Estimating the state and the unknown inputs of nonlinear systems using a multiple model approach

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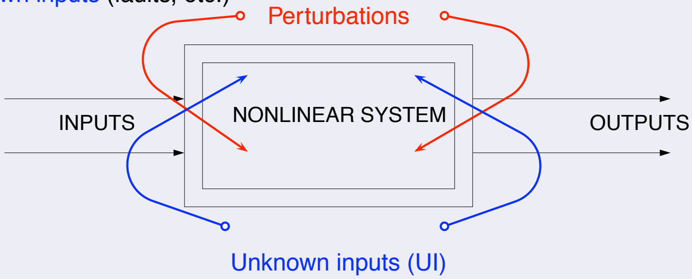
- Dynamic behaviour of most of real systems is nonlinear
- Systems are inevitably subject to external perturbations (noise, etc.) and unknown inputs (faults, etc.)



Goal

Simultaneously states and unknown inputs estimation of a nonlinear system

Motivations

Why?

- State and UI of a system is a key problem in control and/or supervision
- State and UI estimations can be employed for providing fault symptoms

Problems

- Take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)
- Observer design problem for generic nonlinear models is very delicate

Proposed solution

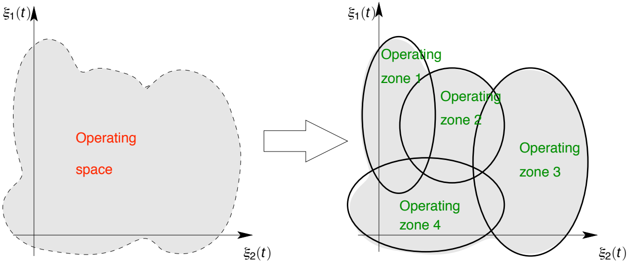
- Multiple model representation of the nonlinear system
- Conception of an unknown input observer (UIO)

Outline

Multiple model Approach

Basis of Multiple model approach

- Decomposition of the operating space into operating zones
- Modelling each zone by a single submodel
- The contribution of each submodel is quantified by a weighting function



Nonlinear system   Multiple model representation

Multiple model = an association of a set of submodels blended by an interpolation mechanism

Why using a multiple model?

- Appropriate tool for modelling complex systems (e.i. black box modelling)
- Linear system tools can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

How the submodels can be interconnected?

Classic structure
Takagi-Sugeno multiple model

- Using a common state vector to the submodels
- Dimension of the submodels must be identical

Proposed structure
Decoupled multiple model

- Using an independent state vector for each submodel
- Dimension of the submodels may be different

Decoupled multiple model : Multiple model with local state vectors

$$
\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i u(t) + D_i \eta(t) + V_i w(t) \\
y_i(t) &= C_i x_i(t) \;, \\
y(t) &= \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t) + E\eta(t) + W w(t)
\end{aligned}
$$

UI Perturbation

$$
\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., L\}
$$

- The multiple model output is given by a weighted sum of the submodel outputs (blending outputs)
- **Dimension of the submodels can be different**

State estimation

Augmented form of the multiple model

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) + \tilde{D}\eta(t) + \tilde{V}w(t), \\ y(t) &= \tilde{C}(t)x(t) + E\eta(t) + Ww(t), \quad x \in \mathbb{R}^n,\ n = \sum_{i=1}^{L} n_i \ . \end{aligned}$$

Linear form

Nonlinear form

Notations

$$x(t) = \left[x_1^T(t) \cdots x_i^T(t) \cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t)) = \mu_i(t) \ ,$$

$$\tilde{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix}, \tilde{D} = \begin{bmatrix} D_1 \\ \vdots \\ D_i \\ \vdots \\ D_L \end{bmatrix}, \tilde{V} = \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_L \end{bmatrix},$$

$$\begin{aligned} \tilde{C}(t) &= \left[\mu_1(t)C_1 \quad \dots \quad \mu_i(t)C_i \quad 0 \quad \dots \quad \mu_L(t)C_L\right], \\ &= \sum_{i=1}^{L} \mu_i(t)\tilde{C}_i, \qquad \tilde{C}_i = \left[0 \quad \dots \quad C_i \quad 0 \quad \dots \quad 0\right] . \end{aligned}$$

Goal

Our objective is to provide a simultaneous estimation of the state and the UI of a system represented by a multiple model

Proportional-Integral Observer

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) + \tilde{B}u(t) + \tilde{D}\hat{\eta}(t) + \tilde{K}\left(y(t) - \hat{y}(t)\right) \ , \\
\dot{\hat{\eta}}(t) &= \tilde{K}_1\left(y(t) - \hat{y}(t)\right) \ , \\
\hat{y}(t) &= \tilde{C}(t)\hat{x}(t) + E\hat{\eta}(t) \ .
\end{aligned}
$$

- This observer uses both proportional and integral action
- The use of this integral action allows a reconstruction of a constant UI

Assumption 1

The unknown input signal $\eta(t)$ is supposed to be a constant signal.

