Fault detection and isolation with robust principal component analysis

Yvon THARRAULT, Gilles MOUROT, José RAGOT

Abstract— **Principal component analysis (PCA) is a powerful fault detection and isolation method. However, the classical PCA which is based on the estimation of the sample mean and covariance matrix of the data is very sensitive to outliers in the training data set. Usually robust principal component analysis was applied to remove the effect of outliers on the PCA model. In this paper, a fast two-step algorithm is proposed. First, the objective was to find a robust PCA model that could be used for outliers detection and isolation. Hence a scale-M estimator [1] is used to determine a robust model. This estimator is computed using an iterative re-weighted least squares (IRWLS) procedure. This algorithm is initialized from a very simple estimate derived from a one-step weighted variance-covariance estimate [2]. Second, structured residuals are used for multiple fault detection and isolation. These structured residuals are based on the reconstruction principle and the existence condition of such residuals is used to determine the detectable faults and the isolable faults. The proposed scheme avoids the combinatorial explosion of faulty scenarios related to multiple faults to consider. Then, this procedure for outliers detection and isolation is successfully applied to an example with multiple faults.**

I. INTRODUCTION

Principal component analysis (PCA) has been applied successfully in the monitoring of complex systems [3], [4]. It enables the determination of the redundancy relationships which are then used to detect and isolate faults. It transforms the data to a smaller set of variables which are linear combinations of the original variables while retaining as much information as possible. In the classical approach, the principal components correspond to the directions in which the projected observations have the largest variance. The principal components, correspond to the eigenvectors of the empirical covariance matrix. From a regression point of view, PCA also constructs the optimal orthogonal linear projections (in terms of mean squared error) from the eigenvectors of the data covariance matrix. The performance of PCA model is then based on the accurate estimation of the covariance matrix from the data which is very sensitive to abnormal observations.

In general, the majority of the training data set is associated with normal operating conditions. The remaining data (faulty data, data obtained during shutdown or startup periods or data issued from different operating mode) are referred to as "outliers". They disturb the correlation structure of the "normal data" and then the PCA model does not accurately represent the process. In practice one often tries to detect outliers using diagnostic tools starting from a classical fitting method. However, classical methods can be affected by outliers so strongly that the resulting fitted model does not allow to detect the true outliers (masking and swamping phenomena). To avoid these effects, the goal of robust PCA methods is to obtain principal components that are not influenced much by outliers.

Several ways of robustifying principal components have been proposed. They can be grouped as follows. A first group of robust PCA methods is obtained by replacing the classical covariance matrix by a robust covariance estimator, such as the minimum covariance determinant (MCD) estimator [5]. This computer-intensive method has a user-defined parameter which has to be optimized with respect to the number of outliers (unknown). A second approach to robust PCA uses Projection Pursuit (PP) techniques. These methods maximize a robust measure of data spread to obtain consecutive directions on which the data points are projected [6], [7]. However, to make these algorithms computationally feasible, the robust directions obtained are approximations of the true ones. Last proposals for robust PCA consists in minimizing a robust scale of the orthogonal distances of each observation to the PCA subspace, similar to least trimmed squares (LTS) estimator, scale-M estimator [1]. These methods are easy to compute but are based on iterative procedures for which it remains the problem of starting values.

Our presentation is devoted to the problem of sensor fault detection and isolation in data. In this paper, a fast two-step algorithm is proposed. First, a scale-M estimator [1] is used to determine a robust model. This estimator is computed using an iterative re-weighted least squares (IRWLS) procedure. This algorithm is initialized from a very simple estimate derived from a one-step weighted variance-covariance estimate [2]. Second, structured residuals are used for multiple fault detection and isolation. These structured residuals are based on the reconstruction principle. The variable reconstruction approach assumes that each set of faulty variables is unknown and suggests to reconstruct these variables using the PCA model from the remaining variables [8]. If the faulty variables are reconstructed, the fault effect is eliminated. This property is useful for fault isolation. Moreover instead of considering the isolation of one up to all sensors, we determine the maximum number of faulty scenarios to take into account by evaluating the existence condition of structured residuals. Note that this number is usually much less than the number of total sensors. The proposed scheme avoids the combinatorial explosion of faulty scenarios related to multiple faults to consider.

