} \tag{7}$$

where $T_p^\infty \in \mathbb{R}$ is the period of the reference cycle and

$$x_i^* = \begin{cases} x(t_i^\infty) & \text{if steady state} \\ \overline{x}^\infty & \text{if transient state} \end{cases} \quad (8)$$

A method based on the Gauss-Newton algorithm and the knowledge of sensitivity functions is proposed. This algorithm yields the optimal time sequence $\mathcal{T}$ which satisfies (7). Let us write the state evolution as a function of switching instants:

$$x(t_i) = \Phi_i(t_i, t_{i-1})x(t_{i-1}), \forall i = 1, \dots, s \quad (9)$$

where $\Phi_i(.,.)$ is the transition matrix of the linear system (1):

$$\Phi_i(t_i, t_{i-1}) = e^{A_{\sigma_i}(t_i - t_{i-1})} \qquad \forall i = 1, \dots, s \quad (10)$$

Error Δx between the trajectory and the reference cycle at instants $\mathcal{T}$ is given by:

$$\Delta x(t_i) = x(t_i) - x_i^*, \qquad \forall i = 1, \dots, s \quad (11)$$

with x_i^* is defined in (8). Since the period of the cycle has a value T_p^∞, time value $t_s = T_p^\infty$. Replacing (11) in (7), a minimization without constraints is obtained:

$$\min_{\mathcal{T}} J(\mathcal{T}) = \min_{\mathcal{T}\backslash t_s} \Delta x(T_p^\infty)^T \mathcal{Q} \Delta x(T_p^\infty) + \sum_{i=1}^{s-1} \Delta x(t_i)^T \mathcal{Q} \Delta x(t_i) \quad (12)$$

This last equation can be also written in a matricial form:

$$\min_{\mathcal{T}} J(\mathcal{T}) = \min_{\mathcal{T}\backslash t_s} \Delta X(\mathcal{T})^T \mathbf{Q} \Delta X(\mathcal{T}) \quad (13)$$

where $\Delta X(\mathcal{T}) : \mathbb{R}^s \to \mathbb{R}^{ns}$ is a vector that contains all state errors at each switching instant:

$$\Delta X(\mathcal{T}) = [\Delta x(t_1)^T, \Delta x(t_2)^T, \dots \Delta x(t_{s-1})^T, \Delta x(T_p^\infty)^T]^T \quad (14)$$

$\mathbf{Q} : \mathbb{R}^{ns \times ns}$ is a diagonal constant matrix composed by the matrix $\mathcal{Q}$, repeated s times.

$$\mathbf{Q} = \begin{bmatrix} \mathcal{Q} & 0_{n\times n} & \dots & 0_{n\times n} \\ 0_{n\times n} & \mathcal{Q} & \dots & 0_{n\times n} \\ \vdots & \vdots & \ddots & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & \dots & \mathcal{Q} \end{bmatrix} \quad (15)$$

Denote τ as the time sequence $\mathcal{T}$ without the value t_s ($\mathcal{T} \setminus t_s$). (13) is a quadratic minimization. Thus, a Gauss - Newton algorithm can be used.

Algorithm 5: **(Determining switching instants)**.
Inputs:

- Values x_0, s, Ω, initial values of switching time sequence τ_{in}.
- Period of the cycle T_p^∞
- Stop parameter $\nu \approx 0$ (Constant).

Algorithm is decomposed into the following steps:

1) Initialize $h \leftarrow 0$, $\tau^0 \leftarrow \tau_{in}$
2) Gauss-Newton iteration:

$$\tau^{h+1} = \tau^h + \mathcal{H}^{-1}(\tau)\frac{\partial J(\tau)}{\partial \tau}, \quad (16)$$

3) Constraint validation: $\forall i = 1, \dots, s$, if $(t_i - t_{i-1}) < 0$, $t_i \leftarrow t_{i-1}$
4) $\forall i = 1, \dots, s$, if $(t_i - t_{i-1}) > T_p^\infty$, $t_i \leftarrow T_p^\infty$
5) Stop test: If $\|\tau^{h+1} - \tau^h\| < \nu$, go to step 7.
6) $h \leftarrow h + 1$ and go back to step 2.
7) End

Remark 6: One of the algorithm assumptions is that the period of the cycle is T_p^∞. Steps 3 and 4 are particular cases. If $t_{i-1} > t_i$, then mode σ_{i-1} is part of the modes sequence with a zero duration. Step 4 is the case when a time sequence has a duration greater than the cycle duration. This case ($t_i > T_p^\infty + t_{i-1}$) for $t_{i-1} > 0$ cannot occur.

