Design of Robust H_∞ Observers for Nonlinear Systems Using a Multiple Model

Rodolfo Orjuela, Benoît Marx, José Ragot and Didier Maquin

Centre de Recherche en Automatique de Nancy (CRAN)
Nancy-Université, CNRS

17th IFAC World Congress
July 6-11, 2008, Seoul, Korea

Goal

State estimation of a nonlinear system subject to perturbations

Why?

- State estimation is often necessary in control and diagnosis
- State estimation of nonlinear systems remains unsolved in a general way

Goal

State estimation of a nonlinear system subject to perturbations

Why?

- State estimation is often necessary in control and diagnosis
- State estimation of nonlinear systems remains unsolved in a general way

Problems

- Take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)
- Observer design problem for generic nonlinear models is very delicate

Goal

State estimation of a nonlinear system subject to perturbations

Why?

- State estimation is often necessary in control and diagnosis
- State estimation of nonlinear systems remains unsolved in a general way

Problems

- Take into consideration the complexity of the system in the whole operating range (nonlinear models are needed)
- Observer design problem for generic nonlinear models is very delicate

Proposed solution

- Multiple model representation of the nonlinear system
- Conception of state estimators based on this representation

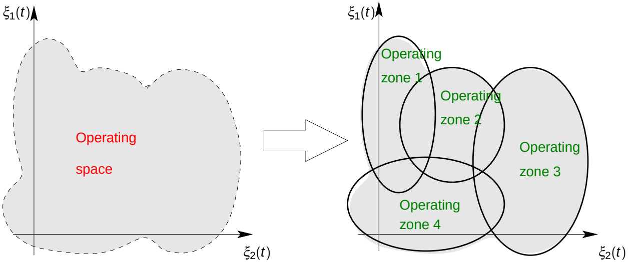
Outline

1. Multiple model approach
   - Basis of Multiple model approach
   - Decoupled multiple model
2. State estimation
   - Proportional observer design
   - Proportional Integral observer design
3. Simulation example
4. Conclusions

Outline

Multiple model Approach

- Decomposition of the operating space into operating zones
- Modelling each zone by a single submodel
- The contribution of each submodel is quantified by a weighting function



Nonlinear system

Multiple model representation

Multiple model = an association of a set of submodels blended by an interpolation mechanism

Why using a multiple model?

- Appropriate tool for modelling complex systems (i.e. black box modelling)
- Tools for linear systems can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

Why using a multiple model?

- Appropriate tool for modelling complex systems (i.e. black box modelling)
- Tools for linear systems can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

How the submodels can be interconnected?

Classic structure Takagi-Sugeno multiple model	**Proposed structure** Decoupled multiple model
Common state vector for all submodels	A different state vector for each submodel
Dimension of the submodels must be identical	Dimension of the submodels may be different

Why using a multiple model?

- Appropriate tool for modelling complex systems (i.e. black box modelling)
- Tools for linear systems can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

How the submodels can be interconnected?

Classic structure
Takagi-Sugeno multiple model

- Common state vector for all submodels
- Dimension of the submodels must be identical

Proposed structure
Decoupled multiple model

- A different state vector for each submodel
- Dimension of the submodels may be different

Why using a multiple model?

- Appropriate tool for modelling complex systems (i.e. black box modelling)
- Tools for linear systems can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

How the submodels can be interconnected?

Classic structure
Takagi-Sugeno multiple model

- Common state vector for all submodels
- Dimension of the submodels must be identical

Proposed structure
Decoupled multiple model

- A different state vector for each submodel
- Dimension of the submodels may be different

Why using a multiple model?

- Appropriate tool for modelling complex systems (i.e. black box modelling)
- Tools for linear systems can be extended to nonlinear systems
- Specific analysis of the system nonlinearity is avoided

How the submodels can be interconnected?

Classic structure
Takagi-Sugeno multiple model

- Common state vector for all submodels
- Dimension of the submodels must be identical

Proposed structure
Decoupled multiple model

- A different state vector for each submodel
- Dimension of the submodels may be different

Decoupled multiple model : Multiple model with local state vectors

$$\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i u(t) \\
y_i(t) &= C_i x_i(t) \ , \\
y(t) &= \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t)
\end{aligned}$$

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \text{ and } 0 \le \mu_i(\xi(t)) \le 1, \forall t, \forall i \in \{1, ..., L\}$$

Decoupled multiple model : Multiple model with local state vectors

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u(t) \\ y_i(t) &= C_i x_i(t)\ , \\ y(t) &= \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t) \end{aligned}$$

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., L\}$$

- The multiple model output is given by a weighted sum of the submodel outputs (blending outputs)

Decoupled multiple model : Multiple model with local state vectors

$$
\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) + B_i u(t) \\
y_i(t) &= C_i x_i(t) \ , \\
y(t) &= \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t)
\end{aligned}
$$