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The authors are with Centre de Recherche en Automatique de Nancy (CRAN), Nancy-Université, CNRS, 2, Avenue de la forêt de Haye. F-54516 Vandoeuvre-lès-Nancy yvon.tharrault@ensem.inpl-nancy.fr

Section 2 is a short reminder, on one hand, of the principal component analysis in the traditional case and, on the other hand, of the proposed robust principal component analysis. A detection and isolation procedure for outliers is proposed in section 3. Then, in section 4, this method is applied to an example emphasizing the generation of fault signatures.

II. PCA FAULT DETECTION AND ISOLATION

Let us consider a data matrix $X \in \Re^{N\times n}$, with row vector x_i^T, which gathers N measurements collected on the n system variables.

A. Classical approach

In the classical *PCA* case, data are supposed to be collected on a system being in a normal process operation. *PCA* determines an optimal linear transformation of the data matrix X in terms of capturing the variation in the data:

$$T = XP \quad \text{and} \quad X = TP^T \tag{1}$$

with $T \in \Re^{N\times n}$ the principal component matrix and the matrix $P \in \Re^{n\times n}$ contains the principal vectors which are the eigenvectors associated to the eigenvalues λ_i of the covariance matrix (or correlation matrix) Σ of X:

$$\Sigma = P\Lambda P^T \qquad \text{with} \qquad PP^T = P^TP = I_n \tag{2}$$

where $\Lambda = diag(\lambda_1 \dots \lambda_n)$ is a diagonal matrix with diagonal elements in decreasing magnitude order.

The relations (1) are useful when the dimension of the representation space is reduced. Once the component number ℓ to retain is determined, the data matrix X can be approximated. For that, the eigenvector matrix is partitioned into the form:

$$P = \begin{pmatrix} \hat{P} & \tilde{P} \end{pmatrix} \qquad \hat{P} \in \Re^{n\times \ell} \tag{3}$$

From the decomposition (1), $\hat{X}$ is the principal part of the data explained by the ℓ first eigenvectors and the residual part $\tilde{X}$ is explained by the remaining components:

$$\hat{X} = X\hat{P}\hat{P}^T = XC_\ell \tag{4}$$

$$\tilde{X} = X - \hat{X} = X(I - C_\ell) \tag{5}$$

where the matrix $C_\ell = \hat{P}\hat{P}^T$ is not equal to the identity matrix, excepted in the case $\ell = n$.

Hence the residual r_i, for $i = 1..N$, is defined as follows:

$$r_i = ||\tilde{P}^T x_i - \tilde{P}^T \mu||^2 \tag{6}$$

where μ correspond to the mean of the data X.

Choosing P as the eigenvectors of the covariance matrix is equivalent as minimizing the function φ of the estimation error with the constraint $P^TP = I$ such as:

$$\varphi = \frac{1}{N}\sum_{i=1}^{N} r_i \tag{7}$$

B. Robust approach

Our approach consists in carrying out *PCA* directly on the data possibly contaminated by outliers. For that, a simple robust estimator, called scale-M estimator, is used. However, this estimator is computed by an iterative procedure. Then, good initialization parameters are needed to avoid local minimum. To initialize this scale-M estimator a robust covariance matrix is first calculated with a low computational cost [9].

1) Robust covariance: Ruiz-Gazen (1996) define a "local" matrix of variance in the sense that the suggested form tends to emphasize the contribution of close observations in comparison with distant observations (outliers). The matrix is defined in the following way:

$$T = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{i,j}(x_i - x_j)(x_i - x_j)^T}{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{i,j}} \tag{8}$$

where the weights $w_{i,j}$ themselves are defined by:

$$w_{i,j} = \exp\left(-\frac{\beta}{2}(x_i - x_j)^T\Sigma^{-1}(x_i - x_j)\right) \tag{9}$$

β being a tuning parameter to reduce the influence of the observations faraway, the authors recommend a value close to 2. For $\beta = 0$, the robust covariance matrix T is equal to 2Σ. And for a high value of β, only the closest observations are taken into account in the robust covariance matrix T.

