Hybrid control of a three-level three-cell dc-dc converter

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***Abstract*— This paper compares three synthesis methods for controlling a three-level three-cell dc-dc converter. The main contribution of this paper is to analyse different strategies: i) The Passivity Based Control that uses the notion of average model, ii) A stabilizing method in which a unique Lyapunov function is introduced and iii) A new predictive control approach, which relies on the use of optimization procedures.**

***Index terms*— Passivity Based Control, Stabilization, Predictive control, limit cycle, neural networks.**

I. INTRODUCTION

Increasing the power of static converters is generally obtained by increasing the voltage because of the efficiency requirements. The studies and development carried out on the capacitor clamped multicell converters over the past ten years revealed excellent characteristics regarding the criteria of the dc-dc converters [1], [2], [3].

The structure of the three-level three-cell dc-dc converter searches to split the voltage constraints and to distribute them on several switches of smaller ratings in series. Since the number of discrete voltage values is directly related to the number of commutation cells, a good approximation of the particular waveform can be obtained. In opposing view, the control of such a converter is more complex. Figure 1 shows a converter, whose function is to feed a passive load (R-L).

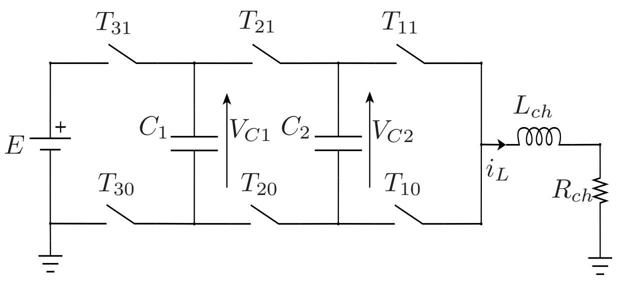

Fig. 1. The three-level three-cell dc-dc converter

Usually, in industrial applications this problem is solved using the classical control theory [4], [5]. The methods and techniques used in this paper will provide three approaches. The first one uses a passivity based control, which is a continuous approach. The second one is a stabilizing approach using the notion of the Lyapunov function. The last one is a new predictive approach over a stable limit cycle for affine hybrid systems.

The present paper is organized as follows: In Section II the physical model of the converter is presented. The Section III is devoted to the modelling for control design. Section IV contains a brief description of the control problem. In Section V the three control strategies are detailed. Section VI shows the simulation results. Finally, a short comparison between these three methods is made in Section VII.

II. PHYSICAL MODEL OF THE THREE-LEVEL THREE-CELL DC-DC CONVERTER

The circuit topology of the three-level three-cell dc-dc converter is represented in figure 1. Three commutation cells can be isolated, each one containing two switches that operate dually thus one boolean control variable $\rho_r \in \{0, 1\}$ with $r = 1, 2, 3$, is used to describe their position. $\rho_r = 1$ means that the upper switch T_{r1} is closed and the lower switch T_{r0} is open. $\rho_r = 0$ means that the upper switch T_{r1} is open and the lower switch T_{r0} is closed.

This system can be written as a hybrid system with the state vector $x = [q_{c1}, q_{c2}, p_L]^T$. q_{C1} and q_{C2} represent the charge of each capacitor and p_L the magnetic flux in the inductance. The state equation is:

$$\begin{bmatrix} \dot{q}_{C1} \\ \dot{q}_{C2} \\ \dot{p}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\rho_2-\rho_1}{L_{ch}} \\ 0 & 0 & \frac{\rho_3-\rho_2}{L_{ch}} \\ \frac{\rho_1-\rho_2}{C_1} & \frac{\rho_2-\rho_3}{C_2} & -\frac{R_{ch}}{L_{ch}} \end{bmatrix} \begin{bmatrix} q_{C1} \\ q_{C2} \\ p_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \rho_3 \end{bmatrix} E \quad (1)$$

and the constitutive relation:

$$\begin{bmatrix} V_{C1} \\ V_{C2} \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C2} & 0 \\ 0 & 0 & \frac{1}{L_{ch}} \end{bmatrix} \begin{bmatrix} q_{C1} \\ q_{C2} \\ p_L \end{bmatrix} \quad (2)$$

where $[V_{C1}, V_{C2}, i_L]^T$ are the voltages on the capacitors and the current in the load. R_{ch} is the resistance of the charge and L_{ch} its inductance. C_1 and C_2 represent the capacitances and E is a constant voltage source.

