Robust observer design for uncertain Takagi-Sugeno model with unmeasurable decision variables: an $\mathscr{L}_2$ approach

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Mediteranean Conference on Automation and Control
25 – 27 juin 2008, Ajaccio, France

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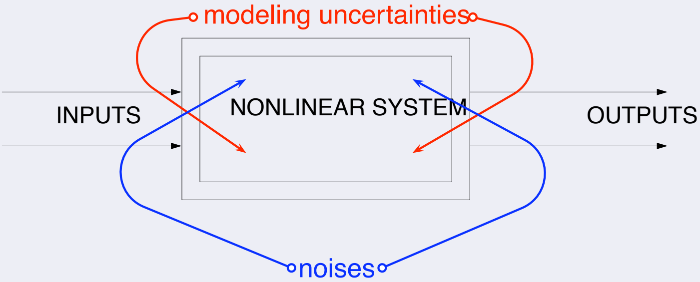
- Dynamic behaviour of most of real systems is nonlinear
- Systems are inevitably subject to modeling uncertainties and noises



Goal

Robust state estimation with regard to modeling uncertainties and noises for nonlinear systems

Motivations

Why?

- State estimation is a heart of control and/or supervision problems
- State estimation can be employed for providing fault symptoms

Problems

- Take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)
- Observer design problem for generic nonlinear models is very delicate

Proposed solution

- Multiple model representation of the nonlinear system
- Conception of a robust observer

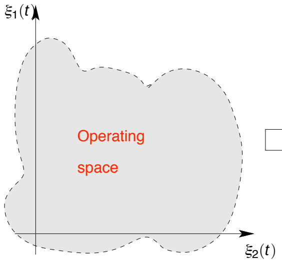
Outline

1. Multiple model approach
   - Basis of Multiple model approach
2. State estimation
   - Preliminaries and notations
   - Observer structure
3. Simulation example
4. Conclusions and future works

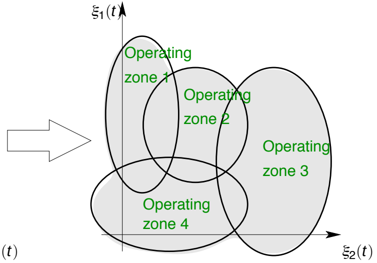
Multiple model Approach

Basis of Multiple model approach

- Decomposition of the operating space into operating zones
- Modelling each zone by a single submodel
- The contribution of each submodel is quantified by a weighting function



Nonlinear system



Multiple model representation

Multiple model = an association of a set of submodels blended by an interpolation mechanism

Objective of this work

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\right) \\ y(t) = Cx(t) + W\omega(t) \end{cases}$$

$$0 \leq \mu_i(\xi(t)) \leq 1 \quad \sum_{i=1}^{r} \mu_i(\xi(t)) = 1$$

Difficulty

- decision variable are not measurable i.e. : $\xi(t) = x(t)$.
- Presence of the norm bounded uncertainties :

$$\Delta A_i(t) = M_i^A \Sigma_A(t) N_i^A$$
$$\Delta B_i(t) = M_i^B \Sigma_B(t) N_i^B$$

with:

$$\Sigma_A^T(t)\Sigma_A(t) \leq I, \quad \forall t$$
$$\Sigma_B^T(t)\Sigma_B(t) \leq I, \quad \forall t$$

State estimation

Preliminaries and notations

Equivalent form of the multiple model

$$\begin{cases} \dot{x} = A_0 x + \sum_{i=1}^{r} \mu_i(x) \left((\bar{A}_i + \Delta A_i) x + (B_i + \Delta B_i) u \right) \\ y = Cx + D\omega \end{cases}$$

Notations

$$A_0 = \frac{1}{r} \sum_{i=1}^{r} A_i$$

and:

$$\bar{A}_i = A_i - A_0$$

Observer structure

Goal

Our objective is to provide a robust estimation of the state of a system represented by a multiple model with modeling uncertainties

Proportional Observer

$$\begin{cases} \dot{\hat{x}} = A_0\hat{x} + \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\overline{A}_i\hat{x} + B_i u + G_i(y - \hat{y})\right) \\ \hat{y} = C\hat{x} \end{cases}$$