2) Scale-M estimator: Two M-estimators are used, one for estimation of the objective function φ (7) and another one for the estimation of the robust residual scale. The general scale-M estimator minimizes the following objective function with the constraint $P^TP = I$ [1]:

$$\frac{1}{N}\sum_{i=1}^{N}\rho\left(\frac{r_i}{\hat{\sigma}}\right) \tag{10}$$

with r_i the residual defined by equation (6), $\hat{\sigma}$ the robust scale of the residual r_i and the function ρ:$\Re^+ \to [0,1]$ is nondecreasing, with $\rho(0) = 0$ and $\rho(\infty) = 1$, and differentiable. $\tilde{P}$ is the eigenvector matrix of the robust covariance matrix C (12) corresponding to its $n-\ell$ smallest eigenvalues. Then the weighted mean μ and the covariance C are defined as follows:

$$\mu = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i} \qquad \text{with } w_i = \dot{\rho}\left(\frac{r_i}{\hat{\sigma}}\right) \tag{11}$$

$$C = \sum_{i=1}^{N} w_i(x_i - \mu)(x_i - \mu)^T \tag{12}$$

Then the scale factor $\hat{\sigma}$ is defined as the solution of:

$$\frac{1}{N}\sum_{i=1}^{N}\rho\left(\frac{r_i}{\hat{\sigma}}\right) = \delta \tag{13}$$

with $\delta \in (0,1)$.

Then an iterative algorithm is necessary to determine all these parameters. To avoid local minimum a good initialization is needed. Here the robust covariance matrix T (8) is used to determined the values of the initial parameters.

The algorithm is described as follows:

1) $it = 1$ and $\sigma_0 = \infty$
2) Compute $\tilde{P}$ the eigenvector matrix of the robust covariance matrix T corresponding to its $n-\ell$ smallest eigenvalues.
3) Compute $a = median(X\tilde{P})$
4) Compute $\sigma = trace(\sqrt{\tilde{P}^T T \tilde{P}})$
5) Do until $it = N_1$ or $\Delta \leq tol$
   a) Compute $r_i = ||\tilde{P}x_i - a||^2$ for $i = 1...N$
   b) If $it > 1$, compute σ from (13)
   c) Set $\Delta = 1 - \sigma/\sigma_0$ and $\sigma_0 = \sigma$
   d) Compute the $w_i = \dot{\rho}(r_i/\hat{\sigma})$ for $i = 1...N$
   e) Compute μ from (11)
   f) Compute C from (12)
   g) $\tilde{P}$ the eigenvector matrix of the covariance matrix C corresponding to its $n-\ell$ smallest eigenvalues.
   h) Compute $a = \tilde{P}^T \mu$
   i) Set $it = it + 1$
6) End do.

In the experiments of this article, ρ, defined by equation (10), was chosen as the bisquare function (r represents the squared distances).

$$\rho(r) = min\{1, 1-(1-r)^3\} \tag{14}$$

The constant δ in equation 13 is chosen as defined by Maronna (2005):

$$\delta = \frac{N-n+\ell-1}{2N} \tag{15}$$

From this new model, detection and isolation of outliers are carried out using the reconstruction principle.

III. Fault detection and isolation

The variable reconstruction approach assumes that a group of variables may be faulty and suggests to reconstruct the assumed faulty variables using the PCA model from the remaining variables [8]. This reconstructed variables are then used to detect and isolate the faults. Moreover these principle allows to determine replacement values for the faulty variables.

A. Data reconstruction

The *PCA* model being known according to (4) and (5), a new measurement vector x (which may contains abnormal values) can be decomposed as below:

$$x = \hat{x} + \tilde{x} \quad , \quad \hat{x} = C_\ell\, x \quad , \quad \tilde{x} = (I - C_\ell)\, x \tag{16}$$

where $\hat{x}$ and $\tilde{x}$ are respectively the projections of x onto the principal space and the residual space.

