EVALUATION OF ESTIMATION QUALITY WITH RESPECT TO SENSORS LOSSES

SIRCOULOMB Vincent(1,2), HOBLOS Ghaleb(2), CHAFOUK Houcine(2), RAGOT José(1)

[1]Institut de Recherche en Systèmes Electroniques EMbarqués (IRSEEM)
Technopôle du Madrillet, avenue Galilée, BP 10024
F - 76 801 Saint Etienne du Rouvray Cedex
{sircoulomb, hoblos, chafouk}@esigelec.fr

[2]Centre de Recherche en Automatique de Nancy (CRAN)
Institut National Polytechnique de Lorraine, UHP, UMR 7039 CNRS
2 avenue de Haye, F - 54 516 Vandœuvre lès Nancy
jose.ragot@ensem.inpl-nancy.fr

Abstract : This paper deals with the estimation quality of nonlinear filters in relation to sensors losses. In a nonlinear state space representation setting, Central Difference Kalman Filter, Ensemble Kaman Filter and Particle Filter are tested on a second order system. Their comportment in relation to the sensors available are then studied, in order to compare the estimation quality by computing criteria such that variances of estimation errors and observability gramian.

Résumé : Cet article étudie la qualité d'estimation de filtres non linaires en fonction de la perte de capteurs. Dans un environnement non linéaire décrit par une représentation d'état, les filtres de Kalman à différence centrale, les filtres de Kalman d'ensemble et les filtres particulaires sont testés sur un système d'ordre deux. Leur comportement relativement aux capteurs disponibles est alors étudié, dans le but de comparer leur qualité d'estimation, en calculant des critères tels que la variance des erreurs d'estimation ainsi que les grammiens d'observabilité.

Keywords : Nonlinear, Kalman Filter, Monte Carlo filter, Observability, Gramian.

Mots clés : Non linéaire, Filtre de Kalman, Filtre de Monte Carlo, Observabilité, Grammien.

1 Introduction

The growing demand for fault tolerance in more and more complex automatic control systems can only be reached using efficient Fault Detection and Isolation (FDI) algorithms and reconfiguration concepts. Consequently, estimation techniques are more and more used in fault detection and data reconciliation of complex industrial processes, like power, aeronautic or chemical ones [Ochi 91] [Ragot 05]. For example, an estimator bank can be used in order to generate residuals sensitive to faults [Sircoulomb 05]. These residuals are then explored to detect and isolate faults, and to reconfigure the control law or the FDI algorithm.

As most of complex systems are nonlinear, acceptable amplitudes of residuals directly depend on the accuracy of the estimators and their robustness, i.e. estimated values must be the closest to the real values. Moreover, if a sensor loss occurs and the system is still observable, the conservation of the FDI algorithm performance will depend on the way the estimators are affected by this lost information. The purpose of this article is to study the estimation quality of nonlinear filters in relation to sensors losses. In this paper, a problem statement is first presented in section 2. For evaluating estimation quality, section 3 proposes some criteria, based on variance and observability Gramian. Then, the nonlinear filters tested are given in section 4. Subsequently, the object of section 5 is the implementation of these estimators on a strongly nonlinear system, in order to analyse and compare their comportment in relation to sensors losses.

2 Problem statement

Consider the following nonlinear discrete time system:

$$\begin{cases} x(k) & = f(x(k-1), u(k-1), w(k-1), k-1) \\ y(k) & = g(x(k), u(k), k) + v(k) \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^{n_x}$ is the state vector, $u(k) \in \mathcal{R}^{n_u}$ the control input vector, $y(k) \in \mathcal{R}^{n_y}$ the measurement vector, $w(k) \in \mathcal{R}^{n_w}$ the process noise, $v(k) \in \mathcal{R}^{n_v}$ the measurement noise and $k \in \mathcal{N}$ the (discrete) time.

Suppose now that a sensor loss is detected and isolated by the FDI algorithm. The accommodation possibility depends on the observability of the system by the new set of sensors (i.e. all sensors except the faulty one). So, it is useful to analyse the observability of the system by each possible sensor subset. This analysis can then be represented by an oriented graph [Staroswiecki et al.]. In such a graph, a node represents a sensor subset and an edge between two nodes means a sensor loss. A color is associated to each node: grey if the system is observable thanks to this sensor subset; otherwise, the color is white. Figure (1) illustrates such a graph for a sensor set a, b, c, d of cardinal 4.