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., L\}$$

- The multiple model output is given by a weighted sum of the submodel outputs (blending outputs)
- **Dimension of the submodels can be different**

Decoupled multiple model : Multiple model with local state vectors

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u(t) \\ y_i(t) &= C_i x_i(t)\ , \\ y(t) &= \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t) + W w(t) \end{aligned}$$

Perturbation

$$\sum_{i=1}^{L} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., L\}$$

- The multiple model output is given by a weighted sum of the submodel outputs (blending outputs)
- **Dimension of the submodels can be different**

State estimation

Augmented form of the multiple model

$$\begin{aligned}\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t)\ ,\\ y(t) &= \tilde{C}(t)x(t) + Ww(t), \quad x \in \mathbb{R}^n,\ n = \sum_{i=1}^{L} n_i\ .\end{aligned}$$

Notations

$$\begin{aligned}x(t) &= \left[x_1^T(t)\cdots x_i^T(t)\cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t)) = \mu_i(t)\ ,\\ \tilde{A} &= \begin{bmatrix} A_1 & 0 & 0 & 0 & 0\\ 0 & \ddots & 0 & 0 & 0\\ 0 & 0 & A_i & 0 & 0\\ 0 & 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & 0 & A_L\end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1\\ \vdots\\ B_i\\ \vdots\\ B_L\end{bmatrix},\\ \tilde{C}(t) &= \left[\mu_1(t)C_1 \quad \ldots \quad \mu_i(t)C_i \quad 0 \quad \ldots \quad \mu_L(t)C_L\right],\end{aligned}$$

Augmented form of the multiple model

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \ , \\ y(t) &= \tilde{C}(t)x(t) + Ww(t), \quad x \in \mathbb{R}^n, \, n = \sum_{i=1}^{L} n_i \ . \end{aligned}$$

Notations

$$\begin{aligned} x(t) &= \left[x_1^T(t) \cdots x_i^T(t) \cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t)) = \mu_i(t) \ , \\ \tilde{A} &= \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix}, \\ \tilde{C}(t) &= \left[\mu_1(t)C_1 \quad \dots \quad \mu_i(t)C_i \quad 0 \quad \dots \quad \mu_L(t)C_L\right], \end{aligned}$$

Augmented form of the multiple model

$$
\begin{aligned}
\dot{x}(t) &= \tilde{A}x(t)+\tilde{B}u(t) \ , \\
y(t) &= \tilde{C}(t)x(t)+Ww(t), \quad x\in\mathbb{R}^n,\, n=\sum_{i=1}^{L} n_i \ .
\end{aligned}
$$

Linear form

Notations

$$
\begin{aligned}
x(t) &= \left[x_1^T(t)\cdots x_i^T(t)\cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t))=\mu_i(t) \ , \\
\tilde{A} &= \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix}, \tilde{B}= \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix}, \\
\tilde{C}(t) &= \begin{bmatrix}\mu_1(t)C_1 & \ldots & \mu_i(t)C_i & 0 & \ldots & \mu_L(t)C_L\end{bmatrix},
\end{aligned}
$$

Augmented form of the multiple model

$$\begin{aligned}\dot{x}(t) &= \tilde{A}x(t)+\tilde{B}u(t)\ ,\\ y(t) &= \tilde{C}(t)x(t)+Ww(t), \quad x\in\mathbb{R}^n,\ n=\sum_{i=1}^{L} n_i\ .\end{aligned}$$

Nonlinear form

Notations

$$\begin{aligned}x(t) &= \left[x_1^T(t)\cdots x_i^T(t)\cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t))=\mu_i(t)\ ,\\ \tilde{A} &= \begin{bmatrix} A_1 & 0 & 0 & 0 & 0\\ 0 & \ddots & 0 & 0 & 0\\ 0 & 0 & A_i & 0 & 0\\ 0 & 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & 0 & A_L\end{bmatrix},\ \tilde{B}=\begin{bmatrix}B_1\\ \vdots\\ B_i\\ \vdots\\ B_L\end{bmatrix},\\ \tilde{C}(t) &= \left[\mu_1(t)C_1 \quad \ldots \quad \mu_i(t)C_i \quad 0 \quad \ldots \quad \mu_L(t)C_L\right],\end{aligned}$$

Augmented form of the multiple model

$$\begin{aligned}\dot{x}(t) &= \tilde{A}x(t)+\tilde{B}u(t) \ , \\ y(t) &= \tilde{C}(t)x(t)+Ww(t), \quad x\in\mathbb{R}^n,\ n=\sum_{i=1}^{L} n_i \ .\end{aligned}$$