The following physical parameters corresponding to a realistic case are used:

$$\begin{aligned} E &= 1.5kV \qquad C_1 = C_2 = 40\mu F \\ L_{ch} &= 1mH \qquad R_{ch} = 10\Omega \end{aligned} \quad (3)$$

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III. Modelling for control design

A. Supelec: Passivity Based Control and Stabilisation Approach

The matrix representation of a switching system in standard PCH (Port Control Hamiltonian) formulation has the following expression:

$$\dot{x} = [J(\rho) - R(\rho)]\frac{\partial H(x)}{\partial x} + G(\rho)u \tag{4}$$

x is the state vector, ρ is the control variable, J the skew-symmetric interconnection matrix, R the symmetric dissipation matrix, H the energy stored in the system, G the power input matrix.

If the constitutive relations of the storage elements are linear, which is most often the case of power converters, the Hamiltonian of the system is:

$$\frac{\partial H(x, \rho)}{\partial x} = Fx = z \tag{5}$$

where $F = F^T > 0$ and in the simple cases, it is a diagonal matrix as in (1). Furthermore, the state equation is affine with respect to boolean control variables[6].

Due to the fact that the state equation is affine with respect to the boolean variable, the matrices $J(\rho)$, $R(\rho)$ and $G(\rho)$ can be written as:

$$J(\rho){=}J_0{+}\sum_1^p \rho_i J_i, \quad R(\rho){=}R_0{+}\sum_1^p \rho_i R_i, \quad G(\rho){=}G_0{+}\sum_1^p \rho_i G_i \tag{6}$$

where ρ_i are the components of the control vector ρ and p is its dimension.

Equation (4), which fundamentally defines an exact state representation for the switching system whose control variables are Boolean, can also be interpreted as its average model provided that the same variables are considered as continuous in the set $[0, 1]$.

B. CRAN: Predictive control approach

The class of systems studied is assumed to be described by the following affine differential equations:

$$\begin{aligned} \dot{x} &= f(x) + g(x)\rho \\ y &= h(x) \end{aligned} \tag{7}$$

where $x \in \mathbb{R}^n$ represents the state, $y \in \mathbb{R}^p$ is the output signal and ρ is an m-dimensional vector that lives in a finite set:

$$\rho \in \mathrm{U} \triangleq \{\rho_1, \rho_2, \ldots \rho_N\} \subset \{0,1\}^m, N \geq 2 \tag{8}$$

with $f(x) \in \mathbb{R}^n$, $g(x) = [g_1(x), \ldots, g_m(x)] \in \mathbb{R}^{n\times m}$, $g_i \in \mathbb{R}^n$, $i \in \overrightarrow{m} \triangleq \{1, \ldots, m\}$ and $h(x) \in \mathbb{R}^m$ are assumed to be functions of x [7]. The control problem is to find a switching policy so that the closed-loop system is internally stable, and y is regulated (as close as possible) around y_0. Due to the physical nature of the switches, the switching frequency will be bounded. Then steady state will correspond not to an equilibrium point, but rather to an oscillating trajectory around a fixed reference which can be a limit cycle.

Definition 1: A switching sequence is a finite or countable ordered set of pairs of time and active subsystem

$$(T, I)^s = \{(t_1, i_1), (t_2, i_2), \ldots, (t_s, i_s)\} \tag{9}$$

where $t_0 \leq t_1 \leq t_2 \leq \ldots t_s \leq \infty$ and each mode $i_j \in \{1, \ldots, m\}$ for $j = 1, \ldots, s, s \leq \infty$. $T = \{t_1, t_2, \ldots, t_s\}$,$I = \{i_1, i_2, \ldots, i_s\}$

IV. The Control Problem

The aim here is to find methods for controlling the system (1)-(2) with the following abilities:

(i) Minimize the oscillations at steady state.

(ii) To ensure robustness with regard to input voltage changes and eventually to load changes.

(iii) To deal with a minimum commutation time due to the physical components.

(iv) A control law with a simple implementation.

The system (1)-(2) is assumed to be fully observed. The reference values will be:

$$V_{C_1 0} = \frac{2}{3}E \qquad V_{C_2 0} = \frac{1}{3}E \tag{10}$$

leading to an optimal behaviour and a constant current in the charge:

$$i_{L0} = 100A \tag{11}$$

All the control laws described in the next section should evaluate the following cases:

1) Start-up from zero initial conditions to the reference as in (10)-(11), using nominal parameter values (3).
2) Response to input voltage variations. The converter is initially in steady state when a step change of the input voltage from $E = 1.5kV$ to $E = 1.2kV$ is applied at $t = 0.01s$ and another step change is applied from $E = 1.2kV$ to $E = 1.8kV$ at $t = 0.02s$.