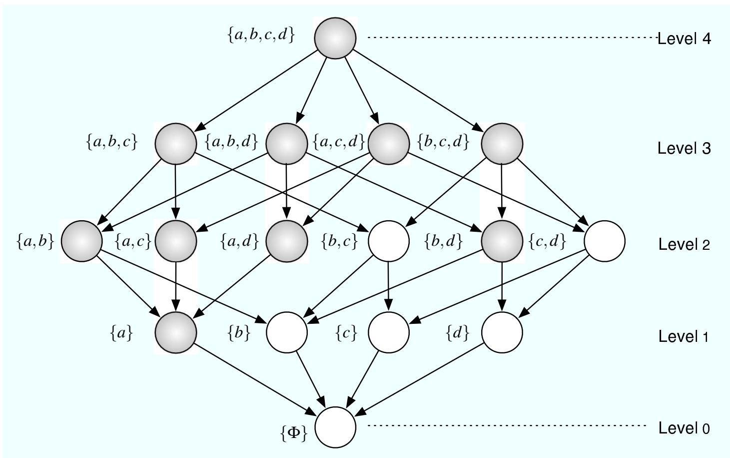

Figure 1: Example of an oriented graph

On the example described on figure (1), as each node of level 3 is grey, the system is still observable even if a sensor becomes faulty, whatever it is. On the one hand, the accommodation task is then possible, and a necessary condition for using a modified FDI algorithm is checked. On the other hand, this sensor loss also means that less information are available for estimating the state and the output of the system, resulting in less accurate estimations. Thus, the residual amplitudes will increase, leading to two different problems. Firstly, the risk of false alarms will augment if the detection threshold is not readapted. Secondly, it proves to be impossible to detect a low amplitude fault, which becomes non significant compared to residual amplitude. The purpose of this article is then to study the estimation quality of some nonlinear filters in relation to sensors losses.

3 Quality estimation criteria

There are different criteria for evaluating the quality of a state filter estimation. By the following, we will only use the filter variance and the observability gramian.

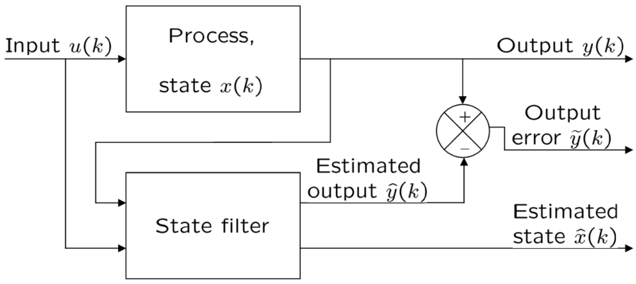

Figure 2: Filtering principle

3.1 Filter variance

In the following, the filter variance is taken as the Euclidian norm of the estimation error $\tilde{y}(k)$ on the system output : $V(k) = \parallel \tilde{y}(k) \parallel$. The way of computing this value is described on figure (2).

When the variances are mentioned without the notion of time dependency, it means that we consider the average of the variance over the simulation length, denoted L:

$$V = \frac{1}{L} \sum_{k=1}^{L} V(k) \tag{2}$$

3.2 Observability gramian

The observability gramian $G(k)$ is a $n_x \times n_x$ positive semi definite symmetric matrix, solution of the following Lyapunov equation [Wu 00]:

$$A(k)G(k) + G(k)A^T(k) + C(k)C^T(k) = 0 \tag{3}$$

where the Jacobians $A(k)$ and $C(k)$ are computed as follow:

$$A(k) = \frac{\partial f}{\partial x} \mid_{x=\hat{x}(k)} \qquad C(k) = \frac{\partial g}{\partial x} \mid_{x=\hat{x}(k)} \tag{4}$$