The reconstruction of variables consists in estimating the reconstructed vector $\hat{x}_R$ by eliminating the effect of the faults. Matrix Ξ_R indicates the reconstruction directions. This matrix is orthonormal with dimension ($n \times$ r), with r the number of component to reconstruct, and is built with 0 and 1, where 1 indicates the reconstructed variables from the other variables (with 0). For example, to reconstruct the set of variables $R = \{2,4\}$ among 5 variables, matrix Ξ_R is formed as follows:

$$\Xi_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

The expression for the reconstruction $\hat{x}_R$ of the vector x is given by:

$$\hat{x}_R = \left(I - \Xi_R(\tilde{\Xi}_R^T \tilde{\Xi}_R)^{-1} \tilde{\Xi}_R^T\right)\, x = G_R\, x \tag{17}$$

where $\tilde{\Xi}_R = (I - C_\ell)\ \Xi_R$

Condition of reconstruction:
The condition for fault detection is the same as for fault reconstruction. Indeed, the fault direction projection onto the residual space have to be different from 0 ($\tilde{\Xi}_R \neq 0$) for allowing fault reconstruction. This condition involves that the number of reconstructed variables r have to respect the inequality (18) and that the columns of the matrix $\tilde{\Xi}_R$ are neither null nor collinear:

$$n - \ell - r \geq 1 \tag{18}$$

B. Structured residual generation

In a diagnosis objective, residuals are generated for fault detection and isolation. The residuals are obtained by projecting the reconstructed variables onto the residual space. Residuals are defined by $\tilde{x}_R$, projection of $\hat{x}_R$ onto the residual space:

$$\begin{aligned} \tilde{x}_R &= (I - C_\ell)\,\hat{x}_R = (I - C_\ell)\, G_R x \\ \tilde{x}_R &= P_R^{(\ell)} x \end{aligned} \tag{19}$$

Property 1. Matrix $P_R^{(\ell)}$ has the following property:

$$P_R^{(\ell)} \Xi_R = 0 \tag{20}$$

Considering (19) and (20), it means that the components of $\tilde{x}_R$ are not sensitive to the components of x belonging to the subset R. This property can be used to identify which components of x are disturbed by faults.

Proof: Considering a measurement x composed with the true value x^*, a noise ε with zero mean and one fault with amplitude d and direction Ξ_F, where F is a subset containing the indices of the fault directions:

$$x = x^* + \varepsilon + \Xi_F d \tag{21}$$

then the residual is:

$$\tilde{x}_R = P_R^{(\ell)}(x^* + \varepsilon + \Xi_F d) = P_R^{(\ell)}(\varepsilon + \Xi_F d) \tag{22}$$

with $P_r^{(\ell)} x^* = 0$ and its expected value is:

$$E(\tilde{x}_R) = P_R^{(\ell)} \Xi_F d \tag{23}$$

- If the reconstruction directions Ξ_R are the same as the fault directions, i.e. if $R = F$, then all components of the vector $P_R^{(\ell)} \Xi_F$ are null and $E(\tilde{x}_R) = 0$
- If the reconstruction directions Ξ_R are different from the fault directions, then all components of the vector $P_R^{(\ell)} \Xi_F$ are a priori not null except the components belonging to the subset R.

The analysis of the residual amplitudes $\tilde{x}_R$ for all possible combinations shows the presence of faults and makes it possible to determine the components of the measurement vector affected by this fault.

For fault detection, a global indicator SPE_R (norm of the projection vector) is calculated for each observation:

$$SPE_R = \| \tilde{x_R} \|^2 \tag{24}$$

A fault is detected, if $SPE > \delta_\alpha^2$ with δ_α^2 the detection threshold of SPE. SPE correspond to the case without reconstruction *i.e.* $R = \{\phi\}$. Then, the faulty variables of the subset $\hat{R}$ are determined as follows:

$$\hat{R} = \underset{R \in \Im}{\arg} \, SPE_R < \delta_\alpha^2 \tag{25}$$

with $\Im$ is the set of all combinations of possible reconstruction directions.

