State estimation and diagnosis : a polytopic representation approach

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Third International Conference
on Systems and Information Processing
ICSIP'13, May 12-14, Guelma, Algertia

1. State estimation and diagnosis

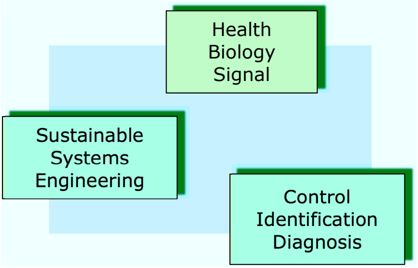
Part 1. Generalities

1. A short presentation of our lab

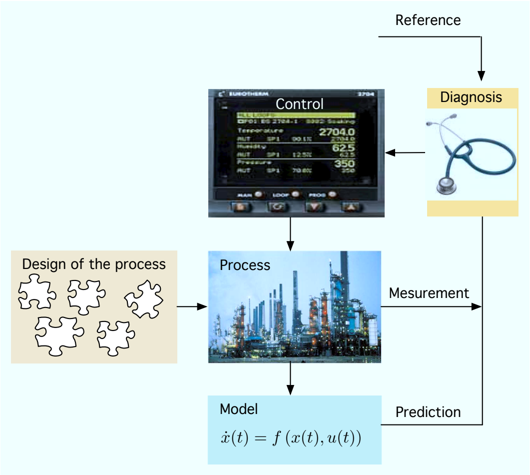
The CRAN was set up in Nancy (France) in 1980. It totals 200 persons. The research activities concentrate on 3 themes :

- Control, Identification, Diagnosis
- Sustainable Systems Engineering
- Health, Biology, Signal

The ≪ Centre de Recherche en Automatique de Nancy ≫ is a Research Centre funded by the "Centre National de la Recherche Scientifique (CNRS)" and two universities in Nancy : UHP (Université Henri Poincaré) and INPL (Institut National Polytechnique de Lorraine).



2. Control and diagnosis



For the elaboration of a global approach for the design and the operation (supervision, maintenance, reconfiguration) of complex automated industrial systems, it is necessary :

- to guarantee the system safety
- to forecast alternate modes

For that purpose, we need :

- to detect functioning modifications
- to detect any fault
- to isolate the faults
- to identify the faults
- to compensate the faults

Moreover, it would be interesting

- to analyze the severity of the faults
- to make a prognosis of the fault evolution

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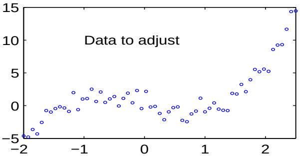
3. A very simple multiple model



Local model 1 : $y_1 = 17.4x + 27.9$
Local model 2 : $y_2 = 11.5x - 17.3$

Weight. function : $\mu_1 = exp\left(-\left(\frac{x+2}{2}\right)^2\right)$

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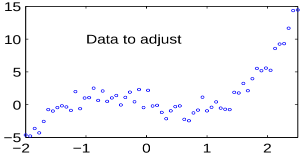
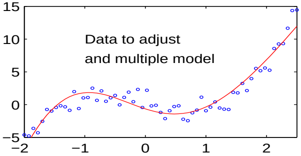
Each of the two local models has a physical meaning.





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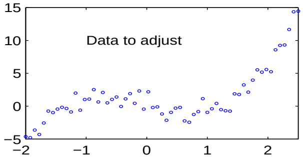

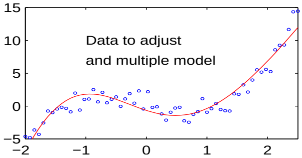

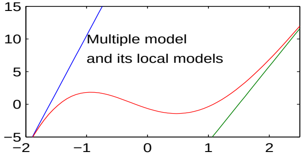

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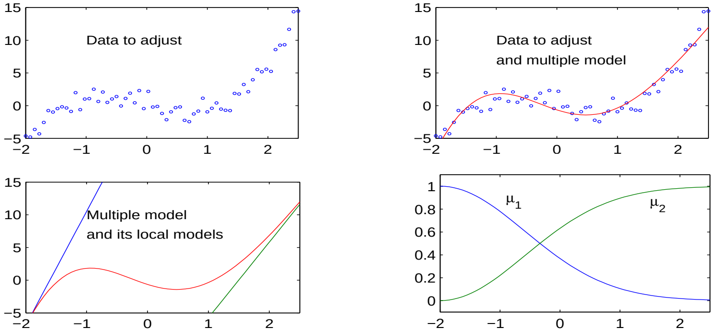

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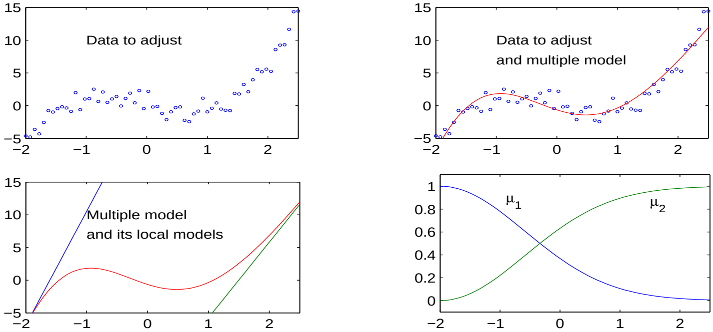

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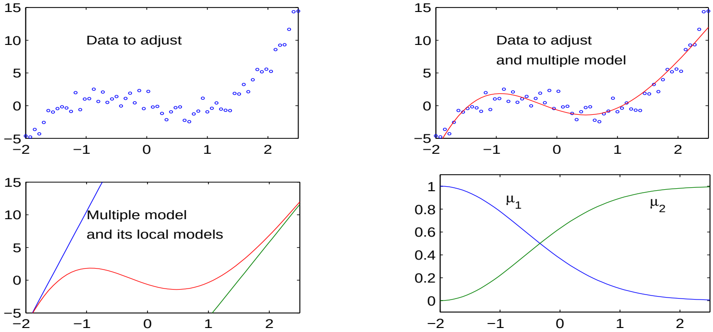

