Fault diagnosis for Takagi-Sugeno nonlinear systems

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Objective of diagnosis

- Fault detection
- Fault isolation
- Fault estimation

Difficulties

- Taking into account the system complexity in a large operating range
- Taking into account the presence of disturbances

Proposed strategy

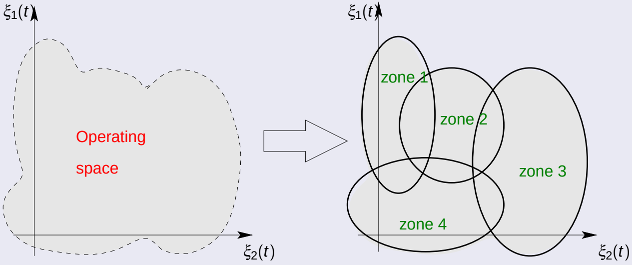
- Takagi-Sugeno representation of nonlinear systems
- Robust observer-based residual generator design for fault diagnosis
- Extension of the existing results on linear systems

Plan

1. Takagi-Sugeno approach for modeling
   - Takagi-Sugeno principle
   - Takagi-Sugeno system
2. Residual generator design
   - First case : Measurable premise variables
   - Second case : Unmeasurable premise variables
   - Robust diagnosis
3. Numerical example
4. Conclusions

Takagi-Sugeno approach for modeling

- Operating range decomposition in several local zones.
- A local model represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system   Multiple Model representation

The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi), \quad 0 \leq \mu_i \leq 1$
- Define an agregation procedure : $M = \sum \mu_i(\xi) M_i$

Interests of Takagi-Sugeno approach

- Simple structure for modeling complex nonlinear systems.
- Possible extension of the theoretical LTI tools for nonlinear systems.

The difficulties

- How many local models ?
- How to define the domain of influence of each local model ?
- On what variables may depend the weighting functions μ_i ?

- Linearisation of an existing non linear model around operating points [1]
- Direct identification of the model parameters [2]
- Non linear transformations of an existing non linear model [3]

[1] R. Murray-Smith, T. A. Johansen, Multiple model approaches to modelling and control. Taylor & Francis, 1997.

[2] K. Gasso, Identification des système dynamiques non linéaires : Approchemultjniodèle, Ph.D., Institut National Polytechnique de Lorraine, France, 2000.

[3] A.M. Nagy, G. Mourot, B. Marx, G. Schutz, J. Ragot, Model structure simplification of a biological reactor, 15th IFAC Symp. on System Identification, SYSID'09, 2009

Basic model

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

- Interpolation mechanism $\sum\limits_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$
- The premise variable $\xi(t)$ can be measurable ($u(t)$, $y(t)$) or unmeasurable ($x(t)$).

A faulty disturbed system

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i d(t) + F_i f(t)) \\ y(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t) + G_i d(t) + R_i f(t)) \end{cases}$$

- $f(t)$: the fault vector (to be detected).
- $d(t)$: the disturbance vector.

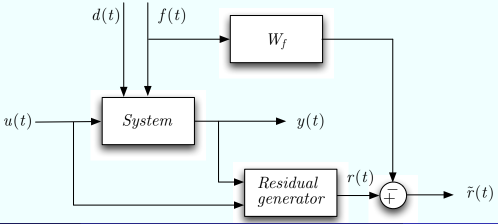
Residual generator design

Properties of residuals

- insensitive to the disturbances d
- robust with respect to modeling errors
- sensitivity with respect to faults f
- computable from the available measurements

The FDI problem depends on the selected structure of the filter W_f

- Fault estimation is obtained with $W_f = I$
- Fault detection problem is considered when $W_f \in \mathbb{R}^{p \times n_f}$
- In either cases, the size of the residual generator is adapted