Estimation errors

$$
\begin{aligned}
e(t) &= x(t) - \hat{x}(t) \ , \\
\dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x})(A_0 - G_i C)e(t) + H_i \tilde{\omega}(t) + \overline{A}_i \delta_i(t) + B_i \Delta_i(t) \ .
\end{aligned}
$$

where:

$$H_i = \begin{bmatrix} -\mu_i(\hat{x}) G_i D & \mu_i(x) \Delta A_i & \mu_i(x) \Delta B_i \end{bmatrix}$$

$$\tilde{\omega}(t)^T = [\omega(t)^T \ \ x(t)^T \ \ u(t)^T]$$

$$\delta_i = \mu_i(x)x - \mu_i(\hat{x})\hat{x}$$

$$\Delta_i = (\mu_i(x) - \mu_i(\hat{x}))u$$

Estimation errors

Assumptions

- **A1.** The system is stable $\Leftrightarrow$ $x(t)$ is bounded
- **A2.** The weighting functions $\mu_i(x)$ are Lipschitz:

$$|\mu_i(x) - \mu_i(\hat{x})| < \gamma_1 |x - \hat{x}|$$

- **A3.** The functions $\mu_i(x)x$ are Lipschitz:

$$|\mu_i(x)x - \mu_i(\hat{x})\hat{x}| < \gamma_2 |x - \hat{x}|$$

- **A4.** The input $u(t)$ of the system is bounded:

$$|u(t)| \leq \beta$$

$\tilde{\omega}(t)$ is bounded

Estimation errors

State estimation error:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{x})(A_0 - G_i C)e(t) + H_i \tilde{\omega}(t) + \overline{A}_i \delta_i(t) + B_i \Delta_i(t)$$

Lyapunov function:

$$V(t) = e(t)^T P e(t), \quad P = P^T > 0$$

The state estimation error is stable with an attenuation $\mu > 0$ of the $\mathscr{L}_2-$norm of the transfer $\tilde{\omega}(t)$ to $e_(t)$ if :

$$\dot{V}(t) + e(t)^T e(t) - \mu^2 \tilde{\omega}(t)^T \tilde{\omega}(t) < 0$$

Estimation errors

By using the following lemma :

Lemma

For two matrices X and Y with appropriate dimensions, the following property holds:

$$X^T Y + XY^T < X^T \Omega^{-1} X + Y \Omega Y^T, \quad \Omega > 0$$

and after some calculations ...

Theorem : Convergence conditions

The state estimation error converges asymptotically to zero, and the $\mathcal{L}_2$ gain of the transfer from $\tilde{\omega}$ to e is minimal if there exists positive and symmetric matrices P and Q, gains K_i, positive scalars $\bar{\mu}$, λ_1, λ_2, ε_2, ε_3, ε_4 and σ solution of the following problem:

$$\min_{P,Q,K_i,\lambda_1,\lambda_2,\varepsilon_2,\varepsilon_3,\sigma,\bar{\mu}} \quad \bar{\mu}$$

s.t. the following conditions for all $i \in \{1,...,r\}$:

$$A_0^T P + PA_0 - K_i C - C^T K_i^T < -Q$$

$$\begin{bmatrix} M & 0 & 0 & 0 & P\overline{A}_i & PB_i & PM_i^A & PM_i^B & K_i D & \gamma_1 \sigma I \\ * & M_{1i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & M_{2i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & M_{3i} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\lambda_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\lambda_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\lambda_2 I \end{bmatrix} < 0$$

$$\sigma - \lambda_2 \beta > 0$$

Theorem : Convergence conditions

where:

$$M = -Q + (\lambda_1 \gamma_2^2 + 1)I$$

$$M_{1i} = (-\bar{\mu} + \varepsilon_2)I$$

$$M_{2i} = -\bar{\mu}I + \varepsilon_3 (N_i^A)^T N_i^A$$

$$M_{3i} = -\bar{\mu}I + \varepsilon_4 (N_i^B)^T N_i^B$$

The gains of the observer are derived from:

$$G_i = P^{-1} K_i$$

and the attenuation level is derived from:

$$\mu = \sqrt{\bar{\mu}}$$

Example

Simulation example

Let us consider the system defined by:

$$A_1 = \begin{bmatrix} -18.5 & 5 & 18.5 \\ 0 & -20.9 & 15 \\ 18.5 & 15 & -33.5 \end{bmatrix}, A_2 = \begin{bmatrix} -22.1 & 0 & 22.1 \\ 1 & -23.3 & 17.6 \\ 17.1 & 17.6 & -39.5 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}$$

$$M_1^A = M_2^A = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0 & 0.02 & 0.11 \\ 0.07 & 0.1 & 0.09 \end{bmatrix}, M_1^B = M_2^B = \begin{bmatrix} 0.1 \\ 0.13 \\ 0.09 \end{bmatrix}$$

$\Sigma_A = \Sigma_B$ are defined in figure 1.