There are also constraints in the commutation time of the switches $T_{k,r}$, $k = 1, 2, 3$, $r = 0, 1$. An upper bound of $16kHz$ on the commutation frequency is assumed.

V. Proposed control approaches

Three control strategies are investigated. Two of them are based on an energetic approach using the notion of stability or passivity. One is a predictive approach with a neural network will be also analysed. While the passivity based control (PBC) and the predictive approach are continuous, the stabilizing method is directly discrete.

A. Supelec: Passivity Based Control

1) General method: PBC is known as an efficient continuous technique for the regulation of switching physical systems that requires the knowledge of an average model. The state equations are usually represented under a PCH form[8].

The stages of the control synthesis:

- The control objective will be to make the observation variable defined by the following prescribed reference:

$$y = y_0 \tag{12}$$

- Let $\widetilde{x}$ be the error with

$$\widetilde{x} = x - x_0 \tag{13}$$

- Using this error vector instead of the original state vector, the average model can be rewritten as:

$$\dot{\widetilde{x}} - (J - R)F\widetilde{x} = GE - [\dot{x}_0 - (J - R)Fx_0] \tag{14}$$

- In PBC method a damping injection is performed by adding some dissipation on the error vector, by means of a matrix denoted by R_1 ($R_1 > 0$). Thus equation (14) becomes:

$$\begin{aligned} &\dot{\widetilde{x}} - (J - (R - R_1))F\widetilde{x} = \\ &= GE - [\dot{x}_0 - (J - R)Fx_0 - R_1 F\widetilde{x}] \end{aligned} \tag{15}$$

  The right hand side of this first order ordinary differential equation has to be null in order to ensure an asymptotic cancellation of the error.
- Finally, the PBC strategy leads to the following system:

$$\dot{x}_0 - [J(\rho) - R(\rho)]Fx_0 - R_1 F(x - x_0) = GE \tag{16}$$

$$y = y_0 \tag{17}$$

  These two equations define the controller dynamics under an implicit form (the variables are the control ρ and x_0).

2) Application to the three-level three-cell DC-DC converter: In that particular case equation (12) imposes the state vector completely so that the obtained controller is a static state feedback. Equations (16) and (17) become:

$$\begin{pmatrix} 0 \\ 0 \\ \rho_3 \end{pmatrix} E + \begin{pmatrix} 0 & 0 & \frac{\rho_2 - \rho_1}{L} \\ 0 & 0 & \frac{\rho_3 - \rho_2}{L} \\ \frac{\rho_1 - \rho_2}{C_1} & \frac{\rho_2 - \rho_3}{C_2} & -\frac{R_{ch}}{L_{ch}} \end{pmatrix} \underbrace{\begin{pmatrix} q_{C10} \\ q_{C20} \\ p_{L0} \end{pmatrix}}_{x_0} + \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \begin{pmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{1}{L} \end{pmatrix} \begin{pmatrix} q_{C1} - q_{C10} \\ q_{C2} - q_{C20} \\ p_L - p_{L0} \end{pmatrix} = 0 \tag{18}$$

Finally under an explicit form, the equation of the controller is:

$$\rho_1 = \varepsilon_1 \frac{q_{C1} - q_{C10}}{C_1} \frac{L_{ch}}{p_{L0}} + \rho_2 \tag{19}$$

$$\rho_2 = \varepsilon_2 \frac{q_{C2} - q_{C20}}{C_2} \frac{L_{ch}}{p_{L0}} + \rho_3 \tag{20}$$

$$\begin{aligned} \rho_3 = &\frac{1}{E} \left(\frac{R_{ch}}{L_{ch}} p_L - \varepsilon_1 \frac{(q_{C1} - q_{C1\_C})\, q_{C1\_C}}{C_1^2} \frac{L_{ch}}{p_{L\_C}} \right) + \\ &\frac{1}{E} \left(-\varepsilon_2 \frac{(q_{C2} - q_{C2\_C})\, q_{C2\_C}}{C_2^2} \frac{L_{ch}}{p_{L\_C}} - \varepsilon_3 \frac{p_L - p_{L\_C}}{L_{ch}} \right) \end{aligned} \tag{21}$$

B. Supelec: Stabilizing Approach

1) General method: The approaches in the literature which are based on Lyapunov function consider, in general, lineary systems with 0 as a common equilibrium point[9]. In the case of power converters, each configuration may or may not have a different equilibrium point and physical considerations enable establishing a common Lyapunov function.