$\frac{\partial J(\tau)}{\partial \tau}$ is the sensitivity function of J with respect to switching instants.
From the chain rule of derivation:

$$\frac{\partial J(\tau)}{\partial \tau} = \mathcal{J}^T(\tau)\frac{\partial J(\tau)}{\partial \Delta X(\tau)} = \mathcal{J}^T(\tau)\mathbf{Q}\Delta X(\tau) \quad (17)$$

where $\mathcal{J}(\tau) : \mathbb{R}^s \to \mathbb{R}^{(ns)\times(s-1)}$ is the Jacobian of the errors with respect to each switching instant,

$$\mathcal{J}(\tau) = \begin{bmatrix} \frac{\partial \Delta x(t_1)}{\partial t_1} & \frac{\partial \Delta x(t_1)}{\partial t_2} & \dots & \frac{\partial \Delta x(t_1)}{\partial t_{s-1}} \\ \frac{\partial \Delta x(t_2)}{\partial t_1} & \frac{\partial \Delta x(t_2)}{\partial t_2} & \dots & \frac{\partial \Delta x(t_2)}{\partial t_{s-1}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial \Delta x(t_s)}{\partial t_1} & \frac{\partial \Delta x(t_s)}{\partial t_2} & \dots & \frac{\partial \Delta x(t_s)}{\partial t_{s-1}} \end{bmatrix} \quad (18)$$

and $\mathcal{H}(\tau)$ is an approximation of the non-singular Hessian matrix defined by $\mathcal{H}(\tau) = \mathcal{J}^T(\tau)\mathbf{Q}\mathcal{J}(\tau)$. Thus, we must compute the sensitivity function of the state errors with respect to each switching instant. The next Proposition yields analytical expressions for the sensitivity functions.

Proposition 7: Sensibility functions of $\Delta x(\tau)$ with respect to switching times are given by:
$\forall j = 1, \dots, s-1$, and $\forall i = 1, \dots, s$

$$\frac{\partial \Delta x(t_i)}{\partial t_j} = \begin{cases} 0 & \text{if j>i} \\ A_{\sigma_i}\Pi(t_i, 0)x(0) & \text{if j=i} \\ \Pi(t_i, t_j)\Delta P_j x(0) & \text{if j<i} \end{cases} \quad (19)$$

where $\Pi(t_i, t_j)$ is the product of transition matrices from $t = t_j$ to $t = t_i$:

$$\Pi(t_i, t_j) = \Phi_i(t_i, t_{i-1})\Phi_{i-1}(t_{i-1}, t_{i-2}) \dots \Phi_{j+1}(t_{j+1}, t_j) \quad (20)$$

ΔP_j is a function given by

$$\Delta P_j = (A_{\sigma_j} - A_{\sigma_{j+1}})\Pi(t_j, 0) \quad (21)$$

Proof: Although $x^\infty(t)$ is a time variable reference, $x^\infty(t_i)$ is constant for a given t_i. From (11) $\frac{\partial \Delta x(t_i)}{\partial t_j} = \frac{\partial x(t_i)}{\partial t_j}$. In order to compute $\frac{\partial x(t_i)}{\partial t_j}$, it is necessary to distinguish three cases:

1) $j > i$: Terms t_j do not appear in the functions $x(t_i)$, then $\frac{\partial x(t_i)}{\partial t_j} = 0$.
2) $j = i$: Consider the left derivative of the state $x(t)$ with respect to switching instants t_j:

$$\frac{\partial x(t_j^-)}{\partial t_j} = \lim_{t \to t_j^-} \frac{\partial x(t)}{\partial t_j}$$

System evolution is computed between $t_j - \epsilon$ and t_j as,

$$x(t_j) = x(t_j - \epsilon) + \int_{t_j - \epsilon}^{t_j} A_{\sigma_j} x(p) dp.$$