Let $\lambda_i(k)$ be the ith eigenvalue of $G(k)^{-1}$ and $\lambda_{max}(k)$ the greatest eigenvalue of $G(k)^{-1}$:

$$\lambda_{max}(k) = \max_{1 \leq i \leq n_x} \lambda_i(k) \tag{5}$$

Lower $\rho(k) = \sqrt{\lambda_{max}(k)}$ is, better the quality of the considered filter is [Staroswiecki 02]. Similarly for the variance, ρ represents the average of $\rho(k)$ over the simulation length:

$$\rho = \frac{1}{L} \sum_{k=1}^{L} \rho(k) \tag{6}$$

4 Nonlinear filtering methods

In a state space representation setting, the most popular estimator is the Kalman filter [Kalman 60], also known as linear Gaussian optimal filter. For nonlinear systems, recent works provided interesting results, with the Unscented Kalman Filter (UKF) [Julier 97], Central Difference Kalman Filter ($CDKF$) [Norgaard 00] and Ensemble Kalman Filter ($EnKF$) [Burgers 98]. These three estimators overperform the classical Extended Kalman Filter (EKF), but are based on empirical developments [Julier 94]. A more general setting is provided by Monte Carlo filters, also called Particle Filters (PF) [Doucet 98]. This kind of tool is more powerful, but also more time-consuming and difficult to synthesize. This section firstly describes the optimal filtering problem. After, the filters tested are presented, and their tunings entered.

4.1 Optimal filtering problem

The optimal state filter is described by the probability density $p(x(k) \mid y(1:k-1))$ [Anderson 79], which can be recursively calculated by the optimal Bayesian filtering equations:

$$p(x(k) \mid y(1:k-1) = \int p(x(k) \mid x(k-1))p(x(k-1) \mid y(1:k-1)dx(k-1) \tag{7a}$$

$$p(x(k) \mid y(1:k)) = \frac{p(y(k) \mid x(k))p(x(k) \mid y(1:k-1))}{\int p(y(k) \mid x(k) \mid y(1:k-1))dx(k)} \tag{7b}$$

where $y(1:k-1)$ represents the output stacked from time 1 to time $k-1$. Then, the state can be calculated thanks to one of the two optimality criteria:

o Least squares:

$$\hat{x}(k) = E(x(k) \mid y(1:k)) = \int x(k)p(x(k) \mid y(0:k)dx(k) \tag{8}$$

o Maximum likelihood:

$$\hat{x}(k) = arg\{\max_{x(k)} p(x(k) \mid y(0:k-1))\} \tag{9}$$

Unfortunately, the equations (7) can't analytically be solved, excepted in the Gaussian case, where it leads to the Kalman filter. In the other cases, these equations can only be computed by Monte Carlo simulation, that's what particle filters do.

4.2 Filters tested

The nonlinear Kalman filters tested in this article are the $CDKF$ and $EnKF$. Concerning the PF, we will restrict our choice to the simplest, which is also the most popular, i.e. using the transition kernel for importance density. The details of the algorithms used by these filters can be found in [Sircoulomb 06]. Concerning the choice of the filters parameters, we also adopt the same values, i.e. 500 particles for the $EnKF$ and 1000 for the PF. The PF will also estimate the state via the least squares optimality, and proceed to resampling step thanks to systematic resampling and entropy based indicator.

5 Results and discussion

In this section, we first describe the system tested and analyse its observability. Then, we compare how accurately each filter can estimate the system state. This comparison is done for each possible sensors subset.

5.1 System under consideration

Description of the system. The system we choose to study is an extension of a second order system commonly used in the particle filtering community [Doucet 98]. It is described by the following equations:

$$\left\{\begin{aligned} x_1(k) &= \tfrac{1}{2}x_1(k-1) + \frac{25x_1(k-1)}{1+x_1^2(k-1)} + 8\cos(1.2k) + x_2(k-1) + w_1(k-1) \\ x_2(k) &= 8\sin(x_1(k-1)) + 8\sin(1.2x_2(k-1)) + w_2(k-1) \\ y_1(k) &= \frac{x_1(k)}{20} + v_1(k) \\ y_2(k) &= x_2(k) + v_2(k) \end{aligned}\right. \tag{10}$$

where $x_1(k)$ and $x_2(k)$ are the two components of the state $x(k)$ at time k, and $y_1(k)$ and $y_2(k)$ the measurements done by two sensors arranged on the system. $v(k) = \begin{pmatrix} v_1(k) & v_2(k) \end{pmatrix}^T$ and $w(k) = \begin{pmatrix} w_1(k) & w_2(k) \end{pmatrix}^T$ are the measurement and process noises. These noises are zero mean, normally distributed, with respective covariance R_{vv} and R_{ww}.