Model of the system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

State observer

$$\text{1st case } \mathscr{O}_1 \begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i \hat{x}(t) + D_i u(t)) \end{cases}$$

$$\text{2nd case } \mathscr{O}_2 \begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{\xi}(t))(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\hat{\xi}(t))(C_i \hat{x}(t) + D_i u(t)) \end{cases}$$

Observer-based residual generator

$$\begin{cases} \dot{\hat{x}}(t) = \sum\limits_{i=1}^{r} \mu_i(\xi)(A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \sum\limits_{i=1}^{r} \mu_i(\xi)(C_i\hat{x}(t) + D_iu(t)) \\ r(t) = M(y(t) - \hat{y}(t)) \quad \textit{computational form of the residuals} \end{cases} \rightarrow \boxed{L_i \text{ and } M}$$

State estimation error

$$e(t) = x(t) - \hat{x}(t)$$

Dynamics of the state estimation error

$$\begin{cases} \dot{e}(t) = A_\xi e(t) + E_\xi d(t) + F_\xi f(t) \\ r(t) = C_\xi e(t) + G_\xi d(t) + R_\xi f(t) \end{cases}$$

$$A_\xi = \sum_{i,j=1}^{r} \mu_i(\xi)\mu_j(\xi)(A_i - L_iC_k) \qquad E_\xi = \sum_{i,j=1}^{r} \mu_i(\xi)\mu_j(\xi)(E_i - L_iG_k)$$

$$F_\xi = \sum_{i,j=1}^{r} \mu_i(\xi)\mu_j(\xi)(F_i - L_iR_k) \qquad C_\xi = \sum_{i=1}^{r} \mu_i(\xi)MC_i$$

$$G_\xi = \sum_{i=1}^{r} \mu_i(\xi)MG_i \qquad R_\xi = \sum_{i=1}^{r} \mu_i(\xi)MR_i$$

$$\begin{cases} \dot{e}(t) = A_\xi e(t) + E_\xi d(t) + F_\xi f(t) \\ r(t) = C_\xi e(t) + G_\xi d(t) + R_\xi f(t) \end{cases}$$

For convenience, the system above is written in the following form

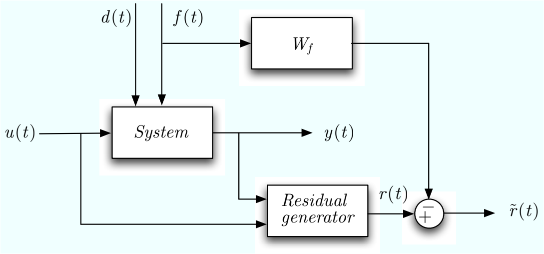
$$\boxed{r = G_{rd} d + G_{rf} f} \quad \Leftarrow \quad \text{Evaluation form of the residual}$$

G_{rd}, the transfer from the disturbances $d(t)$ to $r(t)$, is defined by

$$G_{rd} := \left(\begin{array}{c|c} A_\xi & E_\xi \\ \hline MC_i & G_\xi \end{array} \right)$$

G_{rf}, the transfer from $f(t)$ to $r(t)$, is defined by

$$G_{rf} := \left(\begin{array}{c|c} A_\xi & F_\xi \\ \hline C_\xi & R_\xi \end{array} \right)$$



The introduction of W_f turns the problem of the effect fault maximization on the residual $r(t)$ to a problem of minimization, by introducing the structured residual $\tilde{r}(t)$:

$$\begin{cases} r & = & G_{rd}d + G_{rf}f \\ \tilde{r}(t) & = & r(t) - W_f f(t) \\ & = & G_{rd}d + (G_{rf} - W_f)f \\ W_f : & = & \left(\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right) \end{cases}$$

Fault influence and objective

$$\tilde{r} = G_{rd} d + (G_{rf} - W_f) f$$

Adjust the transfert functions G_{rf} and G_{rd} in order to detect f even if d exist.