The global indicators take into account their corresponding residuals, *i.e.* for example SPE_1, which correspond to $R = 1$, is computed from $\tilde{x}_{11}$, $\tilde{x}_{12}$, ... $\tilde{x}_{1n}$. Then, they use all the sensitivity of the residuals to the faults.

C. Fault isolation

All the directions of reconstruction Ξ_R have to be explored for fault detection and isolation. Solutions for which the faults associated to the reconstruction directions are not detectable are useless. The number of possible reconstructions can then be reduced, and the detectable faults are defined.

The maximum reconstruction number can be calculated as follows:

$$\sum_{r=1}^{n-\ell} \mathbb{C}_n^r \tag{26}$$

with $\mathbb{C}_n^r$ denotes the combination of r from n.
This number takes only into account the number of reconstructions and not the amplitude of the projection of the reconstructed directions onto the residual space. It can be reduced when the matrix of projected fault directions is rank-deficient or near rank-deficient. To detect these cases, the condition number (*Rcond*), defined by equation (27) as the ratio between the smallest singular value and the greatest singular value of the matrix $\tilde{\Xi}_R$, is used:

$$Rcond = \frac{\min\left(\sigma\left(\tilde{\Xi}_R\right)\right)}{\max\left(\sigma\left(\tilde{\Xi}_R\right)\right)} \tag{27}$$

For the near rank-deficient case, fault detection and localisation are possible only if its amplitude is huge. In the following, faults with huge amplitude are not considered as realistic. The process to detect useful directions of reconstruction can be summarized as follows:

1) $r = 1$ (single-fault): calculate all available directions $\tilde{\Xi}_R$. If $\tilde{\Xi}_R^T \tilde{\Xi}_R$ is closed to zero, it means that the fault is not projected onto the residual space and then not detectable. To detect and localise this fault, its projection onto the principal space can be used.
2) $r = r + 1$: calculate for all available directions $\tilde{\Xi}_R$ the values of the condition number *Rcond* (27). If *Rcond* is close to zero, then the r faulty variables of the subset R are not detectable. Therefore, all combinations which take into account these r variables will not be detectable. So they are useless. Moreover all the combinations of $r-1$ variables among the variables of the subset R are only detectable because their fault signatures are identical. Then, it is useful to reconstruct only one combination of these $r-1$ variables from these r variables. Therefore, among all combinations which take into account $r-1$ variables of the subset R, only one combination will be constructed, the others are identical. Moreover all the combinations of $r-2$ variables among the r variables of the subset R are isolable.
3) While $r \leq n - \ell$ go to step 2

This analysis of the structure of the model allows to determine the detectable faults and the isolable faults. The number of useful reconstructions can then be reduced.

IV. Numerical example : Multiple fault case

We consider here the situation in which several faults affect variables at the same time.

A. Data generation

The matrix X includes $N = 128$ observations of a vector x with 8 components generated in the following way:

$$\begin{aligned}
&x_{i,1} = v_i^2 + \sin(0.1i) \\
&x_{i,2} = 2\sin(i/6)\cos(i/4)\exp(-i/N), \quad v_i \sim \mathcal{N}(0,1) \\
&x_{i,3} = \log(x_{i,2}^2), \quad x_{i,4} = x_{i,1} + x_{i,2} \\
&x_{i,5} = x_{i,1} - x_{i,2}, \quad x_{i,6} = 2x_{i,1} + x_{i,2} \\
&x_{i,7} = x_{i,1} + x_{i,3}, \quad x_{i,8} \sim \mathcal{N}(0,1)
\end{aligned} \tag{28}$$

On the data thus generated were added realizations of random variables with centred normal distribution and standard deviations equal to 0.02 as well as faults δx_1, δx_2, δx_3, δx_4, δx_5, δx_6 represented by a bias of amplitude equal to 10% of the amplitudes of the variables. Faults are defined in the following way: observations from 10 to 24 (interval I_1) for the variable x_1, observations from 35 to 49 (interval I_2) for the variables x_2 and x_3, observations from 60 to 74 (interval I_3) for the variables x_4 and x_5, observations from 85 to 99 (interval I_4) for the variable x_1, x_4 and x_6.