The control objective is expressed with the help of an admissible reference, which is a value for the co-state variable $z_0 = Fx_0$ which must satisfy the constraint:

$$0 = (J(\rho_0) - R(\rho_0))\, z_0 + G(\rho_0)\, E \tag{22}$$

if there is a $\rho_0 \in \mathbb{R}^p, 0 \leq \rho_{0i} \leq 1$. According to the properties of this equation and the respective dimension or x and ρ, for one ρ_0, the equilibrium point can be unique or not, and for ρ_0 any point of the state space can be an equilibrium point or not[10]. Also this value corresponds to an equilibrium point for the average model.

For a function V to be a Lyapunov function for a system in a point x_0 it must be positive anywhere except in x_0 and its derivative must always be negative. The candidate Lyapunov function has the following form:

$$V(x, x_0) = \tfrac{1}{2}(x - x_0)^T F (x - x_0) \tag{23}$$

Because the matrix F is unique for all the modes of the system, V is positive and continuous for every x and it is null only in x_0. Its derivative depends on the control variable and using (5) and (6) it can be expressed as following:

$$\begin{aligned} \dot{V}_\rho = &-(z - z_0)^T R(\rho)(z - z_0) + \\ &+ \sum_1^p (z - z_0)^T ((J_i - R_i)\, z_0 + g_i u)(\rho_i - \rho_{0i}) \end{aligned} \tag{24}$$

Due to the fact that $R(\rho)$ is a non-negative matrix, the first term is always negative, and because $0 \leq \rho_{0i} \leq 1$ the sum can be made negative by choosing an appropriate value for each ρ_i. Multiple strategies can be envisaged for attaining this goal. Further on, a minimum switching strategy will be used, which consists in choosing a new value for the control variable each time the trajectory hits the surface defined by $\dot{V} = 0$[10].

Because this strategy requires an infinite bandwidth a dead-zone is created with the help of a parameter ϵ. The new commutation surfaces are thus defined by $\dot{V} = \epsilon$. The period and the amplitude of the oscillations around the reference are determined by this parameter.

2) Application to the three-level three-cell DC-DC converter: When setting the admissible reference, in the case of the multi-level converter, the state is completely defined, and the values for ρ_{0i} are determined by solving equation(22).

Expression (24) becomes:

$$\begin{aligned} \dot{V}_\rho = &-(i_L - i_{L0})^2 R_{ch} + (i_L v_{C20} - i_{L0} v_{C2})(\rho_1 - \rho_{01}) + \\ &+ (i_{L0}(v_{C2} - v_{C1}) + i_L(v_{C10} - v_{C20}))(\rho_2 - \rho_{02}) + \\ &+ (i_{L0} v_{C1} - i_L v_{C20} + E(i_L - i_{L0}))(\rho_3 - \rho_{03}) \end{aligned} \tag{25}$$

C. CRAN: Predictive control Approach

In this work a new scheme is proposed for control of the system (7) using predictive methods and limit cycle analysis. This proposal searches the modes I, the number of modes s and the duration of each modes. Thus, we will focus on the problem of finding the optimal limit cycle, which minimizes a cost function. This allows to select a waveform and to characterize the steady state by a function (e.g., oscillations, harmonics, error). The solution of the optimization problem is used as the cycle of reference.

There is also the tracking problem of the optimal limit cycle. For the problem of controlling a binary system (7), techniques based on a predictive method can be used. It consists in determining the best combination for arriving to a stable limit cycle [3], [11]. Usually, this can be done when a proper mathematical model is available. Using this knowledge, an analysis can be made over a set and the best possibility can be chosen. The methods and techniques proposed in this work will provide a fast approach since a neural network is used as a predictive method. Moreover, this approach only requires evaluation of simple functions.

On the other hand, some restrictions on the commutation time must be faced. In our method, this can be addressed using an optimization procedure with constrains.

Indeed, a general computational scheme to control a system with multiple binary inputs and multiple outputs is presented. The analysis will be divided into two stages: i) An open loop analysis, ii) Close loop analysis.