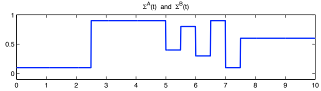


Figure: $\Sigma_A(t)$ et $\Sigma_B(t)$

Simulation example

After solving the optimization problem in theorem, we obtain :

$$G_1 = \begin{bmatrix} 58.66 & -19.55 \\ 41.45 & -13.81 \\ 70.74 & -23.58 \end{bmatrix}, G_2 = \begin{bmatrix} 59.26 & -19.75 \\ 45.00 & -15.00 \\ 70.12 & -23.37 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.14 & -0.05 & -0.00 \\ -0.05 & 0.15 & -0.01 \\ -0.00 & -0.01 & 0.10 \end{bmatrix}$$

The attenuation level is $\mu = 0.02$.

Simulation example

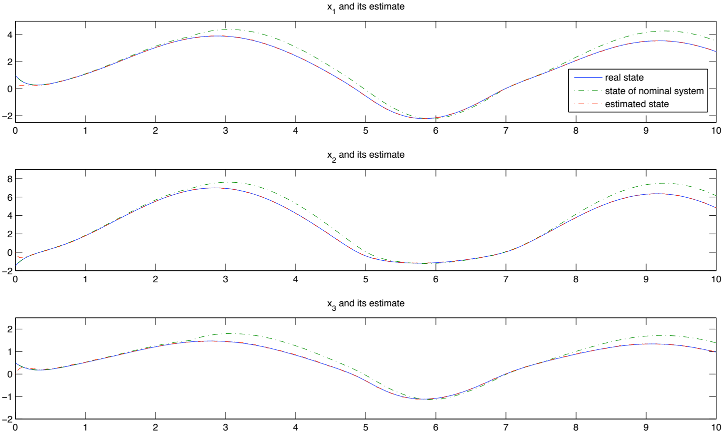
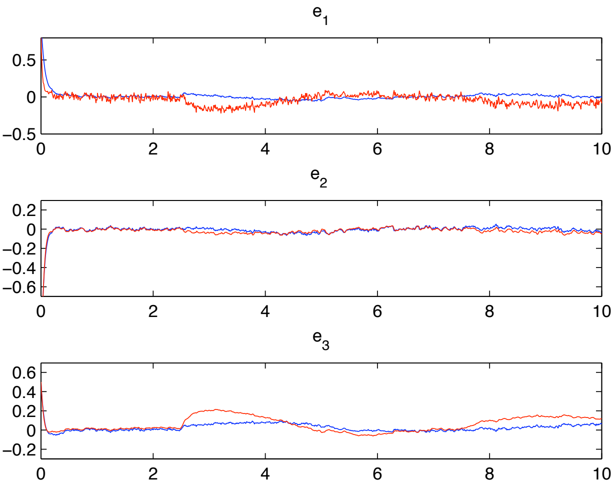
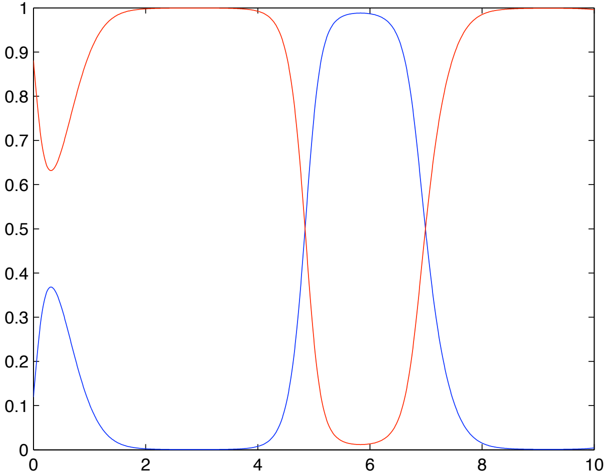


Figure: states of the system and their estimates

Simulation example



Simulation example



Conclusions and perspectives

Conclusions

- An observer is used for estimating the state of nonlinear systems modelled by a multiple model.
- In this multiple model, the decision variable is assumed to be not measurable (One multiple model suffices to develop a banks of observers to detect and isolate actuator and sensor faults.
- Sufficient conditions for ensuring asymptotic convergence and robust performances of the estimation error are proposed.

Perspectives

- The suggested observer can be used in the diagnosis of failures in complex systems.
- The conservatism of the convergence conditions given in the theorem can be reduced by using an types of Lyapunov functions (Polytopic functions, ...).

Thank you for your attention!