From the contaminated data, the robust *PCA* model, with four principal axes ($\ell = 4$), was chosen.

B. Choice of the tuning parameters

To highlight advantages of this method, a discussion on the influence of the β value for fault detection and isolation with and without scale-M estimator is carried out. The results are shown with the global indicator SPE_1 (24). Two extreme cases are distinguished, firstly data are disturbed by small faults which involved small perturbation on the covariance matrix (figure 1), and secondly data are disturbed by huge

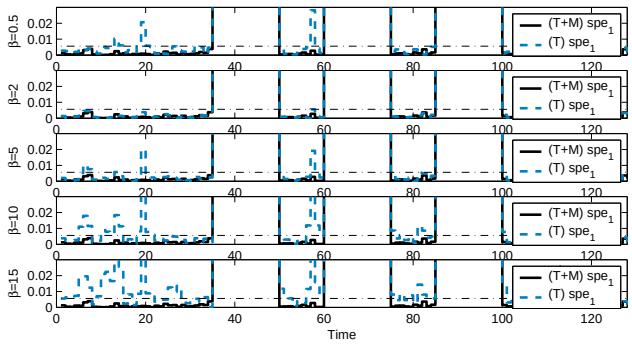

Fig. 1. SPE_1 with small outliers

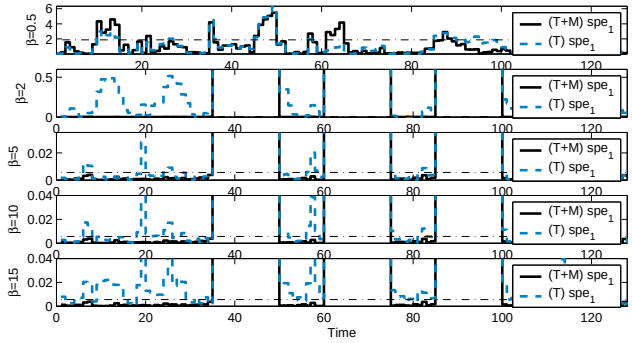

Fig. 2. SPE_1 with huge outliers

faults which involved large perturbation on the covariance matrix (figure 2). Legend *(T+M)* denotes the use of a model based on the robust covariance estimation and the scale-M estimator whereas legend *(T)* means the use of a model based only on the robust covariance estimation. For β close to zero, the robust covariance matrix is close to the classical covariance matrix. In the small fault case, the fault detection is identical for small values of β and better with the scale-M estimator for large values of β. In the huge fault case, for $\beta = 0.5$, it is not possible to detect faults. The robust covariance matrix is badly estimated, and the scale-M estimator is badly initialized. For the other values of β it is possible to detect the fault only with the addition of the scale-M estimator. To conclude, adding a scale-M estimator allows to reduce the importance of the β value. Then, in the following, $\beta = 2$, as recommended by the authors of the robust covariance matrix, is chosen to ensure the detection of all faults.

C. Useful reconstruction

From the size of the residual space, we cannot reconstruct more than four variables simultaneously. The maximum number of reconstructions is then equal to 162 (26). Now, the norm of the projections of fault directions onto the residual space are studied (table I). From this table, let us note that the last variable is not detectable. Indeed the variable x_8 is uncorrelated with the other variables. To detect and isolate a fault on this variable, it is better to work in the principal

TABLE I

EXISTENCE CONDITION OF RESIDUALS

$\tilde{\Xi}_1^T\tilde{\Xi}_1$	$\tilde{\Xi}_2^T\tilde{\Xi}_2$	$\tilde{\Xi}_3^T\tilde{\Xi}_3$	$\tilde{\Xi}_4^T\tilde{\Xi}_4$	$\tilde{\Xi}_5^T\tilde{\Xi}_5$	$\tilde{\Xi}_6^T\tilde{\Xi}_6$	$\tilde{\Xi}_7^T\tilde{\Xi}_7$	$\tilde{\Xi}_8^T\tilde{\Xi}_8$
0.84	0.72	0.46	0.71	0.41	0.40	0.46	**0.00**