Its derivative gives: $\frac{\partial x(t_j^-)}{\partial t_j} = A_{\sigma_j} x(t_j)$ Since $j = i$, further switches are not considered, and:

$$\frac{\partial x(t_j)}{\partial t_j} = \frac{\partial x(t_j^-)}{\partial t_j} = A_{\sigma_j} x(t_j) \tag{22}$$

3) $j < i$: Contrary to the case $j = i$, further switches must be taken into account. Let us consider the right derivative of the state $x(t)$ with respect to switching instant t_j:

$$\frac{\partial x(t_j^+)}{\partial t_j} = \lim_{t \to t_j^+} \frac{\partial x(t)}{\partial t_j}$$

State evolution $x(t)$ is computed once again as $x(t) = x(t_j^+) + \int_{t_j}^{t} A_{\sigma_{j+1}} x(p)dp$. Since $x(t_j^+) = x(t_j^-) = x(t_j)$ and from (22), the limit $t \to t_j^+$ becomes:

$$\frac{\partial x(t_j^+)}{\partial t_j} = (A_{\sigma_j} - A_{\sigma_{j+1}}) x(t_j) \tag{23}$$

From (9), $x(t_j)$ is a function of the initial state:

$$x(t_j) = \Pi(t_j, 0)x(0).$$

Using again the chain rule: sensitivity funcion of $x(t_i)$ with respect to $x(t_{i-1})$ can be computed from (9),

$$\frac{\partial x(t_i)}{\partial x(t_{i-1})} = \Phi_i(t_i, t_{i-1}).$$

Therefore,

$$\frac{\partial x(t_i)}{\partial t_j} = \Pi(t_i, t_j)(A_{\sigma_j} - A_{\sigma_{j+1}}) x(t_j) \tag{24}$$

■

V. Closed loop construction

Algorithm 5 has as input parameters x_0 (State measure), Ω (modes sequence) and s (modes number). If the last two variables are *a-priori* known, the closed loop control is directly built. Although it is a particular case, it is the case of the most used DC/DC converters (e.g., buck, boost and buck-boost) because their models are not complex and Ω can be easily determined. On the other hand, when Ω and s are not known, two possible solutions are:

1) Test algorithm 5 with all modes sequences with size s. s also varies between 1 and s_{max} ($1 \leq s \leq s_{max}$). Take the sequence which gives the minimum value of $J(\mathcal{T})$. Numerically speaking, solution becomes a very expensive process to apply it on-line. Thus, control is hardly available in real time. Nevertheless, this process can be carried out off-line.
2) Use different values of x_0. For each x_0, compute $\mathcal{T}$, Ω and s as it is indicated in the point 1, but off-line. A table is then created. This table has as inputs the different values of x_0 and as outputs the employed mode σ_0 and its duration $t_1 - t_0$. Then, an interpolation method is used to establish an input-output function. This function is applied on-line. One of the approaches which has shown good performances concerning robustness and computation time is the NN. Since all solutions for a whole initial conditions set are obtained, NN may interpolate them. Indeed, it can imitate the controller behavior. NN are considered as universal approximators because they can *learn* the behavior of a non-linear function. Hence, they can compute on-line the control value. In [12], a NN is used for implementing a predictive control.

VI. Examples

Two examples are considered: The buck-boost converter and the multilevel converter. The first converter is presented only for claryfing the method in a very simple system. The second converter is a real converter for which the control law has been tested in simulation and in a real platform.

A. Buck-boost converter

Consider a Buck-Boost converter in a Continuous Conduction Mode (CCM) whose topology is shown in figure 2.

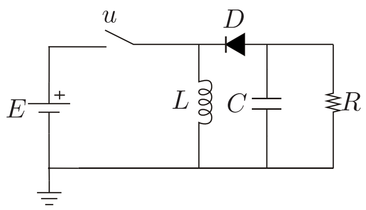

Fig. 2. Buck-Boost converter

The state variables are the the capacitor voltage v_c and the inductor current i_L. Its dynamics is given by the equation:

$$\dot{x} = \begin{cases} \mathcal{A}_1 x + B_1 \text{ for } u = 1 \text{ (closed switch - Mode 1)} \\ \mathcal{A}_2 x + B_2 \text{ for } u = 0 \text{ (open switch - Mode 2)} \end{cases}$$