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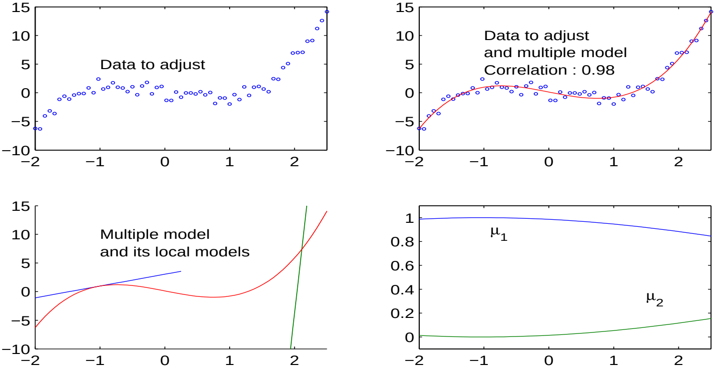
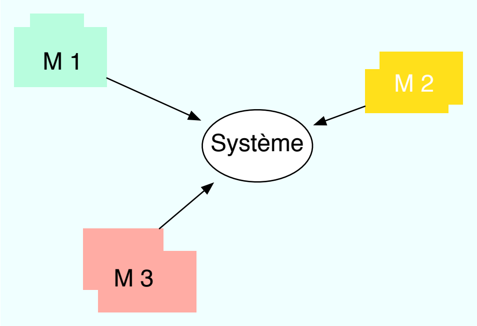


$$\left.\begin{array}{l}\text{Local model 1 : } y_1 = 0.0001x + 1.32 \\ \text{Local model 2 : } y_2 = 67.4x - 98.77 \\ \text{Weight. function : } \mu_1 = exp\left(-\left(\dfrac{x+0.8}{7.1}\right)^2\right)\end{array}\right\} y(x) = \mu_1(x).y_1(x) + \mu_2(x).y_2(x)$$

The two local models have no physical meaning. However, the approximation of the experimental data is quite satisfactory.

- The multimodel approach supposes the definition of a set of local models which make possible to replace the unique model by a set of simpler models.
- Each local model describes the behavior of the considered process around a specific operating point.
- Moreover, it is possible to move from one operating point to another one by using an adequate interpolation mechanism.



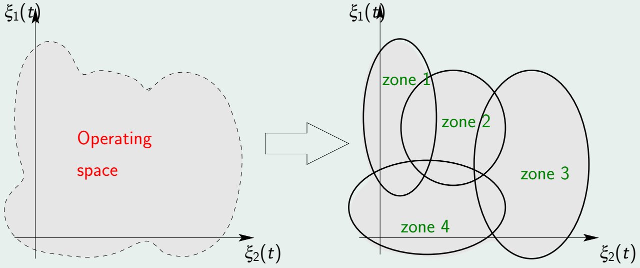
2. General multi-model approach

5. Some thesis of our laboratory

1. A. Boukhris. Identification de systèmes non linéaires par une approche multi-modèle. Application à la modélisation de la relation pluie-débit, 1998.
2. C. Loverini. Identification de systèmes dynamiques non-linéaires à l'aide de représentations multi-modèles, 1999.
3. K. Gasso. Identification des systèmes dynamiques non-linéaires : approche multi-modèle, 2000.
4. I. Bara. Estimation d'état des systèmes linéaires à paramètres variants, 2001.
5. M. Chadli. Analyse des systèmes non linéaires décrits par des structures multimodèles, 2002.
6. A. Akhenak. Conception de multi-observateurs pour des multimodèles. Application au diagnostic, 2004.
7. E. Cherrier. Estimation de l'état et des entrées inconnues pour une classe de systèmes non linéaires, 2006.
8. E. Domlan. Diagnostic des systèmes à changement de régime de fonctionnement, 2006.
9. A. Hocine. Estimation d'état des systèmes à commutation par l'approche multi-modèle : application au diagnostic, 2006.
10. R. Orjuela. Identification et diagnostic des systèmes représentés par un multimodèle, 2008.
11. D. Ichalal. Estimation et diagnostic de systèmes non linéaires décrits par un modèle de Takagi-Sugeno, 2009.
12. A.M. Nagy. Analyse et synthèse de multimodèles pour le diagnostic : application à une station d'épuration, 2011.
13. F. Ankoud. Apport de l'effet parc pour la surveillance et le diagnostic des matériels en centrale nucléaire, 2012.
14. S. Bezzaoucha. Diagnostic et contrôle tolérant aux fautes de systèmes non linéaires sous forme multimodèle (thèse en cours), 2013.

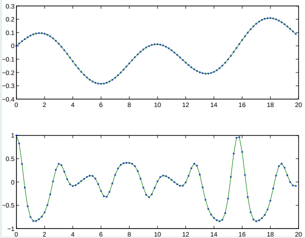
6. Principle of multiple models (Takagi-Sugeno)

- Operating range decomposition in several local zones.
- A local model represents the behavior of the system in a specific zone and for a specific regime.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system Multiple Model representation

We collect data on a system



We already have a model

$y = x^3.sin(x) - x.log(x^2+4)$

Direct identification of the parameters

Mesures : $x_k, y_k, k = 1 \dots N$

$$Model \begin{cases} y = \mu_1.y_1(x) + \mu_2.y_2(x) \\ y_1 = a_1.x + b_1 \quad y_2 = a_2.x + b_2 \\ \mu_1 = \exp(x-m) \quad \mu_2 = 1 - \mu_1 \end{cases}$$

$$\rightarrow \quad a_1, b_1, a_2, b_2, m$$

Problem to be solved :

- number of local models
- structure of the weighting functions

Transformation of the existing model

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- determination of the variable bounds
- structure of the weighting functions

6. Obtaining the Takagi-Sugeno's model

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6. Get the structure of a dynamic MM system

- Non linear state equation

$$\begin{cases} \dot{x}_1 = x_1 + x_1.x_2 + 2.u \\ \dot{x}_2 = x_1^2 + u \end{cases} \qquad \leftarrow \qquad \dot{x} = f(x, u)$$

- State vector

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- The system is nonlinear, difficult to use, but ...
- A quasi LPV form

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} & = & \begin{pmatrix} 1 & x_1 \\ x_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} u \\ \dot{x} & = & A(x).x + B.u \\ A(x) & = & \begin{pmatrix} 1 & x_1 \\ x_1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{cases}$$

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6. Exercise : get the structure of a MM system