Principle of the method

Adjust the observer gains (L_i, M) such as to minimize

$$G_{rf} - W_f \quad and \quad G_{rd}$$

Practical implementation

Obtain L_i and M which minimize

$$\Phi = a\gamma_f + (1-a)\gamma_d, \quad a \in [0 \;\; 1]$$

subjected to the following constraints

$$\begin{aligned} \|G_{rf} - W_f\|_\infty &< \gamma_f \\ \|G_{rd}\|_\infty &< \gamma_d \\ \dot{e}(t) &= A_\xi e(t) + E_\xi d(t) + F_\xi f(t) \; stable \end{aligned}$$

- Select a positive parameter $a \in [0,1]$ and a weighting function $W_f \in \mathscr{S}$.
- The residual generator exists if there exist :

  matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$
  gain matrices K_i and M
  positive scalars $\bar{\gamma}_f$ and $\bar{\gamma}_d$

solution of

$$\min_{L_i, M, P_1, P_2, K_i, \bar{\gamma}_f, \bar{\gamma}_d} \quad a\bar{\gamma}_f + (1-a)\bar{\gamma}_d \quad \text{s.t.}$$

$$\begin{pmatrix} X_{ik}^1 & 0 & P_1F_i - K_iR_k & C_k^T M^T \\ (\bullet) & X_f^2 & P_2B_f & -C_f^T \\ (\bullet) & (\bullet) & -\bar{\gamma}_f I & R_k^T M^T - D_f^T \\ (\bullet) & (\bullet) & (\bullet) & -I \end{pmatrix} < 0$$

$$\begin{pmatrix} X_{ik}^1 & P_1E_i - K_iG_k & C_k^T M^T \\ (\bullet) & -\bar{\gamma}_d I & G_k^T M^T \\ (\bullet) & (\bullet) & -I \end{pmatrix} < 0$$

where

$$\begin{cases} X_{ik}^1 = A_i^T P_1 + P_1 A_i - K_i C_k - C_k^T K_i^T \\ X_f^2 = A_f^T P_2 + P_2 A_f \end{cases}$$

$$\forall i, k = 1, \ldots, r$$

r being the number of local models.

- The gains L_i are derived from

$$L_i = P_1^{-1} K_i, \quad i = 1, \ldots, r$$

and the attenuation levels are given by

$$\gamma_d = \sqrt{\bar{\gamma}_d} \quad \gamma_f = \sqrt{\bar{\gamma}_f}$$

Step 1 : Faulty case without disturbances $r = G_{rf} f$.

The maximization problem can be formulated as a minimization one by solving $\|G_{rf} - W_f\|_\infty < \gamma_f$.

$$G_{rf} - W_f := \left(\begin{array}{cc|c} A_\xi & 0 & F_\xi \\ 0 & A_f & B_f \\ \hline C_\xi & -C_f & R_\xi - D_f \end{array}\right)$$

1. Using the bounded real lemma [Boyd 1994], we obtain

$$\left.\begin{array}{l} \begin{pmatrix} X_{ik}^1 & 0 & P_1F_i - P_1L_iR_k & C_k^T M^T \\ (\bullet) & X_f^2 & P_2B_f & -C_f^T \\ (\bullet) & (\bullet) & -\gamma_f^2 I & R_k^T M^T - D_f^T \\ (\bullet) & (\bullet) & (\bullet) & -I \end{pmatrix} < 0 \\ X_{ik}^1 = A_i^T P_1 + P_1 A_i - P_1 L_i C_k - C_k^T L_i^T P_1 \\ X_f^2 = A_f^T P_2 + P_2 A_f^T \end{array}\right\} \quad i,k = 1,\ldots,r$$

2. The change of variables $K_i = P_1 L_i$ and $\bar{\gamma}_f = \gamma_f^2$ allows to obtain the first LMI of the theorem 1.

Step 2 : Faulty free case with disturbances $r = G_{rd}d$.