where

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \qquad \mathcal{A}_2 = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$
$$\mathcal{B}_1 = [\frac{E}{L}, 0]^T \qquad \mathcal{B}_2 = [0, 0]^T$$

where $R = L = C = E = 1$. DCM is not taken into account. Thus, the diode do not influence the system response. It is really not hard to extend this method to DCM as it has been shown in [13] for a SEPIC converter. The criterion is to minimize oscillations with respect to an average value $x_{ref} = [v_{c,ref}, i_{l,ref}]^T = [2, -1]^T$. The weight matrix is $Q = diag[1, 1]$, $T_{p,max} = 1s$. and the maximum modes number $s_{max} = 2$. The switch has a dwell time $t_{min} = 0.25s$. For this example $r = 1$. Hence, there are only 4 modes sequences. After finding the solution of the optimization problem, the optimal switching sequence is $\mathcal{T}^{\infty} = \{0, 0.2509, 0.5\}$, $\Omega^{\infty} = \{1, 2\}$. In order to write the system in an autonomous form, another state variable x_{n+1} is added with $x_{n+1}(0) = 1$. Thus, matrices become:

$$A_{\sigma_i} = \begin{bmatrix} \mathcal{A}_{\sigma_i} & \mathcal{B}_{\sigma_i} \\ 0 & 0 \end{bmatrix} \tag{25}$$

The system is now an autonomous system with $\dot{x}(t) = A_{\sigma_i} x(t)$. The values of the reference cycle at the switching instants are:

$$x^{\infty}(t_2) \in Ker \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - e^{A_2(t_2^{\infty} - t_1^{\infty})} e^{A_1 t_1^{\infty}} \right) \tag{26}$$
$$x^{\infty}(t_1) = e^{A_1(t_2^{\infty} - t_1^{\infty})} x^{\infty}(t_2)$$

The function to minimize is eq. (13) where

$$\begin{aligned}\Delta x(t_2) &= e^{A_2(T_p-t_1)}x(t_1) - x^{\infty}(t_2)\\ \Delta x(t_1) &= e^{A_1(t_1)}x(0) - x^{\infty}(t_1)\end{aligned} \tag{27}$$

$\mathbf{Q} = diag[\mathcal{Q},\mathcal{Q}]$, with $\mathcal{Q} = diag[100, 1, 0]$. $\frac{\partial J(\tau)}{\partial \tau}$ is computed as $\mathcal{J}(\tau)^T \mathbf{Q} \Delta X(\tau)$ where $\mathcal{J}(T) = \left[\frac{\partial \Delta x(t_1)^T}{\partial t_1}, \frac{\partial \Delta x(t_2)^T}{\partial t_1}\right]^T$ and from Proposition 7, sensitivity functions are:

$$\begin{aligned}\frac{\partial \Delta x(t_1)}{\partial t_1} &= A_1 \Pi(t_1, 0)x(0)\\ \frac{\partial \Delta x(t_2)}{\partial t_1} &= \Pi(t_2, t_1)\Delta P_1 x(0)\end{aligned} \tag{28}$$

where $\Pi(t_1, 0) = e^{A_1 t_1}$, $\Pi(t_2, t_1) = e^{A_2(T_p-t_1)}$ and $\Delta P_1 = (A_1 - A_2)\Pi(t_1, 0)$.

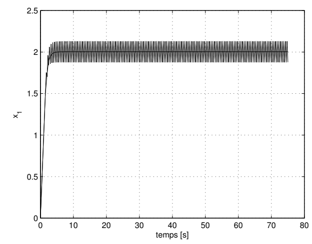

Fig. 3. $x_1(t)$: Capacitor voltage.

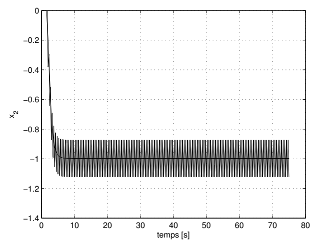

Fig. 4. $x_2(t)$: Inductor current.

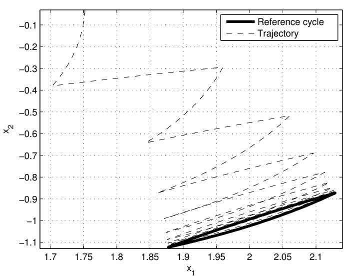

Fig. 5. Reference cycle and trajectory (x_1, x_2) near this cycle.