1) Open loop analysis: The problem of searching a limit cycle is addressed using a performance index J. The aim is to obtain the best sequence $(T, I)^s$, $1 < s < s_{max}$. Although J is a function which highly depends on the application, in this article we are concerned by the least oscillation criterion defined:

$$J(s, I, \tau) = \min_{s,I,\tau} \sum_{j=1}^{s} \| \overline{x}_j - x_0 \|_Q^L \tag{26}$$

where $\| \cdot \|^L$ represents the L-norm for $L = 1, 2, \infty$, Q is a weighting matrix which in the case of $L = 2$ is characterized by $Q = Q^T \geq 0$ and in the case of $L = 1, \infty$ is a full column rank matrix, s is the number of modes in a sequence according to definition 1, $\overline{x}_j$ is the average value of the state in the mode i_j and x_{ref} is the reference (10)-(11) for the model, the duration of the modes are represented by $\tau_j = t_{j+1} - t_j$, $j = 1, \ldots, s$

The limit cycle has to be ensured using boundary condition [12]:

$$x(t_1) = x(t_f) \tag{27}$$

This is a constraint on the state which defines the existence of a cycle with a performance index (26). However, this is not enough for most of the problems. The following time conditions also need to be verified :

$$\sum_{j=1}^{s} \tau_j < T_p \tag{28}$$

Equation (28) is a period constraint where T_p is a superior border for the duration of the cycle. There are also constraints due to the physical nature of the switches:

$$\begin{aligned} \delta_k(t_j) \geq t_{min} |\rho_k(t_j) - \rho_k(t_{j+1})| \qquad & \forall j = 1, \ldots, s \\ \dot{\delta_k}(t_j) = 1 \qquad & \forall k = 1, \ldots, N \\ \delta_k(t_{j+1}) = 0 \qquad if \qquad & |\rho_k(t_j) - \rho_k(t_{j+1})| \neq 0 \end{aligned} \tag{29}$$

The equation (29) is a boundary condition for the duration of each mode i_j, where t_{min} is a constant and represents the minimum time allowed, δ is the elapsed time from the last activation of the switch. The solution of system (7) with the feasible sequence $(T, I)^s$ which solves the constrained mathematical problem (26)-(27)-(28)-(29) is a candidate to be used as the reference $R(t)$ in close loop.

We highlight that this sequence $(T, I)^s$ is far from being useful if a proper characterization of the close loop behavior is not made. Nevertheless it can be used as the reference $R(t)$, instead of using a constant value for the average model.

2) Close loop analysis: In this subsection, a brief description of the control formulation in close loop which guarantees the robustness in the steady state and the performances in the transitory regime will be presented. The predictive control amounts to find the control sequence $(T, I)^s$, minimizing a performance index [11]:

$$\min_{I,\tau} \sum_{j=1}^{s} \|\tau_j - \tau_j^*\|_Q^L + \|x(t_j) - R(t_j)\|_Q^L \tag{30}$$

The reference $R(t)$ is found using an open loop analysis obtained as in the previous subsection. Although the formulation (30) can be properly used for all the systems written as (7). Thus, the solution of (30) is searched off-line and it is interpolated using an artificial neural network model (ANN) [13]. Then, we use an on-line ANN already trained, which gives us the partition of the state space defining a sliding mode with respect to the error ε [14].

Consider the vector ε as the input of the network:

$$\varepsilon(t_j) = \begin{bmatrix} R(t_j) - x(t_j) \\ \tau_j - \tau_j^* \end{bmatrix} \tag{31}$$

the output of the net is given by:

$$o_j = f\left(\sum_{k=1}^{m+r}\left(w_{jk} f\left(\sum_{h=1}^{n} v_{kh}\varepsilon_h\right)\right)\right) \qquad j = 1, \ldots, m \tag{32}$$

where w_{jk}, v_{kh} are the weights of the network. The dimension of the input layer corresponds to the number of state variables to control. The output layer size is defined by the number of configurations so that each node is associated with one mode and with the times of the mode. The training data for the network are the error vector ε and $x(t)$ which is the solution of (30). Different algorithms can be used to train the ANN [13].

VI. Simulation results

In this section simulation results for the two tests proposed in the section IV are presented. All the simulations have been performed using Matlab-Simulink .