In faulty case without disturbances $r = G_{rd}d$. The maximization problem can be formulated as a minimization one by solving $\|G_{rd}\|_\infty < \gamma_f$.

$$G_{rd} := \left(\begin{array}{c|c} A_\xi & E_\xi \\ \hline MC_i & G_\xi \end{array}\right)$$

1. Using the bounded real lemma [Boyd 1994], we obtain

$$\left.\begin{array}{l} \begin{pmatrix} X_{ik}^1 & P_1E_i - P_1L_iG_k & C_k^TM^T \\ (\bullet) & -\gamma_f^2 I & G_k^TM^T \\ (\bullet) & (\bullet) & -I \end{pmatrix} < 0 \\ X_{ik}^1 = A_i^TP_1 + P_1A_i - P_1L_iC_k - C_k^TL_i^TP_1 \end{array}\right\} \quad i,k = 1,\ldots,r$$

2. The change of variables $K_i = P_1L_i$ and $\bar{\gamma}_d = \gamma_d^2$ allows to obtain the second LMI of the theorem 1.

Step 3 : Faulty case with disturbances $r = G_{rf}f + G_{rd}d$.

The problem is expressed as a minimization of the linear combination $a\gamma_f + (1-a)\gamma_d$ where $a \in [0 \;\; 1]$.

- System

$$\mathscr{S}\begin{cases} \dot{x}(t)=\sum_{i=1}^{r}\mu_i(x)\,(A_ix(t)+B_iu(t)+E_id(t)+F_if(t)) \\ y(t)=\sum_{i=1}^{r}\mu_i(x)\,(C_ix(t)+D_iu(t)+G_id(t)+R_if(t)) \end{cases}$$

- Observer

$$\mathscr{O}\begin{cases} \dot{\hat{x}}(t)=\sum_{i=1}^{r}\mu_i(\hat{x})(A_i\hat{x}(t)+B_iu(t)+L_i(y(t)-\hat{y}(t))) \\ \hat{y}(t)=\sum_{i=1}^{r}\mu_i(\hat{x})(C_i\hat{x}(t)+D_iu(t)) \\ r(t)=M(y(t)-\hat{y}(t)) \end{cases}$$

- The system can also be presented in the following form

$$\mathscr{S}\begin{cases} \dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x)\mu_j(\hat{x})(\tilde{A}_{ij}x(t)+\tilde{B}_{ij}u(t)+E_id(t)+F_if(t)) \\ y(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x)\mu_j(\hat{x})(\tilde{C}_{ij}x(t)+\tilde{D}_{ij}u(t)+G_id(t)+R_if(t)) \end{cases}$$

where

$$\begin{array}{llll} \tilde{A}_{ij}=A_j+\Delta A_{ij} & \tilde{C}_{ij}=C_j+\Delta C_{ij} & \tilde{B}_{ij}=B_j+\Delta B_{ij} & \tilde{D}_{ij}=D_j+\Delta D_{ij} \\ \Delta X_{ij}=X_i-X_j & X_i\in\{A_i,B_i,C_i,D_i\} & i,j=1,\ldots,r & \end{array}$$

- After calculating the state estimation error, the following is obtained

$$\begin{cases} \dot{e}(t) = \tilde{A}_{x\hat{x}}e(t) + \Delta\tilde{A}_{x\hat{x}}x(t) + \tilde{B}_{x\hat{x}}\tilde{d}(t) + \tilde{F}_{x\hat{x}}f(t) \\ r(t) = \tilde{C}_{x\hat{x}}e(t) + \Delta\tilde{C}_{x\hat{x}}x(t) + \tilde{G}_{x\hat{x}}\tilde{d}(t) + \tilde{R}_{x\hat{x}}f(t) \end{cases}$$

  (For details see the paper).