Nonlinear form

Notations

$$\begin{aligned}x(t) &= \left[x_1^T(t)\cdots x_i^T(t)\cdots x_L^T(t)\right]^T \quad \text{and} \quad \mu_i(\xi(t))=\mu_i(t) \ , \\ \tilde{A} &= \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix}, \\ \tilde{C}(t) &= \begin{bmatrix}\mu_1(t)C_1 & \ldots & \mu_i(t)C_i & 0 & \ldots & \mu_L(t)C_L\end{bmatrix}, \\ &= \sum_{i=1}^{L}\mu_i(t)\tilde{C}_i, \qquad \tilde{C}_i=\begin{bmatrix}0 & \ldots & C_i & 0 & \ldots & 0\end{bmatrix} \ .\end{aligned}$$

Goal

State estimation of a system represented by a multiple model

Proportional Observer

$$\begin{aligned}\dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) + \tilde{B}u(t) + \tilde{K}(y(t) - \hat{y}(t)) \ , \\ \hat{y}(t) &= \tilde{C}(t)\hat{x}(t) \ .\end{aligned}$$

- This observer uses a proportional action

Assumption 1

The perturbation is a bounded energy signal, i.e. $\|w(t)\|_2^2 < \infty$.

Goal

State estimation of a system represented by a multiple model

Proportional Observer

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) + \tilde{B}u(t) + \tilde{K}\left(y(t) - \hat{y}(t)\right) \ , \\
\hat{y}(t) &= \tilde{C}(t)\hat{x}(t) \ .
\end{aligned}
$$

- This observer uses a proportional action

Assumption 1

The perturbation is a bounded energy signal, i.e. $\|w(t)\|_2^2 < \infty$.

Estimation error

$$e(t) = x(t) - \hat{x}(t) \; ,$$

Estimation error

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) \ , \\ \dot{e}(t) &= \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t) \ , \end{aligned}$$

Estimation error

$$
\begin{aligned}
e(t) &= x(t) - \hat{x}(t) \ , \\
\dot{e}(t) &= \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t) \ , \\
&= A_{obs}(t)e(t) - \tilde{K}Ww(t) \ .
\end{aligned}
$$

Estimation error

$$
\begin{aligned}
e(t) &= x(t) - \hat{x}(t) \ , \\
\dot{e}(t) &= \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t) \ , \\
&= A_{obs}(t)e(t) - \tilde{K}Ww(t) \ .
\end{aligned}
$$

Comments

- The impact of $w(t)$ on $e(t)$ is directly modified by $\tilde{K}$.
- Goal: finding the gain matrix $\tilde{K}$ such that the influence of $w(t)$ on $e(t)$ is attenuated.

Estimation error

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) \ , \\ \dot{e}(t) &= \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t) \ , \\ &= A_{obs}(t)e(t) - \tilde{K}Ww(t) \ . \end{aligned}$$

Comments

- The impact of $w(t)$ on $e(t)$ is directly modified by $\tilde{K}$.
- Goal: finding the gain matrix $\tilde{K}$ such that the influence of $w(t)$ on $e(t)$ is attenuated.

Difficulties

- The matrix $A_{obs}(t)$ is time-varying ($\mu_i(t)$ are used !)
- Blending between submodels must be taken into account

Performance constraints

We introduce the following $\mathscr{H}_\infty$ performance constraints:

$$\begin{aligned} \lim_{t\to\infty} e(t) = 0 \quad &\text{for} \quad w(t) = 0 \ , \\ \|e(t)\|_2^2 \leq \gamma^2 \|w(t)\|_2^2 \quad &\text{for} \quad w(t) \neq 0 \text{ and } e(0) = 0 \,, \end{aligned}$$

where γ is the L_2 gain from $w(t)$ to $e(t)$ to be minimised.

Theorem

The optimal PO for the decoupled multiple model is obtained if there exists a matrix $P = P^T > 0$ and a matrix G minimizing $\bar{\gamma} > 0$ under the following LMIs

$$\begin{bmatrix} P\tilde{A} - G\tilde{C}_i + (P\tilde{A} - G\tilde{C}_i)^T + \mathrm{I} & -GW \\ -(GW)^T & -\bar{\gamma}\mathrm{I} \end{bmatrix} < 0, \quad i = 1 \ldots L$$

The observer gain is given by $\tilde{K} = P^{-1}G$ and the L_2 gain from $w(t)$ to $e(t)$ is given by $\gamma = \sqrt{\bar{\gamma}}$.