Observability of the system under test. As each component of $x(k)$ is linearly measured, it is obvious that the system is observable. Now, let $\mathcal{O}_1(k)$ and $\mathcal{O}_2(k)$ be the observability matrix of the linearized system, observed with only sensor 1 (sensor 2). These matrices are given on equation (11).

$$\mathcal{O}_1(k) = \begin{pmatrix} c_1 & 0 \\ c_1 a_{11} & c_1 a_{12} \end{pmatrix} \quad \mathcal{O}_2(k) = \begin{pmatrix} c_2 & 0 \\ c_2 a_{21} & c_2 a_{22} \end{pmatrix} \tag{11}$$

where $A(k) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $C(k) = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$ are the Jacobians described in section 32 which are here defined by:

$$a_{11} = \frac{x_1^4(k) - 48x_1^2(k) + 51}{2x_1^4(k) + 4x_1^2(k) + 2} \quad a_{21} = 8\cos(x_1(k)) \quad c_1 = \frac{1}{20}$$
$$a_{12} = 1 \quad a_{22} = 9.6\cos(x_2(k)) \quad c_2 = 1$$

Thanks to Kalman criterion, we can say that the system is locally observable around $\hat{x}(k)$ by the sensor i if $\mathcal{O}_1(k)$ is a full rank matrix (i.e. $rank\ \mathcal{O}_1(k) = n_x = 2$). One can easily check that $x(k) \in \mathcal{R}^2$, $rank\ \mathcal{O}_1(k)) = 2$ and $rank\ \mathcal{O}_2(k)) = 2 \Leftrightarrow \mathrm{x}_1(k) \neq \frac{\pi}{2} + n\pi, n \in \mathcal{Z}$. We will suppose that these very particular values are never reached by $x_1(k)$. So, as it is locally observable at every point of the state space, the system (11) is globally observable when using one of the two sensors (figure 3).

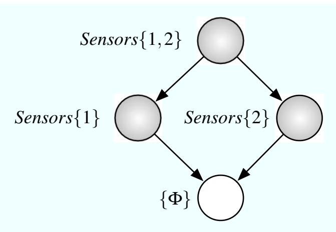

Figure 3: Observability graph of the system tested

Comparison of filters performances. Consider the following covariance matrices: $R_{ww} = I$, $R_{vv} = aI$, where $a \in \mathcal{R}^*$ and I denotes the identity matrix of appropriate size. The noise covariances are supposed known. Consequently, the filter covariances can be set to these values. By the following, V_{Filter} (6), with $Filter = \{CDKF, EnKF, PF\}$, will denote the variance of the considered filter. ρ_{Filter} (7) will stand for the square root of the greatest eigenvalue of the observability gramian inverse matrix, calculated thanks to the values given by the considered filter.

Results obtained using the two sensors. The different values of V_{Filter} and ρ_{Filter} in relation to a are given on tables (1) and (2).

a	1	10	50	100	500	1000
V_{CDKF}	7.78	9	12.15	14.84	28	39.65
V_{EnKF}	0.8	2.9	7.63	11.25	25.88	37.91
V_{PF}	1.31	3.07	7.59	11.32	25.94	37.96

Table 1: Filters variance in function of measurement noise covariance, using sensors 1 and 2

a	1	10	50	100	500	1000
ρ_{CDKF}	8.82	9.72	10.22	10.05	9.7	9.39
ρ_{EnKF}	8.54	9.28	8.85	10.22	10.11	9.59
ρ_{PF}	8.56	9.74	9.49	9.64	9.97	10.31

Table 2: Filters ρ in function of measurement noise covariance, using sensors 1 and 2

Results obtained using one sensor. Now, let study the quality estimation of the filters in relation to a, but when using only one sensor. The results obtained with sensor 1 are presented in tables (3) and (4), and those given with sensor 2 are exposed in tables (5) and (6).

a	1	10	50	100	500	1000
V_{CDKF}	7.83	9.27	12.71	15.28	29.54	41.04
V_{EnKF}	6.66	8.09	10.93	14.23	29.21	40.94
V_{PF}	6.85	8.15	10.96	14.24	29.25	40.92