- Let define the augmented state vector $\tilde{x} = [e^T \quad x^T]^T$. The residual vector r is then given by

$$r = G_{rd}\tilde{d} + G_{rf}f$$

  where

$$G_{rd} = \left(\begin{array}{cc|c} \tilde{A}_{x\hat{x}} & \Delta\tilde{A}_{x\hat{x}} & \tilde{B}_{x\hat{x}} \\ 0 & A_x & \tilde{B}_x \\ \hline \tilde{C}_{x\hat{x}} & \Delta\tilde{C}_{x\hat{x}} & \tilde{G}_{x\hat{x}} \end{array}\right)$$

  and

$$G_{rf} = \left(\begin{array}{cc|c} \tilde{A}_{x\hat{x}} & \Delta\tilde{A}_{x\hat{x}} & \tilde{F}_{x\hat{x}} \\ 0 & A_x & F_x \\ \hline \tilde{C}_{x\hat{x}} & \Delta\tilde{C}_{x\hat{x}} & \tilde{R}_{x\hat{x}} \end{array}\right)$$

- The FDI problem is the same that the problem exposed previously (Theorem 2 in the paper).

Given a positive parameter a and a weighting function W_f. The residual generator exists if there exist matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$ and gain matrices K_i and M and positive scalars $\bar{\gamma}_1$ and $\bar{\gamma}_2$ solution of the following optimization problem

$$\min_{L_i, M, P_1, P_2, K_i, \bar{\gamma}_f, \bar{\gamma}_d} \quad a\bar{\gamma}_f + (1-a)\bar{\gamma}_d \ \text{ s.t.}$$

$$\begin{pmatrix} * & X_f^3 & P_3 B_f & -C_f^T & \\ * & * & * & -\bar{\gamma}_f I & (MR_k - D_f)^T \\ * & * & * & * & -I \end{pmatrix} < 0, \ \forall i,j,k,l = 1,\ldots,r$$

$$\begin{pmatrix} X_{jl}^1 & \Xi_{ijkl} & P_1 \Delta B_{ij} - K_j \Delta D_{ij} & P_1 E_i - K_j G_k & C_l^T M^T \\ * & X_i^2 & P_2 B_i & P_2 E_i & \Delta C_{kl}^T M^T \\ * & * & -\bar{\gamma}_{\tilde{d}} I & 0 & \Delta D_{kl}^T M^T \\ * & * & * & -\bar{\gamma}_{\tilde{d}} I & G_k^T M^T \\ * & * & * & * & -I \end{pmatrix} < 0, \ \forall i,j,k,l = 1,\ldots,r$$

where

$$X_{jl}^1 = A_j^T P_1 + P_1 A_j - K_j C_l - C_l^T K_j^T, \qquad X_i^2 = A_i^T P_2 + P_2 A_i, \qquad X_f$$

The gains L_i are derived from $L_i = P_1^{-1} K_i \ \ i = 1, \ldots, r$ and the attenuation levels are given by

$$\gamma_d = \sqrt{\bar{\gamma}_d} \quad \gamma_f = \sqrt{\bar{\gamma}_f}$$

- An alarm is generated by comparison between $r(t)$ and the threshold defined by

$$J_{th} = \gamma_d \rho$$

where :
γ_d is the attenuation level of the disturbance $d(t)$
ρ the bound of $d(t)$

- The decision logic is given by

$$\begin{cases} |r_i(t)| < J_{th} \Rightarrow & \text{no fault} \\ |r_i(t)| > J_{th} \Rightarrow & \text{fault} \end{cases}$$

The fault vector $f(t)$ take into account the actuator and the sensor faults :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i d(t) + F_i f(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t) + G_i d(t) + R_i f(t)) \end{cases}$$

$$f(t) = \begin{pmatrix} f_a(t) \\ f_s(t) \end{pmatrix}$$

If $f_a(t)$ does not affect the output of the system, there is a nul column in R_i which then is not of full rank. The proposed solution consist in :

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(C_i x(t) + D_i u(t) + \underbrace{\begin{pmatrix} G_i & -b\varepsilon_i \end{pmatrix}}_{\dots} \underbrace{\begin{pmatrix} d(t) \\ \frac{f_a(t)}{b} \end{pmatrix}}_{disturbance} + \underbrace{\begin{pmatrix} \varepsilon_i & R_i^1 \end{pmatrix}}_{\dots} \underbrace{\begin{pmatrix} f_a(t) \\ f_s(t) \end{pmatrix}}_{fault} \right)$$