A. Supelec: Passivity Based Control

The following values for the damping parameters have been used: $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 2$. A pulse width modulation (PWM) at $5KHz$ is used.

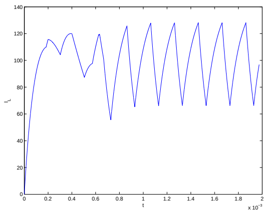

Fig. 2. Current in the load

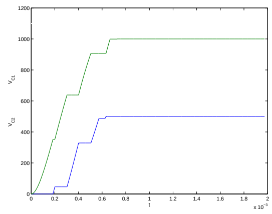

Fig. 3. Capacitor voltages

1) Start-up of the converter: The figure 2 shows the evolution of the current i_L. The duration of the settling time is of $0.8ms$ and in the stationary regime the amplitude of the oscillations is of about 23%. Their amplitude is directly related to the frequency of the PWM and can be reduced if the frequency is higher. The smallest duration between two succesive commtations of a control variable is of $0.075ms$ which corresponds to a frequency of $13.3kHz$, lower than the $16kHz$ constraint.

As for the evolution of the capacitor voltages, there is no overshot and the settling time is equal to $0.7ms$. When the control is achieved, because the three Boolean control variables have always the same values, there are no oscillations of the capacitors voltages (cf. figure 2).

2) Input voltage variations: Figures 4-5 show the current i_L and the capacitor voltage v_{C1} and v_{C2}. The control objective (10)-(11) is accomplished. Furthermore there are no oscillations of v_{C1} and v_{C2}, but there are oscillations for i_L. It can be noticed that these oscillations are proportional to E and the maximum value of 40% for their amplitude is measured when $E = 1.8kV$.

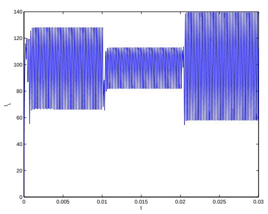

Fig. 4. Current in the load

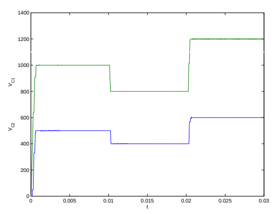

Fig. 5. Capacitor voltages

B. Supelec: Stabilizing Approach

Different commutation principles have been tested, and the one yielding the smallest commutation frequency is the one where only one pair of switches is commuted in each commutation instance. The parameter ϵ is chosen so that the oscillation frequency is as small as possible.

1) Start-up of the converter: The figures 6 and 7 show the current i_L and the capacitor voltages v_{C1} and v_{C2}. The control objective is achieved with a settling time of $0.7ms$. All the three variables present oscillations around the constant values (10)-(11), under 20% for the current and under 10% for the voltages. For the maximal possible value of ϵ (6000) the shortest duration between two consecutive commutations of the same control variable is of $0.057ms$, which corresponds to a frequency of $17kHz$.

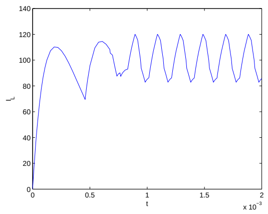

Fig. 6. Current in the load

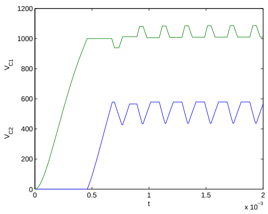

Fig. 7. Capacitor voltages

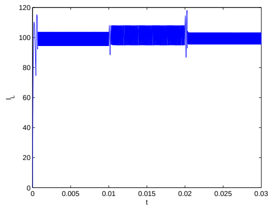

Fig. 8. Current in the load

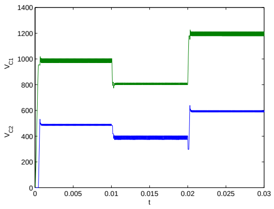

Fig. 9. Capacitor voltages

2) Input voltage variations: The figures 8 and 9 show the results for this case. The main drawback is that the maximum feasible value for the parameter ϵ, when the input voltage is $E = 1.2kV$, is less than 1000, which gives a minimum commutation frequency of around $68kHz$.