Table 3: Filters variance in function of measurement noise covariance, using sensor 1

a	1	10	50	100	500	1000
ρ_{CDKF}	106	126.5	123.62	127.2	113.77	108.09
ρ_{EnKF}	376	321.97	260.27	254.5	209.66	242.3
ρ_{PF}	156	175.35	181.44	268.59	235.05	277.67

Table 4: Filters ρ in function of measurement noise covariance, using sensor 1

5.1.1 Analysis of the results obtained

From these tables, we can say that the results provided by the PF and $EnKF$ are quite similar, whatever the situation. Moreover, in a normal functioning (i.e. with two sensors available), they overperform the $CDKF$ for a low measurement noise covariance, leading to the same results than those presented in [Sircoulomb 06]. The variations of filters variance and values of ρ are provided by figures (4) and (5).

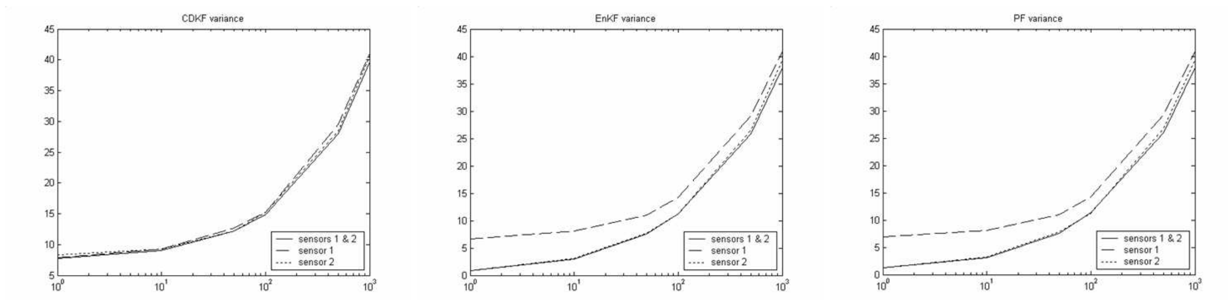

Figure 4: Evolution of filters variance for each sensors combination, in respect to a

On the one hand, according to the evolution of ρ and the filters variance, we can clearly see that the loss of sensor 1 doesn't affect very much the filters accuracy. On the other hand, a default on sensor 2 will penalize the estimations, as attest the elevation of ρ value, and PF and $EnKF$ variance augmentation. Lastly, thanks to figure (5), we can also notice that the $CDKF$ doesn't seem to be affected by any sensor loss. These results are confirmed on figures (6) and (7), for a low covariance level (: the PF and $EnKF$ estimations are very precise if both sensors are available (figures 6c)and 7c), and a little bit affected if only sensor 2 is available (Figures 6b and 7b). If only sensor 1 is safe, the PF' and $EnKF$ accuracy for estimating tend to be of the same quality than the $CDKF$ one (Figure 6a). Contrary to the $CDKF$ one, their estimations of are always close to 0 (Figure 5a).

a	1	10	50	100	500	1000
V_{CDKF}	8.37	9.32	12.15	15.21	28.48	40.68
V_{EnKF}	0.84	3.07	7.73	11.24	26.57	39.37
V_{PF}	1.31	3.26	7.81	11.3	26.72	39.41

Table 5: Filters variance in function of measurement noise covariance, using sensor 2

a	1	10	50	100	500	1000
ρ_{CDKF}	13.22	12.67	14.94	13.64	12.07	15.63
ρ_{EnKF}	13.61	14.01	12.88	15.13	14.36	15.47
ρ_{PF}	14.69	14.52	13.65	14.86	14.91	16.45