- LPV form

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- A useful decomposition : if $x_1 \in [a \quad b]$, it is always possible to write :

$$\boxed{x_1 = \frac{x_1 - a}{b - a} b + \frac{b - x_1}{b - a} a}$$

- In a more concise way :

$$x_1 \quad = \quad \mu_1(x)\ b + \mu_2(x)\ a, \qquad \mu_1(x) = \frac{x_1 - a}{b - a} \quad \mu_2(x) = \frac{b - x_1}{b - a}$$

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6. Exercise : get the structure of a MM system

- LPV form

$$\begin{cases} \dot{x}_1 = x_1 + x_1.x_2 + 2.u \\ \dot{x}_2 = x_1^2 + u \end{cases} \qquad A(x) = \begin{pmatrix} 1 & x_1 \\ x_1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

- A useful decomposition : if $x_1 \in [a \quad b]$, it is always possible to write :

$$\boxed{x_1 = \frac{x_1 - a}{b - a} b + \frac{b - x_1}{b - a} a}$$

- In a more concise way :

$$x_1 \quad = \quad \mu_1(x) \ b + \mu_2(x) \ a, \qquad \mu_1(x) = \frac{x_1 - a}{b - a} \quad \mu_2(x) = \frac{b - x_1}{b - a}$$

- Thus :

$$\begin{cases} A(x) & = & \begin{pmatrix} 1 & \mu_1(x) \ b + \mu_2(x) \ a \\ \mu_1(x) \ b + \mu_2(x) \ a & 0 \end{pmatrix} \\ A(x) & = & \mu_1(x) \begin{pmatrix} 1 & b \\ b & 0 \end{pmatrix} + \mu_2(x) \begin{pmatrix} 1 & a \\ a & 0 \end{pmatrix} \end{cases}$$

- Finally, the MM form obtained without loss of information :

$$\dot{x} = A(x).x + B.u \quad \rightarrow \quad \boxed{\dot{x} = \sum_{i=1}^{2} \mu_i(x) (A_i x + B_i u)}$$

7. Multi-model or Takagi-Sugeno's model

Basic model

- Structure of the model

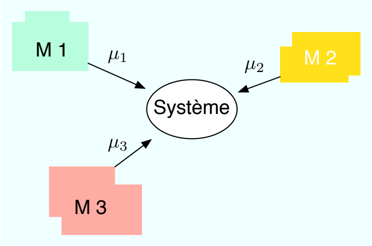
$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = C\, x(t) \end{cases}$$

- Interpolation mechanism

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$$

$$0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$$

- The premise variable $\xi(t)$ can be measurable or not : y(t), u(t), x(t)

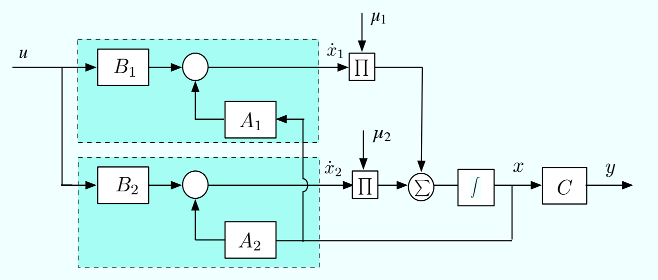


7. Multi-model with coupling effect

Structure with coupling effect

$$\begin{cases} \dot{x}(t) = \left(\sum_{i=1}^{r} \mu_i(\xi(t)) A_i \right) \; x(t) + \left(\sum_{i=1}^{r} \mu_i(\xi(t)) B_i \right) \; u(t) \\ y(t) = C \; x(t) \end{cases}$$

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \quad \text{and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$$

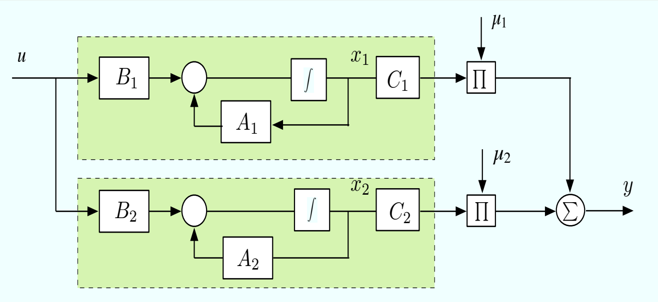


Structure with decoupling effect

- Structure of the decoupled multi-model

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u(t) \\ y_i(t) = C_i x_i(t) \end{cases}, \qquad i = 1, \dots, r$$

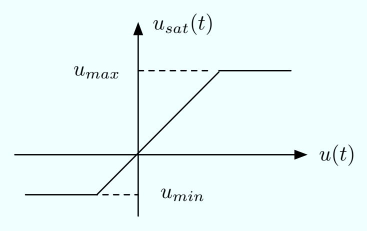
$$\begin{cases} x(t) = \sum_{i=1}^{r} \mu_i(\xi) x_i(t) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi) y_i(t) \end{cases} \quad \text{with} \quad \begin{cases} 0 \leq \mu_i(\xi) \leq 1 \\ \sum_{i=1}^{r} \mu_i(\xi) = 1 \end{cases}$$



8. Multi-model : saturated control

$$\dot{x}(t) = Ax(t) + Bu_{sat}(t)$$

$$u_{sat} = \begin{cases} u(t) & \text{if} \quad u_{min} \leq u(t) \leq u_{max} \\ u_{max} & \text{if} \quad u(t) \geq u_{max} \\ u_{min} & \text{if} \quad u(t) \leq u_{max} \end{cases}$$



$$\begin{cases} u_{sat}(t) & = \mu_1(t)u_{min} + \mu_2(t)u(t) + \mu_3(t)u_{max} \\ \mu_1(t) & = \dfrac{1 - sign(u(t) - u_{min})}{2} \\ \mu_2(t) & = \dfrac{sign(u(t) - u_{min}) - sign(u(t) - u_{max})}{2} \\ \mu_3(t) & = \dfrac{1 + sign(u(t) - u_{max})}{2} \end{cases}$$

$$u_{sat}(t) = \sum_{i=1}^{3} \mu_i(t)(a_i u(t) + b_i)$$

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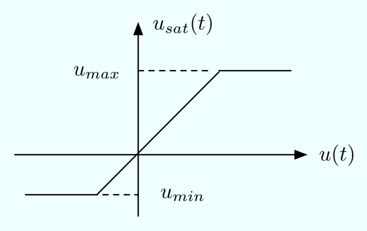
$$u_{sat} = \begin{cases} u(t) & \text{if} \quad u_{min} \leq u(t) \leq u_{max} \\ u_{max} & \text{if} \quad u(t) \geq u_{max} \\ u_{min} & \text{if} \quad u(t) \leq u_{max} \end{cases}$$