TABLE II

EXISTENCE CONDITION OF RESIDUALS

Rcond	1	2	3	4	5	6	7
1		0.88	0.72	0.88	0.57	0.57	0.72
2			0.79	0.73	0.42	0.68	0.80
3				0.80	0.75	0.76	**0.01**
4					0.68	0.41	0.80
5						0.79	0.75
6							0.75

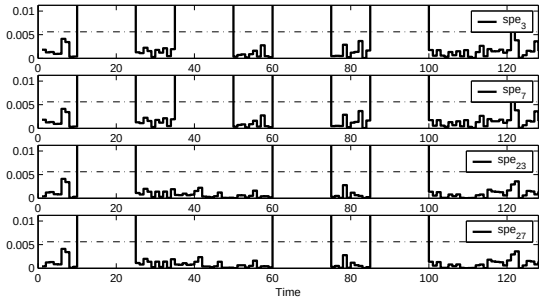

Fig. 3. *SPE* for $R = \{3\}$, $R = \{7\}$, $R = \{2,3\}$ and $R = \{2,7\}$

space. To analyse the multiple fault directions, the indicator *Rcond* (27) is calculated for all available directions (for $r = 2$ to 4). The result for the reconstruction of two variables is shown in the table II. For example, for $R = \{1,2\}$, the values of *Rcond* is the intersection of the first row and the second column of the table II.

Let us notice that for $R = \{3,7\}$, *Rcond* is close to zero. It means that the fault signatures for a fault on x_3 or on x_7 are identical ($SPE_3 = SPE_7$). So we can only detect the fault and conclude that the variables x_3 or x_7 or x_3 and x_7 are faulty. To illustrate this case, figure 3 shows the reconstructions of the variables $R = \{3\}$, $R = \{7\}$, $R = \{2,3\}$ and $R = \{2,7\}$ with the detection threshold. This figure shows that $SPE_3 = SPE_7$ and $SPE_{23} = SPE_{27}$, then only one *SPE* of each combination is useful to detect a fault. In the following, only the combinations with 3 are considered (not with 7). On the interval I_2 (figure 3), SPE_3 is not equal to zero, so there is an other fault at the same time. Moreover, this figure shows that for SPE_{23}, on the interval I_2, the *SPE* is zero. We conclude that on the interval I_2, variables x_2 and x_3 or/and x_7 are faulty.

For all the directions of reconstruction these indicators are calculated. Another case where *Rcond* is close to zero with $R = \{2,4,5,6\}$ is detected. Then all the combinations of 3 variables among the variables of the subset R are only detectable and their fault signatures are identical ($SPE_{245} = SPE_{246} = SPE_{256} = SPE_{456}$). Therefore only one *SPE* is useful to detect a fault, for example SPE_{245}. Thus, a fault can be detected but not isolated, the faulty variables are among the variables x_2, x_4, x_5 and x_6.

From the 162 reconstruction possibilities, only 91 are really reconstructible. Among these reconstructible directions, only 21 combinations are useful to isolate the faulty variables. For the others, a set of variable is considered as faulty but it is not possible to determine the faulty variables in the set.