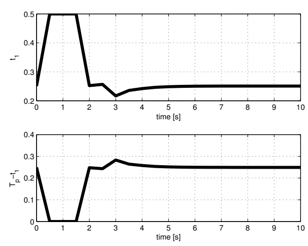

Fig. 6. Duration of each mode in function of the period $T_p = 0.5$s.

The evolution of x_1 and x_2 are shown in Figs. 3 and 4. In figure 5, the reference cycle is observed. The duration $\tau_1 = t_1$ and $\tau_2 = T_p - t_1$ are presented in Fig.6 Note that in this example, the system reaches the reference cycle. Since Buck-Boost converter has only two modes, the modes sequence Ω is known for the transient state and algorithm 5 easily converges to the desired steady state in only one iteration.

B. Multilevel converter

Increasing the power of converters is generally obtained by increasing the voltage. The studies and development carried out on the capacitor converters over the past ten years have revealed excellent characteristics with respect to other DC/DC converters [14], [15], [16]. Fig.7 shows a four-level three-cells converter. Its function is to supply energy to a passive R-L load. The state vector of the converter in Fig. 7 is composed by the circuit voltages and currents. Thus,

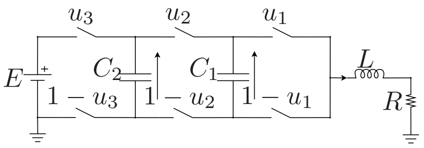

Fig. 7. Four-level three-cell converter

$x(t) = [v_{c_1}(t), v_{c_2}(t), i_L(t)]^T$. The differential equation which describes this system is:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix} = \begin{bmatrix}-\frac{x_3(t)}{C_1} & \frac{x_3(t)}{C_1} & 0\\ 0 & -\frac{x_3(t)}{C_2} & \frac{x_3(t)}{C_2}\\ \frac{x_1(t)}{L} & \frac{x_2(t)-x_1(t)}{L} & \frac{E-x_2(t)}{L}\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -\frac{R}{L}x_3(t)\end{bmatrix} \tag{29}$$

where $x_1(t)$, $x_2(t)$ are the voltages in each capacitor and $x_3(t)$ is the load current. This model is affine to the control u. $u_i(t)$, $i \in \{1, 2, 3\}$ represents the switches position dually operating. It means that if $u_i(t)$ is equal to 1, upper switch is closed and lower switch is open. If $u_i(t) = 0$, lower switch is closed and upper switch is open. The goal of the converter is to obtain a constant average current value. In order to balance the voltages in the switches, the reference for the capacitor voltages are given by $v_{c_2,ref} = \frac{2}{3}E, v_{c_1,ref} = \frac{1}{3}E$. Thus, the reference vector is $\bar{x}^{\infty} = [v_{c_1,ref}, v_{c_2,ref}, i_{L,ref}]^T$. Simulation and experimental parameters are $C_1 = C_2 = 40\mu F$, $L = 10mH$. and $R = 10\Omega$. The nominal input supply is $E = 30$ V and $i_{L,ref} = 1A$.. Table I shows all the possible operation modes. Using the algorithm to determine de steady state, $T_{p,max} = 0.4ms$, $s_{max} = 12$, $t_{min} = 1/45kHz = 0.022ms$ and $\mathcal{Q} = diag[10, 5, 20000]$. This matrix yields less oscillation in the load current compared to voltages. The optimal switching sequence is:

$$\begin{aligned}\Omega^{\infty} =& \{0, 1, 3, 7, 2, 0, 4, 7, 4\}\\ \mathcal{T}^{\infty} =& \{0, 0.066ms, 0.088ms, 0.11ms, 0.132ms, 0.154ms,\\ & 0.22ms, 0.242ms, 0.264ms, 0.286ms\}\end{aligned} \tag{30}$$

with an initial condition $x(0) = [9.9247, 19.2928, 0.9823]^T$. For this example, the modes sequence is not *a-priori* known. Thus a NN must be used. For this example, a grid of 1000 points has been employed in each state variable. For each point, the predictive control was computed as the solution of the tracking problem. A NN interpolates the mode with respect