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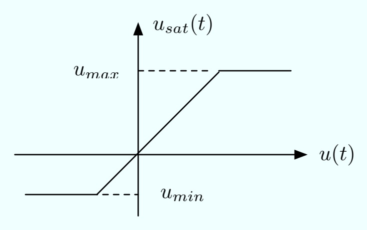
$$u_{sat}(t) = \sum_{i=1}^{3} \mu_i(t)(a_i u(t) + b_i)$$

8. Multi-model : saturated control

Transform a saturation function into multi-model form



$$\begin{cases} \dot{x}(t) & = Ax(t) + Bu_{sat}(t) \\ u_{sat}(t) & = \sum_{i=1}^{3} \mu_i(t)(a_i u(t) + b_i) \end{cases}$$

$$\begin{cases} \dot{x}(t) & = Ax(t) + B\sum_{i=1}^{3} \mu_i(t)(a_i u(t) + b_i) \\ \dot{x}(t) & = \sum_{i=1}^{3} \mu_i(t)(Ax(t) + B_i u(t) + E_i) \end{cases}$$

System

$$\dot{x}(t) = (A_0 + \theta(t)A_1)x(t)$$
$$\theta(t) \in [\theta_1 \quad \theta_2]$$

Transform varying parameter into multi-model

$$\begin{aligned}\theta(t) &= \frac{\theta_2 - \theta(t)}{\theta_2 - \theta_1}\,\theta_1 + \frac{\theta(t) - \theta_1}{\theta_2 - \theta_1}\,\theta_2 \\ &= \mu_1(\theta(t))\theta_1 + \mu_2(\theta(t))\theta_2 \\ &= \sum_{i=1}^{2} \mu_i(t)\theta_i\end{aligned}$$

System into multi-model form

$$\begin{aligned}\dot{x}(t) &= (A_0 + \theta(t)A_1)x(t) \\ &= (A_0 + \sum_{i=1}^{2} \mu_i(t)\theta_i A_1)x(t) \\ &= \sum_{i=1}^{2} \mu_i(t)(A_0 + \theta_i A_i)x(t) \\ &= \sum_{i=1}^{2} \mu_i(t)\overline{A}_i x(t)\end{aligned}$$

8. Multi-model : time varying parameters

System

$$\dot{x}(t) = (A_0 + \theta(t)A_1)x(t)$$
$$\theta(t) \in [\theta_1 \quad \theta_2]$$

Transform varying parameter into multi-model

$$\begin{aligned}
\theta(t) &= \frac{\theta_2 - \theta(t)}{\theta_2 - \theta_1}\ \theta_1 + \frac{\theta(t) - \theta_1}{\theta_2 - \theta_1}\ \theta_2 \\
&= \mu_1(\theta(t))\theta_1 + \mu_2(\theta(t))\theta_2 \\
&= \sum_{i=1}^{2} \mu_i(t)\theta_i
\end{aligned}$$

System into multi-model form

$$\begin{aligned}
\dot{x}(t) &= (A_0 + \theta(t)A_1)x(t) \\
&= (A_0 + \sum_{i=1}^{2} \mu_i(t)\theta_i A_1)x(t) \\
&= \sum_{i=1}^{2} \mu_i(t)(A_0 + \theta_i A_i)x(t) \\
&= \sum_{i=1}^{2} \mu_i(t)\overline{A}_i x(t)
\end{aligned}$$

8. Multi-model : time varying parameters

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$$\dot{x}(t) = (A_0 + \theta(t)A_1)x(t)$$
$$\theta(t) \in [\theta_1 \quad \theta_2]$$

Transform varying parameter into multi-model

$$\begin{aligned}\theta(t) &= \frac{\theta_2 - \theta(t)}{\theta_2 - \theta_1}\ \theta_1 + \frac{\theta(t) - \theta_1}{\theta_2 - \theta_1}\ \theta_2 \\ &= \mu_1(\theta(t))\theta_1 + \mu_2(\theta(t))\theta_2 \\ &= \sum_{i=1}^{2} \mu_i(t)\theta_i\end{aligned}$$

System into multi-model form

$$\begin{aligned}\dot{x}(t) &= (A_0 + \theta(t)A_1)x(t) \\ &= (A_0 + \sum_{i=1}^{2} \mu_i(t)\theta_i A_1)x(t) \\ &= \sum_{i=1}^{2} \mu_i(t)(A_0 + \theta_i A_i)x(t) \\ &= \sum_{i=1}^{2} \mu_i(t)\overline{A}_i x(t)\end{aligned}$$

9. Stability analysis of a multi-model

An important concept for state estimation and control systems

Theorem

The multimodel

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi) A_i x(t)$$

is globally asymptotically stable if there exists $P = P^T > 0$ such that :

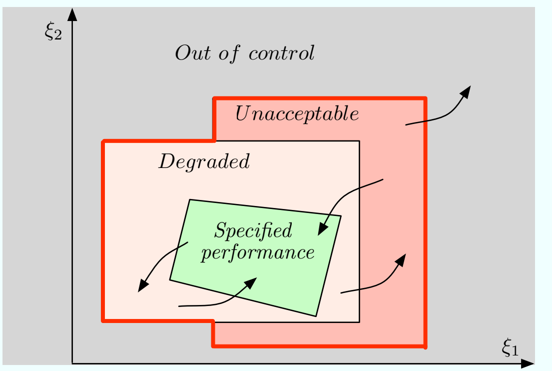
$$A_1^T P + PA_1 < 0$$
$$A_2^T P + PA_2 < 0$$
$$\ldots$$
$$A_r^T P + PA_r < 0$$

Usefulness

- not only for stability test
- but also for control design
- and also for observer design

3. Process diagnosis

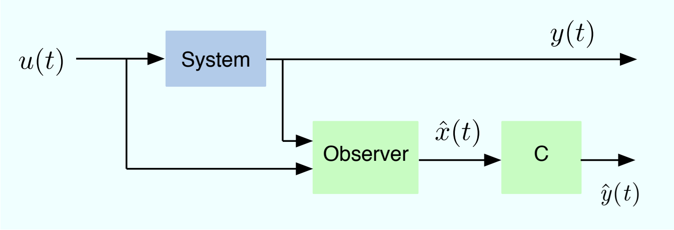
The link between Observer / Diagnosis / Control