Assumption 2

The perturbation is bounded energy signal, i.e. $\|w(t)\|_2^2 < \infty$.

Estimation errors

$$
\begin{aligned}
e(t) &= x(t) - \hat{x}(t) \ , \\
\dot{e}(t) &= \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) + (\tilde{D} - \tilde{K}E)\varepsilon(t) + (\tilde{V} - \tilde{K}W)w(t) \ . \\
\varepsilon(t) &= \eta(t) - \hat{\eta}(t) \ , \\
\dot{\varepsilon}(t) &= \dot{\eta}(t) - \tilde{K}_1\tilde{C}(t)e(t) - \tilde{K}_1 E\varepsilon(t) - \tilde{K}_1 W w(t) \ .
\end{aligned}
$$

Augmented form

By using the following augmented state :

$$\Sigma(t) = \begin{bmatrix} e(t) \\ \varepsilon(t) \end{bmatrix} \in \mathbb{R}^{n+q} \quad ,$$

the equations of the state and UI estimation errors can be gathered as follows:

$$\dot{\Sigma}(t) \quad = \quad \tilde{A}_a(t)\Sigma(t) + (V_a - K_a W)w(t) \quad ,$$

where

$$\begin{aligned} \tilde{A}_a(t) &= \sum_{i=1}^{L} \mu_i(t)\Phi_i \quad , \\ \Phi_i &= A_a - K_a \mathscr{C}_i \quad , \end{aligned}$$

and

$$A_a = \begin{bmatrix} \tilde{A} & \tilde{D} \\ 0 & 0 \end{bmatrix}, K_a = \begin{bmatrix} \tilde{K} \\ \tilde{K}_1 \end{bmatrix}, \mathscr{C}_i = \begin{bmatrix} \tilde{C}_i^T \\ E^T \end{bmatrix}^T, V_a = \begin{bmatrix} \tilde{V} \\ 0 \end{bmatrix} \quad .$$

Goal

The robust observer design problem can thus be formulated as finding the matrix gain $K_a \in \mathbb{R}^{(n+p)\times p}$ such that the influence of $w(t)$ on $\Sigma(t)$ is attenuated.

Performance constraints

Now let us consider the following objective signal:

$$z(t) = H\Sigma(t) \quad ,$$

where H is a prescribed constant matrix and the following $\mathscr{H}_\infty$ performance constraints:

$$\begin{aligned} \lim_{t\to\infty} \Sigma(t) = 0 \quad & \text{for} \quad w(t) = 0 \quad , \\ \|z(t)\|_2^2 \le \gamma^2 \|w(t)\|_2^2 \quad & \text{for} \quad w(t) \neq 0 \text{ and } z(0) = 0 , \end{aligned}$$

where γ is the L_2 gain from $w(t)$ to $z(t)$ to be minimised.

Convergence conditions

Idea

(i) Robust performances are guaranteed with

(ii)
$$\dot{V}(t) < -2\alpha V(t) - z^T(t)z(t) + \gamma^2 w^T(t)w(t) \quad .$$

$$\int_0^\infty (\dot{V}(t) + 2\alpha V(t))dt \quad < \quad -\int_0^\infty z^T(t)z(t)dt + \gamma^2 \int_0^\infty w^T(t)w(t)dt\,,$$

and by taking into consideration the fact that $V(\infty) > 0$ and $V(0) = 0$, then : $\|z(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2 \quad ,$

(iii) Exponential convergence is guaranteed with $\dot{V}(t) < -2\alpha V(t)$

(iv) Using the following Lyapunov function

$$V(t) = \Sigma^T(t)P\Sigma(t), \quad P > 0 \quad P = P^T \quad ,$$

(v) Using the estimation error equations and after some algebraic manipulations...

Convergence conditions

Theorem

The PI observer for the decoupled multiple model is obtained if there exists a symmetric, positive definite matrix P and a matrix M minimizing $\bar{\gamma} > 0$ under the following LMIs

$$\begin{bmatrix} \Delta_i + \Delta_i^T + H^T H & \Gamma \\ \Gamma^T & -\bar{\gamma}, \mathrm{I} \end{bmatrix} < 0, \quad i = 1 \ldots L$$

where

$$\begin{aligned} \Delta_i &= P(A_a + \alpha \mathrm{I}) - M\mathscr{C}_i \ , \\ \Gamma &= PV_a - MW \ , \end{aligned}$$

for a prescribed $\alpha > 0$. The observer gain is given by $K_a = P^{-1}M$ and the L_2 gain from $w(t)$ to $z(t)$ is given by $\gamma = \sqrt{\bar{\gamma}}$.