Mode	u_1	u_2	u_3
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

TABLE I
EQUIVALENCE BETWEEN MODES AND THE VALUE OF EACH u_i

to the error. This network has 50 neurons at the hidden layer. Learning algorithm is a back-propagation. The performance tests are:

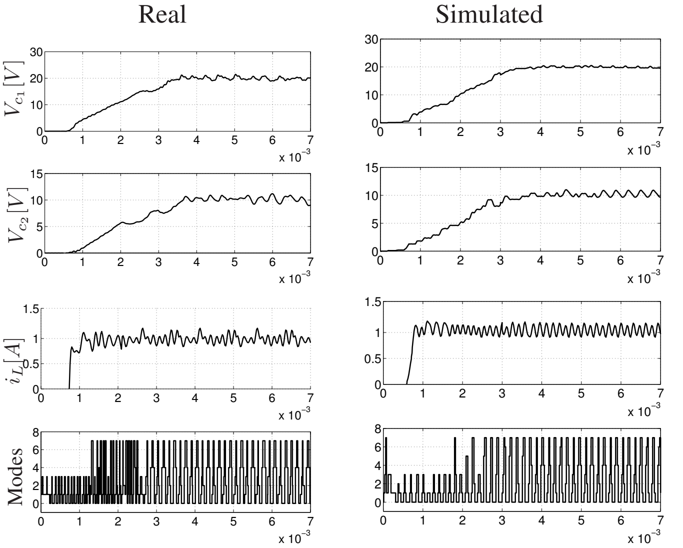

Fig. 8. Start-up test. Comparaison between the simulated results and the experimental results.

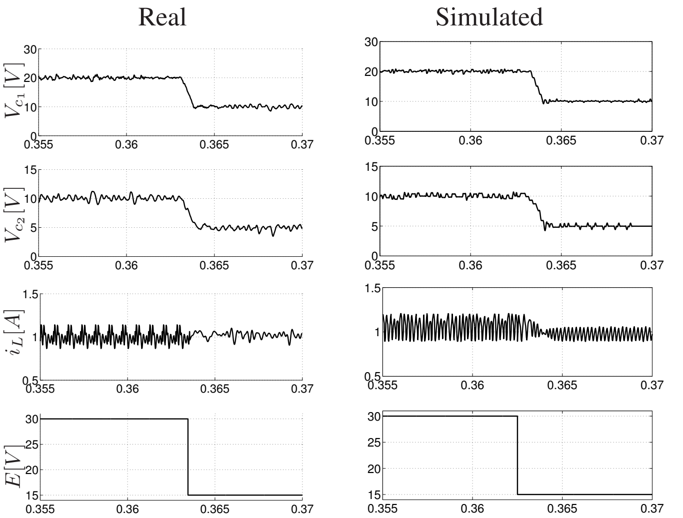

Fig. 9. Input supply variation. Comparaison between the simulated results and the experimental results. An input supply variation changes from $E = 30$ to $E = 15$ V.

1) Start-up test. Fig. 8 shows the system response with nominal parameters in simulation and in a real prototype. The control law reaches the reference in ≈ 3 ms. The sampling frecuency is 45 kHz.
2) Line variation test. Input supply instantaneously changes during operation. Fig. 9 show the steady state when a voltage variation is applied: From $E = 30$ V to $E = 15$ V and back again. Figs. also show that control law can compensate the different input supplies variation. The sampling frecuency is 45 kHz.

The system reaches a cycle in steady state even if an input supply variation is applied.

VII. Conclusions

In this article, a solution is proposed to control DC/DC power converters. The proposed method is composed by three parts: i) Computing of an optimal cycle in open loop. ii) Tracking a trajectory with a control strategy which is function of the distance with respect to the cycle. iii) If necessary, the implementation uses a NN to verify the real-time constraints. A Buck-Boost and a multilevel converter is used as example. They show the good performance of the method. One conclusion is that even if method requires the resolution of many optimization algorithms, they are used off-line (if necessary). With interpolation, a state feedback control law can be established. Control method allows to follow a known cycle and to control the waveform. Although it does not exist proof concerning stability, the method is flexible and easy to implement. Simulations and experimental results show robustness with respect to input supply disturbances. However, there exist several points to study in the future: i) Take into account the DCM in DC/DC converters. ii) Variate the prediction horizon. iii) Study a predictive-adaptive control.

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