- Estimate the state of the system
- Estimate the performance of the system
- Decide to take an action

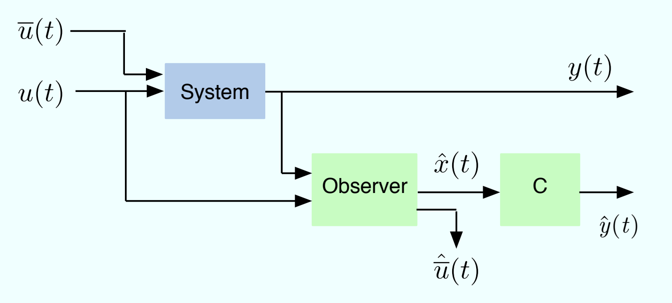
10. Observer

- State and output observer

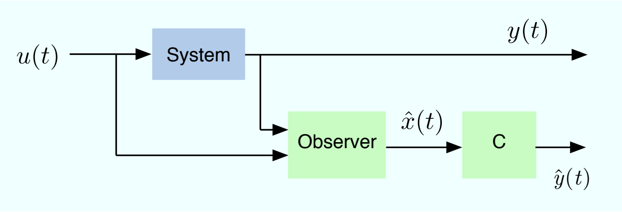


10. Observer

- State and output observer
- State and output observer, with unknown input



10. The observer design



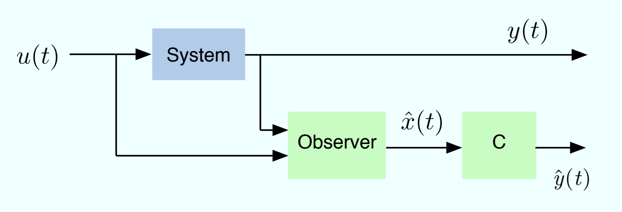
Model of the system

$$\mathscr{S}\begin{cases}\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right)\\ y(t) = Cx(t)\end{cases}$$

State observateur

$$\mathscr{O}\begin{cases}\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)\right)\\ \hat{y}(t) = C\hat{x}(t)\end{cases}$$

10. The observer design



Model of the system

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10. State estimation. First situation : ξ is known

- System

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi)(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi) C_i x(t) \end{cases}$$

- Observer : state estimation

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi)\left(A_i \hat{x}(t) + B_i u(t) + L_i\left(y(t) - \hat{y}(t)\right)\right) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi) C_i \hat{x}(t) \end{cases}$$

- State error

$$\begin{cases} \tilde{x}(t) = x(t) - \hat{x}(t) \\ \dot{\tilde{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi)(A_i - L_i C)\tilde{x}(t) \end{cases} \quad \rightarrow \quad \boxed{L_i \text{ to compute}}$$

- Conditions for stability $\rightarrow$ LMI Toolbox MATLAB

$$(A_i - L_i C)^T P + P(A_i - L_i C) < 0, \qquad i = 1, \ldots, r$$

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- Observer : state estimation

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi)\left(A_i \hat{x}(t) + B_i u(t) + L_i\left(y(t) - \hat{y}(t)\right)\right) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi) C_i \hat{x}(t) \end{cases}$$

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$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi) \left(A_i \hat{x}(t) + B_i u(t) + L_i \left(y(t) - \hat{y}(t)\right)\right) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi) C_i \hat{x}(t) \end{cases}$$

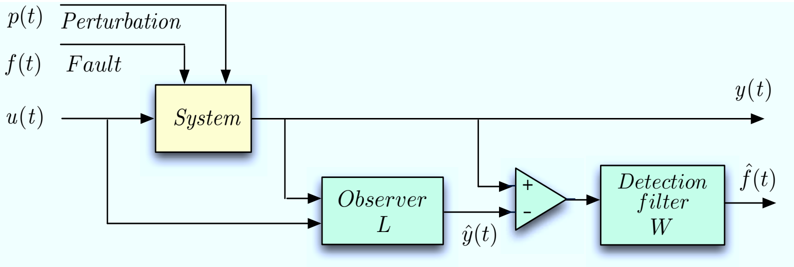
- State error

$$\begin{cases} \tilde{x}(t) = x(t) - \hat{x}(t) \\ \dot{\tilde{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi)(A_i - L_i C)\tilde{x}(t) \end{cases} \quad \rightarrow \quad \boxed{L_i \text{ to compute}}$$

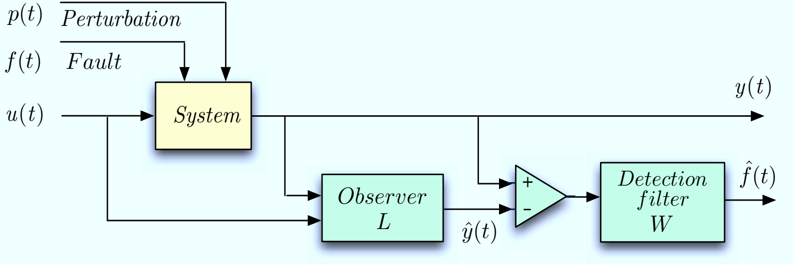
- Conditions for stability $\rightarrow$ LMI Toolbox MATLAB

$$(A_i - L_i C)^T P + P(A_i - L_i C) < 0, \qquad i = 1, \ldots, r$$

11. Principles of process diagnosis



Goal	estimate the state of a system
	detect the occurrence of faults $f(t)$
Difficulty	influence of some perturbations $p(t)$
Residual	signal indicating the occurrence of a fault $f(t)$ $r(t) = W(y(t) - \hat{y}(t)), \quad r \in \mathscr{R}^h, W(.) \in \mathscr{R}^{h \times p}$ where $W(.)$ is a filter that must be properly designed
Fault detection	analyze residuals in order to detect a fault $f(t)$:
Fault isolation	locate where the fault occurred
Fault characterization	try to estimate $\hat{f}(t)$ from $r(t)$.