ε_i is chosen as small as possible

- The proposed algorithm of robust diagnosis is illustrated by an academic example. Let consider the nonlinear system defined by

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix} \quad A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, F_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

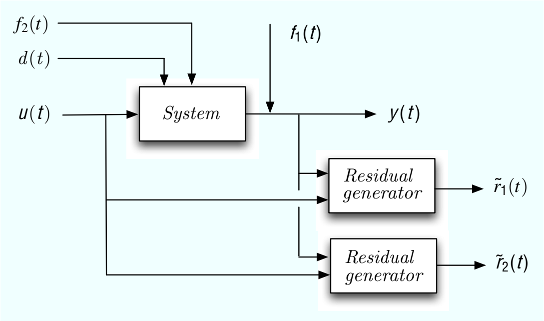
$$E_2 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, R_{1,2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- The weighting functions μ_i are defined as follows

$$\begin{cases} \mu_1(u(t)) = \frac{1 - \tanh((u(t)-1)/10)}{2} \\ \mu_2(u(t)) = 1 - \mu_1(u(t)) \end{cases}$$

For each fault a dedicated residual has been designed.

Generator	sensitive to
1	f_1
2	f_2



A first simulation is performed for fault detection and isolation. W_f is a diagonal first order low-pass filter.

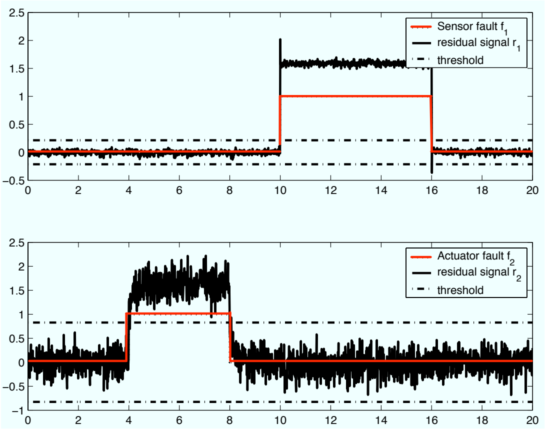


FIG.: Faults and corresponding residual signals

A second simulation is performed for fault estimation. W_f is then an identity matrix.

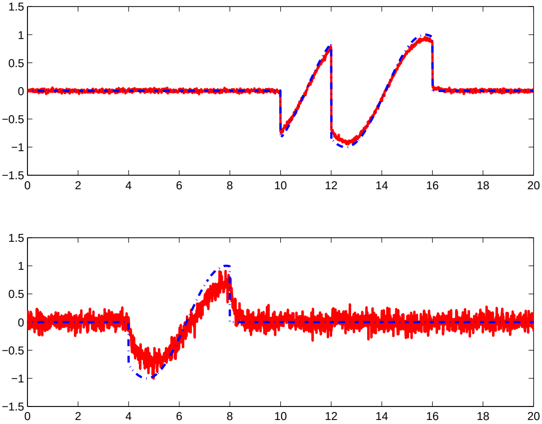


FIG.: Comparison of the faults (dashed lines) and residual signals (solid lines)

Conclusions

- Robust residual generator for nonlinear systems represented by a Takagi-Sugeno structure.
- In many situation $T_{-}S$ structure may represent exactly non linear systems.
- Study of two cases : measurable and unmeasurable premise variables.
- The problem of Fault detection, isolation and estimation is expressed via an optimization problem subject to LMI constraints.

Perspectives

- Study and reduction of the conservatism in the second theorem with unmeasurable premise variables (using other Lyapunov functions).
- Synthesis of the weighting transfer function W_f.
- Extension to fault tolerant control.

Thank you for your attention !