Table 6: Filters ρ in function of measurement noise covariance, using sensor 2

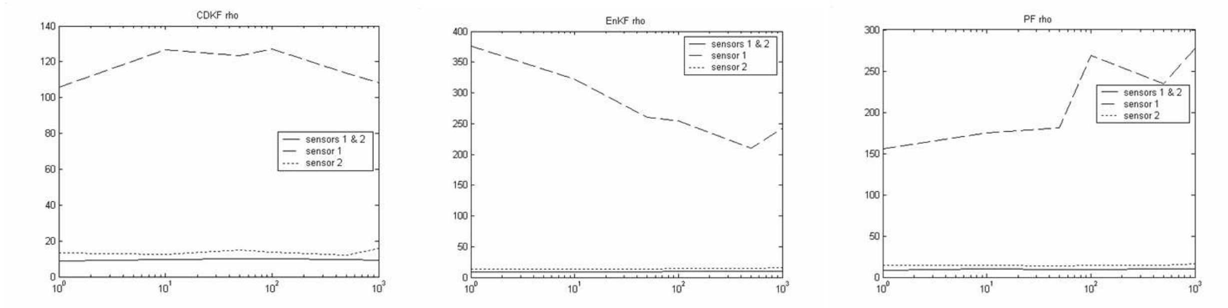

Figure 5: Evolution of filters value of ρ for each sensors combination, in respect to a

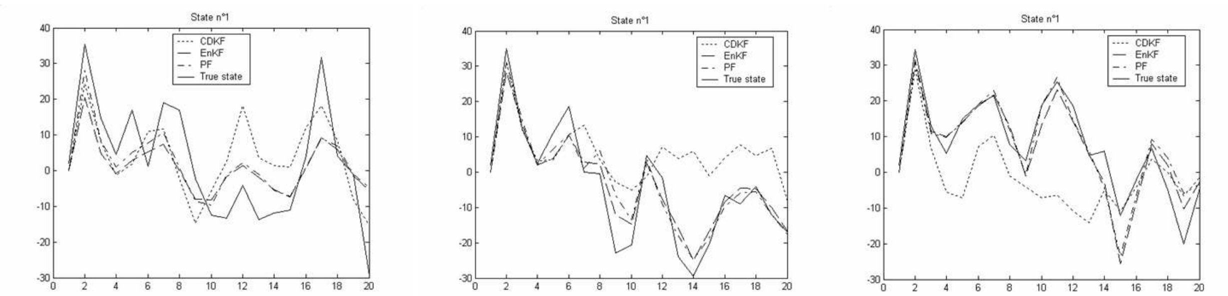

Figure 6: Real and estimated, 1st state. From left to right: with sensor 1, sensor 2, both sensors

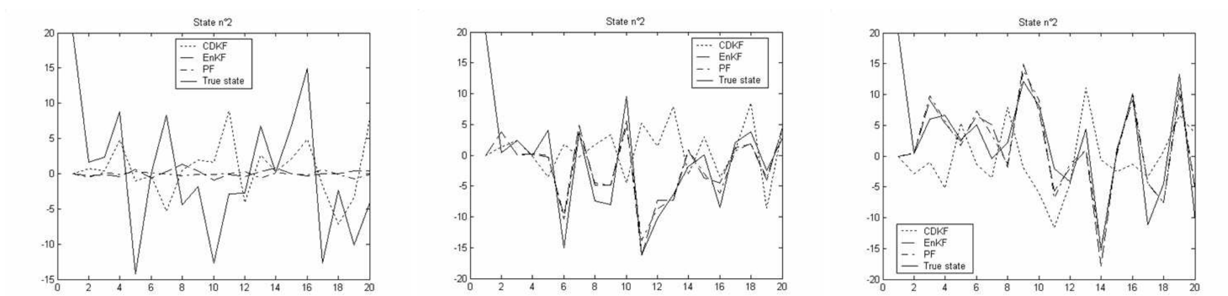

Figure 7: Real and estimated, 2nd state. From left to right: with sensor 1, sensor 2, both sensors

6 Conclusion

On the system tested, the filters accuracies are not affected by the loss of sensor 1. But, the loss of sensor 2 depreciates the quality estimations of the PF and $EnKF$, especially on the second state component. The $CDKF$ is the only one filter, which conserves its performances in relation to sensors losses. So, the $EnKF$ is proving to be the best choice, because on one hand, it always provides better or equivalent results than the $CDKF$ (especially in the case of low measurement noise). On the second hand, its estimation quality is approximately the same than the PF one, with only half of particles (so a less important computation time), and on top of that, is simpler to parameterize. As outlook of this work, two points can be distinguished. First, it is to improve the $EnKF$ by computing the square root of the covariance matrices instead of theses matrices. Second, it is to study the sensitivity of generated residuals using these filters with respect to sensors faults, with an adaptive threshold.

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