C. CRAN: Predictive Control Approach

For this approach an optimal limit cycle solving the mathematical program (26) is found with:

$$G(x) = \begin{bmatrix} -\frac{i_L}{C_1} & \frac{i_L}{C_1} & 0 \\ 0 & -\frac{i_L}{C_2} & \frac{i_L}{C_2} \\ \frac{V_{C1}}{L} & \frac{V_{C2}-V_{C1}}{L} & \frac{E-V_{C2}}{L} \end{bmatrix} \tag{33}$$

$$f(x) = \begin{bmatrix} 0 & 0 & -\frac{R}{L} i_L \end{bmatrix}^T$$

A period $T_p = 3ms$ is imposed and the available modes are shown in the table I

i_j	u_1	u_2	u_3
1	0	0	0
2	0	0	1
3	0	1	1
4	0	1	0
5	1	1	0
6	1	1	1
7	1	0	1
8	1	0	0

TABLE I

TABLE OF MODES

A sequence $(T, I)^*$ is searched, for $s_{max} = 7$, with $L = 2$ and

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

the sequence is:

$$(T, I)^* = \{(0.0625ms, 1), (0.1733ms, 6)\} \tag{34}$$

It can be noticed, from equation (34) that the optimal period which minimizes the oscillations is $T_p = 0.1733$ ms. It is also verified that the commutation constraints $t_{min} = 1/16e3 = 0.0625ms$ for each switching component is successful.

The equation (34) and the solution of the space state is used as the reference for the close loop law with the ANN. We use a back-propagation algorithm to train the ANN.

1) Start-up of the converter: Figure 10 shows the voltage on the capacitors and the reference. Figure 11 shows the current of the inductance and its reference. There are no oscillations and no overshoot of the voltage on the capacitors. As for the current, the overshoot reaches 40% and the amplitude of oscillations reaches 20%.

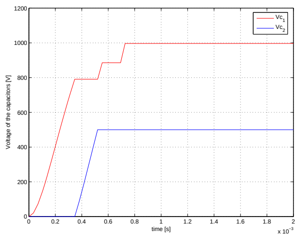

Fig. 10. Voltage on the capacitors

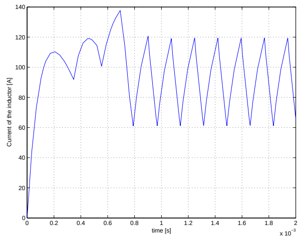

Fig. 11. Current in the charge

2) Input voltage variations: Figures 12 and 13 summarize this test. The control objective is successful. As above there are no oscillations of the voltage on the capacitors whereas the amplitude of oscillations for the current is directly related to the value of E.

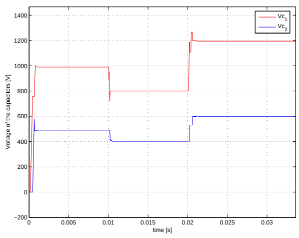

Fig. 12. Voltage on the capacitors

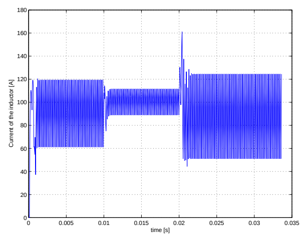

Fig. 13. Current in the charge

VII. Comparison and Conclusions

The method presented by CRAN and the passivity approach use a fixed sampling period, whereas the stabilizing approach is a method with a variable switching frequency leading to a more complex realisation. If a comparison in term of performances is to be made, it can be observed that the three approaches lead to similar results, but some differences can nevertheless be observed.

Concerning the current in the load, the limit cycle is reached after around $1ms$ and the amplitudes of the oscillations are similar: 24% in the passivity method, 10% in the stability approach and 18% in the predictive approach. Concerning the voltage on the capacitors, in PBC and predictive approach no oscillation is observed. During the limit cycle only two modes are available, corresponding to the three lower switches open or closed whereas in stabilizing approach there are small oscillations. It can be also underlined that the constraint concerning the limitation of frequency commutation to $16kHz$ is verified except for the stabilizing approach ($17kHz$). Concerning the input voltage variation, the main drawback of the stabilizing approach is that the constraint related to the commutation frequency can not be ensured ($68kHz$). For PBC, when $E = 1.2kV$ the maximum commutation frequency for a switching component is equal to $27kHz$. To avoid this problem, the PWM frequency must be reduced to $2kHz$ but in that case large oscillations appear for the current in the load (80%). In the predictive approach this is easily addressed because the commutation time is a constraint of the problem.

Finally the main difference between the three approaches lies in the control design itself. So, it would be interesting to test them on a real process in order to verify the feasibility in terms of time computation and the robustness to uncertainty as for the modelling of the operative part.

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