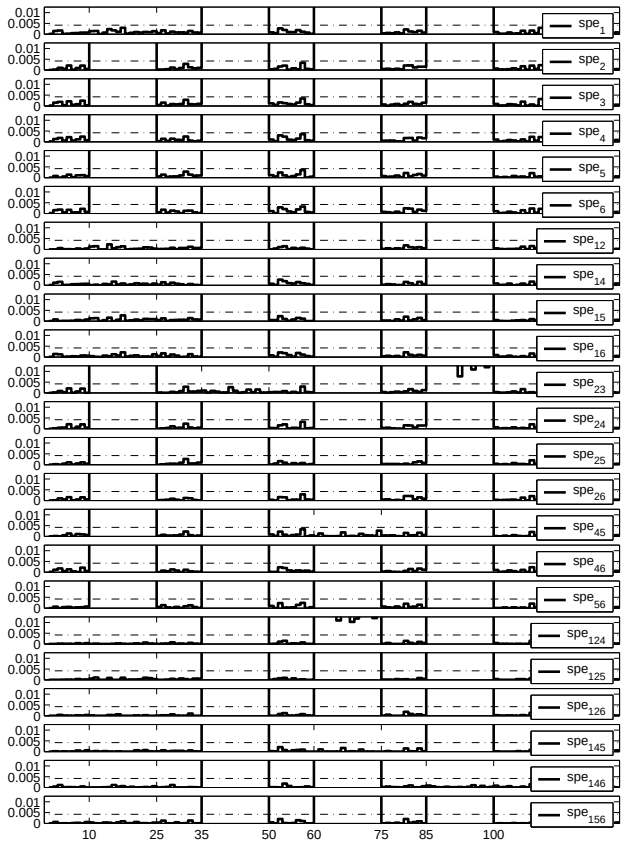

Fig. 4. *SPE* for some reconstruction directions

D. Fault detection

The N reconstructed data were then projected onto the residual space. For each observation fault indicators SPE_R (24) were calculated.

The first graph of the figure 4 shows the global indicator SPE_1 (24). For the observations of the interval I_1 this norm close to the value 0 thus shows the absence of outliers in the variables used for the reconstruction, i.e. all the variables except x_1. Let us note that the three other groups of observations (I_2, I_3, I_4) are affected by faults, and we don't know exactly which components of the measurement vector are faulty. Finally, by taking into account the fault presence in the four intervals, the examination of the first graph of the figure 4 concludes that:

- in each interval I_2, I_3, I_4, a variable other than x_1 is faulty or more than one variable is faulty.

Other projections are built and are interpreted in a similar way. Figure 4 shows the global indicator for some reconstruction directions. The table III summarizes the conclusions resulting from the SPE_R analysis (figure 4). The SPE_1 relates to the reconstructed residuals without using the first variable, the symbol 0 denotes the fault absence in the considered interval. The diagnosis is then:

- in the interval I_1, x_1 is faulty
- in the interval I_2, x_2 and x_3 or/and x_7 are faulty, the fault is not isolable
- in the interval I_3, x_4, x_5 are faulty
- in the interval I_4, x_1, x_4, x_6 are faulty

TABLE III
FAULT SIGNATURES

	I_1	I_2	I_3	I_4
SPE_1	0	×	×	×
SPE_{23}	×	0	×	×
SPE_{45}	×	×	0	×
SPE_{146}	0	×	×	0

V. CONCLUSION

Principal components analysis reduces the data representation space and enables the determination of the redundancy relationships which are then used to detect and isolate the faults. Usually PCA is constructed with fault-free data from a decomposition in eigenvalues and eigenvectors of a covariance matrix. However, real data set being usually not fault-free, the covariance matrix is then disturbed by outliers. In order to reduce the sensitivity of the model to outliers, a fast two-step algorithm is proposed. First, a scale-M estimator is used to determine a robust model. This estimator is computed using an iterative re-weighted least squares (IRWLS) procedure. This algorithm is initialized from a very simple estimate derived from a one-step weighted variance-covariance estimate. Therefore, a model robust with respect to outliers has been constructed. Second, structured residuals, based on the reconstruction principle, are generated for multiple faults detection and isolation. For fault isolation, the proposed scheme avoids the combinatorial explosion of faulty scenarios related to multiple faults. Indeed, instead of considering all combinations of one up to all sensors, we limit the maximum number of faulty scenarios to consider by evaluating the existence condition of structured residuals. Therefore the detectable faults and the isolable faults are determined as well as the different faulty scenarios for which it is not possible to distinguish the faulty variables.

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