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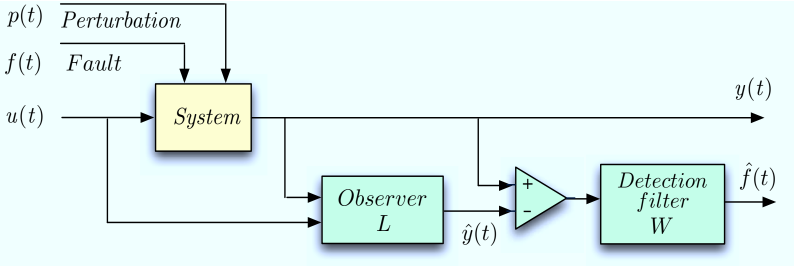

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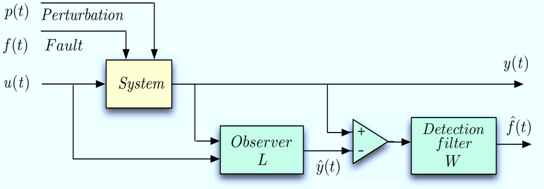
11. Process diagnosis in the multiple-model framework



A faulty disturbed system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i p(t) + F_i f(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + \qquad + G_i p(t) + R_i f(t)) \end{cases}$$

- $f(t)$: fault vector
- $p(t)$: perturbation or disturbance vector.

Problem to solve

- Estimate the state of the system
- How to detect and localize the fault f ?
- Adjust the control u in order to minimize the influence of the fault f ?
- Try to be insensitive to the perturbation p

11. Process diagnosis in the multiple-model framework

- Generally, a fault detection system consists of two main parts :
  - a residual generator
  - a residual evaluator.
- Observer based residual generator :

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{q} \mu_i(\xi(t))(A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases}$$

  where $r(t)$ is the residual signal which will be designed in order to indicate the presence of the fault $f(t)$.
- Objective of the design : adjust the gain L_i and M in order to :
  - minimise the transfer from the perturbations $p(t)$ to the residual signal $r(t)$
  - maximisethe transfer of the faults $f(t)$ to the residual signal $r(t)$.

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- Objective of the design : adjust the gain L_i and M in order to :
  - minimise the transfer from the perturbations $p(t)$ to the residual signal $r(t)$
  - maximisethe transfer of the faults $f(t)$ to the residual signal $r(t)$.

- State reconstruction error :

$$\tilde{x}(t) = x(t) - \hat{x}(t)$$

- Residual :

$$r(t) = M(y(t) - \hat{y}(t))$$

- After straightforward calculation, we deduce :

$$\begin{cases} \dot{\tilde{x}}(t) = A_\xi \tilde{x}(t) + E_\xi p(t) + F_\xi f(t) \\ r(t) = C_\xi \tilde{x}(t) + G_\xi p(t) + R_\xi df(t) \end{cases}$$

where the matrices A_ξ, E_ξ, F_ξ , C_ξ, G_ξ and R_ξ depend on the observer-detecteur parameters L_i et M.

- State reconstruction error :

$$\tilde{x}(t) = x(t) - \hat{x}(t)$$

- Residual :

$$r(t) = M(y(t) - \hat{y}(t))$$

- After straightforward calculation, we deduce :

$$\begin{cases} \dot{\tilde{x}}(t) = A_\xi \tilde{x}(t) + E_\xi p(t) + F_\xi f(t) \\ r(t) = C_\xi \tilde{x}(t) + G_\xi p(t) + R_\xi df(t) \end{cases}$$

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11. Process diagnosis in the multi-model framework

- Expression of the residuals :

$$r(t) = G_{rd}\,p(t) + G_{rf}\,f(t)$$

$$G_{rp} = \left[\begin{array}{c|c} A_\xi & E_\xi \\ \hline C_\xi & G_\xi \end{array}\right]$$

$$G_{rf} = \left[\begin{array}{c|c} A_\xi & F_\xi \\ \hline C_\xi & R_\xi \end{array}\right]$$

- Detection : adjust the gains of the observer in order to maximize the gain of G_{rf}.
- Robustness : adjust the gains of the observer in order to reduce the gain of G_{rp}.
- In order to avoid a mixt problem max_min, let us consider an ideal residue :

$$\tilde{r}(t) = G_{rp}\,p(t) + (G_{rf} - W_f)\,f(t)$$

- To ensure a compromise between the goals of detection and rejection, the design consists in obtaining L_i and M which minimize the quantity $a\gamma_f + (1-a)\gamma_d$ where $a \in [0\ 1]$ subjected to the following constraints :

$$\begin{cases} \| G_{rf} - W_f \|_\infty < \gamma_f \\ \| G_{rp} \|_\infty < \gamma_d \\ \text{Residual generator is stable} \end{cases}$$

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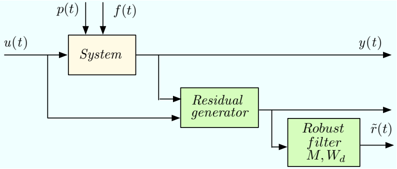

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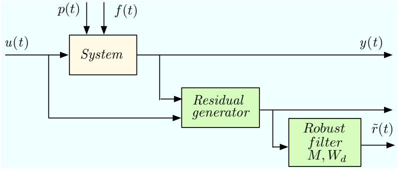

11. Diagnosis approach using multi-model

- System perturbed by faults

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi)\left(A_i\ x(t) + B_i\ u(t) + E_i\ p(t) + F_i\ f(t)\right) \\ y(t) = Cx(t) + \qquad\qquad + G\ p(t) + R\ f(t) \end{cases}$$

- Let consider a 3rd order MM with two faults and one perturbation

$$E_1 = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 \\ 0.3 \\ 0.5 \end{bmatrix}, G = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- Fault $f_1(t)$ acts on the 1st sensor. Fault $f_2(t)$ affects the 3 states. The disturbance $d(t)$ acts on the 3 states and the 2 sensors.
- The weighting functions μ_i :

$$\begin{cases} \mu_1(u(t)) = \dfrac{1 - \tanh((u(t) - 1)/10)}{2} \\ \mu_2(u(t)) = 1 - \mu_1(u(t)) \end{cases}$$

11. Diagnosis approach using multi-model

- System perturbed by faults

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi)(A_i\ x(t) + B_i\ u(t) + E_i\ p(t) + F_i\ f(t)) \\ y(t) = Cx(t) + \qquad + G\ p(t) + R\ f(t) \end{cases}$$

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- System perturbed by faults