Comments

- Convergence velocity of the estimation error is adjusted by $\overline{\alpha}$
- Attenuation level is adjusted by $\overline{\gamma}$

Example

Example

Consider the following decoupled multiple model with $L = 2$ submodels, the parameters are given by:

$$A_1 = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.2 & -0.9 & 0.8 \\ 0 & -0.8 & -0.7 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.25 & 0 \\ -0.4 & -0.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.5 & 0.4 & 0.3 \end{bmatrix}^T, \qquad B_2 = \begin{bmatrix} -0.5 & 0.7 \end{bmatrix}^T,$$

$$C_1 = \begin{bmatrix} 0.8 & 0.5 & 0.7 \\ 0.4 & -0.7 & -0.2 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.9 & 0.6 \\ 0.5 & -0.4 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \\ 0 & -0.2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.1 & 0.3 \\ -0.1 & 0.5 \end{bmatrix},$$

$$V_1 = \begin{bmatrix} 0.0 & -0.1 & 0.0 \\ 0.2 & -0.3 & 0.1 \end{bmatrix}^T, \qquad V_2 = \begin{bmatrix} 0.0 & -0.1 \\ 0.2 & 0.0 \end{bmatrix}^T,$$

$$E = \begin{bmatrix} 0.1 & 0.2 \\ 0.5 & -0.3 \end{bmatrix}, \qquad W = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Simulation example

The eigenvalues of the submodels are

$$\lambda_1 = \begin{bmatrix}-0.19 & -0.80 \pm 0.78i\end{bmatrix} \text{ and } \lambda_2 = \begin{bmatrix}-0.3 & -0.25\end{bmatrix} \;,$$

The weighting functions are normalised Gaussian functions

$$\mu_i(\xi(t)) = \omega_i(\xi(t)) / \sum_{j=1}^{L} \omega_j(\xi(t)) \quad \text{with} \quad \omega_i(\xi(t)) = \exp\left(-(\xi(t) - c_i)^2/\sigma^2\right),$$

with $\sigma = 0.5$, $c_1 \; = \; 0.25$ et $c_2 = 0.75$. The decision variable is $\xi(t) = u(t)$.

Choosing a decay rate $\alpha = 0.1$, conditions of the proposed theorem are fulfilled with:

$$K_a = \begin{bmatrix} 2.56 & -0.08 & -1.82 & 2.28 & 3.80 & 3.18 & 2.94 \\ 0.95 & -0.64 & -1.29 & 0.81 & 1.64 & 3.34 & 1.07 \end{bmatrix}^T$$

with a minimal attenuation level given by $\gamma = 1.29$.

Simulation example

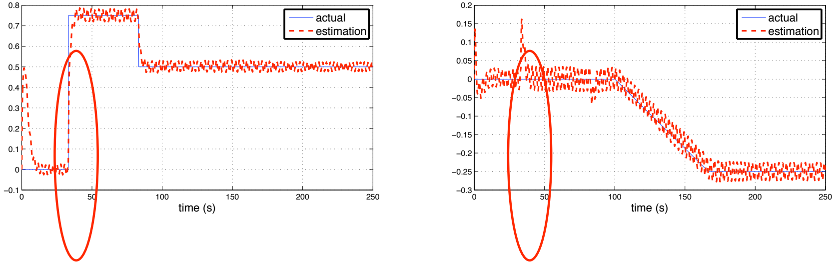
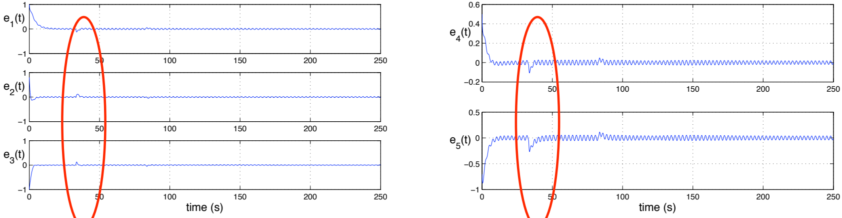


Figure: $\eta_i(t)$ and its estimate



Conclusion and perspectives

Conclusion

- A PI observer is used for estimating the state and the constant unknown inputs of nonlinear systems modelled by a decoupled multiple model.
- In this multiple model, the dimension of each submodel may be different (flexibility in a black box modelling stage can be provided).
- Sufficient conditions for ensuring exponential convergence and robust performances of the estimation error are proposed.

Perspectives

- The suggested observer can be used in the detection and the isolation of sensor and actuator failures of complex systems.
- The use of several integral actions (Multi-Integral Observer) can be a way to take into consideration a more general class of UI (non constant UI).

Thank you!