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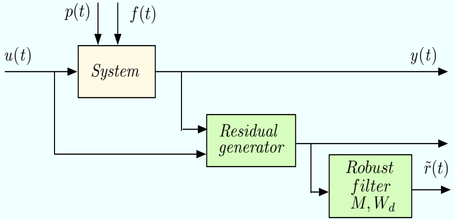


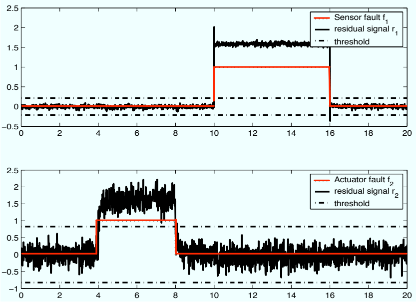


FIGURE: Faults and residuals

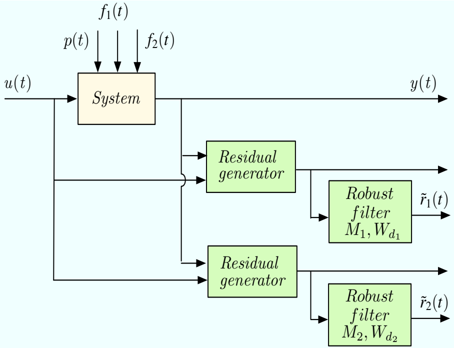
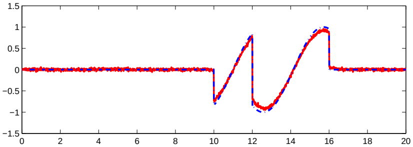


FIGURE: Dedicated filter for detection

A second simulation is performed for fault estimation.
W_f is then an identity matrix.

$$\begin{cases} r(t) & = & G_{rp}\ p(t) + G_{rf}\ f(t) \\ \tilde{r}(t) & = & G_{rp}\ p(t) + (G_{rf} - I)\ f(t) \end{cases}$$



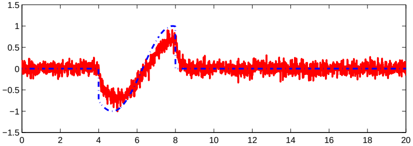
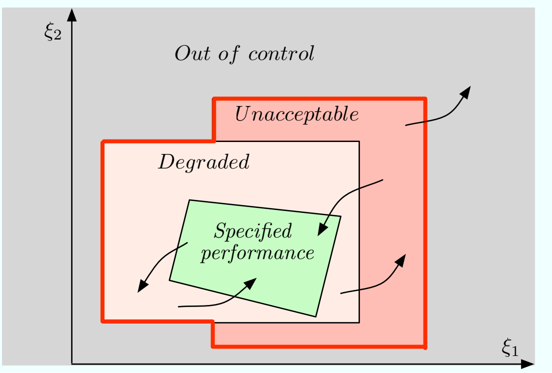
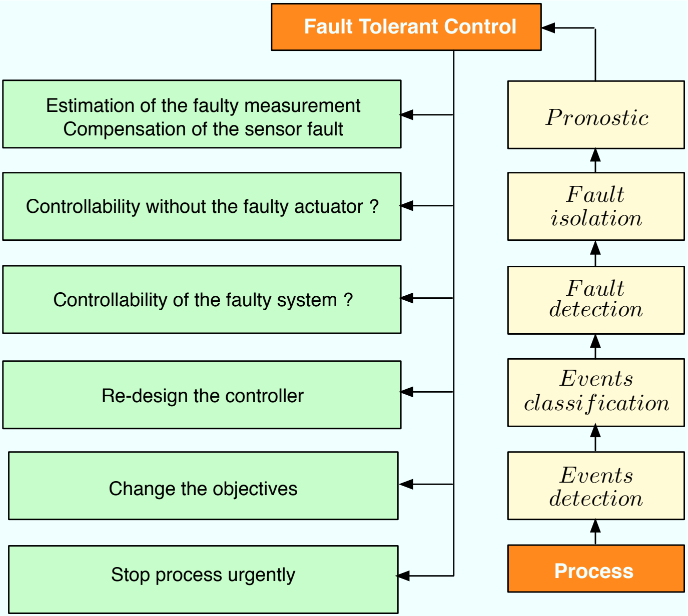


FIGURE: Comparison of the faults (dashed lines) and residual signals (solid lines)

The link between Observer / Diagnosis / Control



12. The different steps of Fault Tolerant Control



12. Fault Tolerant Control (FTC)

In the presence of faults, FTC possess the ability to :

- detect and accommodate the faults
- maintain overall system stability
- maintain ≪ acceptable ≫ performances

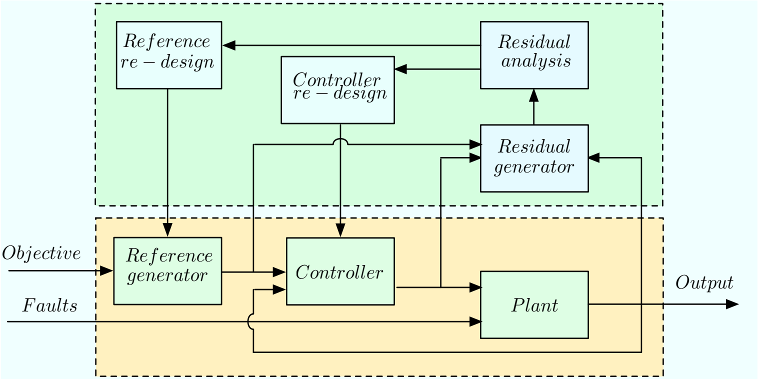


FIGURE: Reconfiguration structure

- System

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i(u(t) + f(t)) \\ y(t) &= Cx(t) + Rf(t) \end{cases}$$

- Reference model

$$\begin{cases} \dot{x}_r(t) &= \sum_{i=1}^{q} \mu_i(x_r(t))(A_i x_r(t) + B_i u_r(t)) \\ y_r(t) &= Cx_r(t) \end{cases}$$

- Control law

$$\begin{cases} u = \sum_{i=1}^{q} (-\hat{d} + K_{1i}(x_r - \hat{x}) + u_r) \\ \dot{\hat{x}} = \sum_{i=1}^{q} \mu_i(\hat{x})(A_i \hat{x} + B_i(u + \hat{d}) + H_{1i}(y - \hat{y}) \\ \dot{\hat{f}} = \sum_{i=1}^{q} \mu_i(\hat{x}) H_{2i}(y - \hat{y}) \end{cases}$$

- System

$$\begin{cases} \dot{x}(t) & = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i(u(t) + f(t)) \\ y(t) & = Cx(t) + Rf(t) \end{cases}$$

- Reference model

$$\begin{cases} \dot{x}_r(t) & = \sum_{i=1}^{q} \mu_i(x_r(t))(A_i x_r(t) + B_i u_r(t)) \\ y_r(t) & = Cx_r(t) \end{cases}$$

- Control law

$$\begin{cases} u = \sum_{i=1}^{q} (-\hat{d} + K_{1i}(x_r - \hat{x}) + u_r) \\ \dot{\hat{x}} = \sum_{i=1}^{q} \mu_i(\hat{x})(A_i \hat{x} + B_i(u + \hat{d}) + H_{1i}(y - \hat{y}) \\ \dot{\hat{f}} = \sum_{i=1}^{q} \mu_i(\hat{x}) H_{2i}(y - \hat{y}) \end{cases}$$

12. Fault Tolerant Control (FTC)

- System

$$\begin{cases} \dot{x}(t) & = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i(u(t) + f(t)) \\ y(t) & = Cx(t) + Rf(t) \end{cases}$$

- Reference model

$$\begin{cases} \dot{x}_r(t) & = \sum_{i=1}^{q} \mu_i(x_r(t))(A_i x_r(t) + B_i u_r \\ y_r(t) & = Cx_r(t) \end{cases}$$

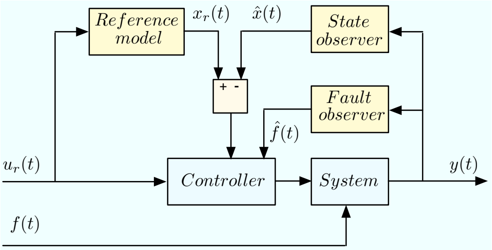


FIGURE: Fault tolerant control structure

- Control law

$$\begin{cases} u = \sum_{i=1}^{q} (-\hat{d} + K_{1i}(x_r - \hat{x}) + u_r) \\ \dot{\hat{x}} = \sum_{i=1}^{q} \mu_i(\hat{x})(A_i \hat{x} + B_i(u + \hat{d}) + H_{1i}(y - \hat{y}) \\ \dot{\hat{f}} = \sum_{i=1}^{q} \mu_i(\hat{x}) H_{2i}(y - \hat{y}) \\ \hat{y} = C\hat{x} + R\hat{f} \end{cases}$$

$$A_1 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix} \; A_2 = \begin{bmatrix} -3 & 2 & 2 \\ 0 & -3 & 0 \\ 5 & 2 & -4 \end{bmatrix} \; B_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \; B_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

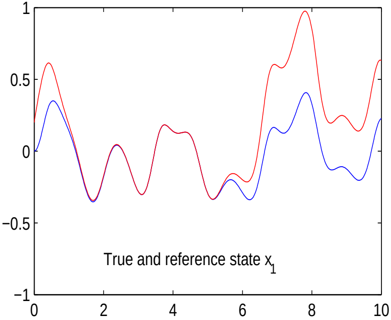


FIGURE: Reference state, faulty state without FTC

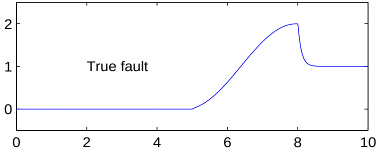


FIGURE: Fault

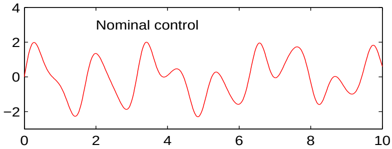


FIGURE: Nominal control

12. Fault Tolerant Control

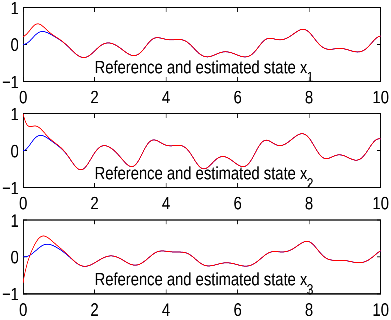


FIGURE: Reference state, faulty states with FTC

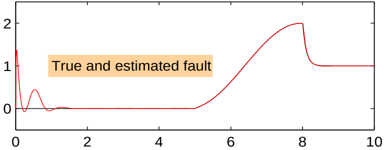


FIGURE: Fault and estimated fault

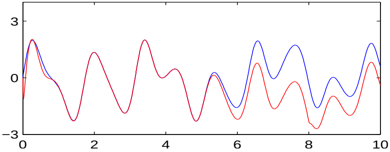


FIGURE: Nominal control and FTC

13. To conclude

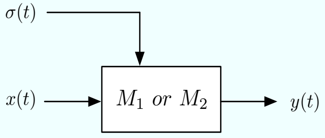
What is possible to do with multiple models ?

- Complexity reduction without loss of information
- State observer of a non linear system
- Unknown input observer
- Fault detection, isolation and estimation
- Fault tolerant control

Perspectives

- Reducing the effect of measurement noise
- Dealing with large scale systems

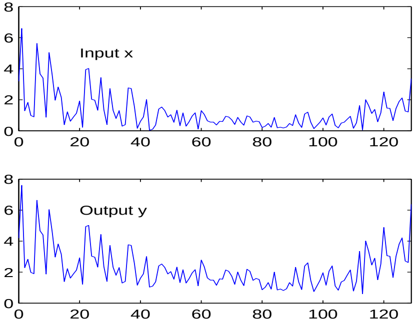
- Model of the system



$$y(t) = f_{\sigma(t)}(x(t))$$

- Problem : knowing only the input $x(t)$ and the output $y(t)$ of the system, is it possible to find the switching between the two modes M_1 and M_2 ?
- Hypothesis

$$\begin{cases} M_1 & y(t) = a_1.x(t) + b_1 \\ M_2 & y(t) = a_2.x(t) + b_2 \end{cases}$$



- Where is the switching time instant ?
- What are the values of the models parameters ?

Idea ?

Thank you for your attention

Vitrail de J. Grüber, Ecole de Nancy, 1904

Additionnal Informations

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