Idea

(i) Robust performances are guaranteed if there exists a Lyapunov function $V(t)$ satisfying

$$\dot{V}(t) < -e^T(t)e(t) + \gamma^2 w^T(t)w(t)$$

(ii) Using the following Lyapunov function

$$V(t) = e^T(t)Pe(t), \quad P = P^T > 0$$

(iii) Using the estimation error equation

$$\dot{e}(t) = \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t)$$

and some algebraic manipulations, the result follows

Idea

(i) Robust performances are guaranteed if there exists a Lyapunov function $V(t)$ satisfying

$$\dot{V}(t) < -e^T(t)e(t) + \gamma^2 w^T(t)w(t)$$

(ii) Using the following Lyapunov function

$$V(t) = e^T(t)Pe(t), \quad P = P^T > 0$$

(iii) Using the estimation error equation

$$\dot{e}(t) = \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t)$$

and some algebraic manipulations, the result follows

Idea

(i) Robust performances are guaranteed if there exists a Lyapunov function $V(t)$ satisfying

$$\dot{V}(t) < -e^T(t)e(t) + \gamma^2 w^T(t)w(t)$$

(ii) Using the following Lyapunov function

$$V(t) = e^T(t)Pe(t), \quad P = P^T > 0$$

(iii) Using the estimation error equation

$$\dot{e}(t) = \sum_{i=1}^{L} \mu_i(t)(\tilde{A} - \tilde{K}\tilde{C}_i)e(t) - \tilde{K}Ww(t)$$

and some algebraic manipulations, the result follows

Comments on the P observer

- Only one degree of freedom is available for observer design
- Perturbation attenuation *versus* dynamics performances

Comments on the P observer

- Only one degree of freedom is available for observer design
- Perturbation attenuation *versus* dynamics performances

Goal of the Proportional Integral observer

- Introduce robustness in the state estimation
- Two degrees of freedom for the observer design can be used
- Attenuation level can be accomplished with good dynamics performances

Comments on the P observer

- Only one degree of freedom is available for observer design
- Perturbation attenuation *versus* dynamics performances

Goal of the Proportional Integral observer

- Introduce robustness in the state estimation
- Two degrees of freedom for the observer design can be used
- Attenuation level can be accomplished with good dynamics performances

Multiple model structure

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \ , \\ \dot{z}(t) &= \tilde{C}(t)x(t) + W\omega(t) \ , \\ y(t) &= \tilde{C}(t)x(t) + W\omega(t) \ , \end{aligned}$$

where $$z(t) = \int_0^t y(\xi)d\xi.$$

Comments on the P observer

- Only one degree of freedom is available for observer design
- Perturbation attenuation *versus* dynamics performances

Goal of the Proportional Integral observer

- Introduce robustness in the state estimation
- Two degrees of freedom for the observer design can be used
- Attenuation level can be accomplished with good dynamics performances

Multiple model structure

Supplementary variable

$$
\begin{aligned}
\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \ , \\
\dot{z}(t) &= \tilde{C}(t)x(t) + W\omega(t) \ , \\
y(t) &= \tilde{C}(t)x(t) + W\omega(t) \ ,
\end{aligned}
$$

where $$z(t) = \int_0^t y(\xi)d\xi.$$

Compact multiple model structure

$$\begin{aligned} \dot{x}_a(t) &= \tilde{A}_1(t)x_a(t) + \bar{C}_1\tilde{B}u(t) + \bar{C}_2W\omega(t) \ , \\ y(t) &= \tilde{C}(t)\bar{C}_1^T x_a(t) + W\omega(t) \ , \\ z(t) &= \bar{C}_2^T x_a(t) \ . \end{aligned}$$

Compact multiple model structure

$$\begin{aligned}\dot{x}_a(t) &= \tilde{A}_1(t)x_a(t)+\bar{C}_1\tilde{B}u(t)+\bar{C}_2W\omega(t) \ , \\ y(t) &= \tilde{C}(t)\bar{C}_1^T x_a(t)+W\omega(t) \ , \\ z(t) &= \bar{C}_2^T x_a(t) \ .\end{aligned}$$

Proportional Integral Observer

$$\begin{aligned}\dot{\hat{x}}_a(t) &= \tilde{A}_1(t)\hat{x}_a(t)+\bar{C}_1\tilde{B}u(t)+K_P(y(t)-\hat{y}(t))+K_I(z(t)-\hat{z}(t)) \ , \\ \hat{y}(t) &= \tilde{C}(t)\bar{C}_1^T\hat{x}_a(t) \ , \\ \hat{z}(t) &= \bar{C}_2^T\hat{x}_a(t) \ .\end{aligned}$$

Proportional action

Integral action

Notations

$$x_a(t)=\begin{bmatrix}x(t)\\z(t)\end{bmatrix},\bar{C}_1=\begin{bmatrix}I\\0\end{bmatrix},\bar{C}_2=\begin{bmatrix}0\\I\end{bmatrix},\tilde{A}_1(t)=\sum_{i=1}^{L}\mu_i(t)\bar{A}_i,\bar{A}_i=\begin{bmatrix}\tilde{A} & 0\\ \tilde{C}_i & 0\end{bmatrix}$$

Estimation error

$$e_a(t) = x_a(t) - \hat{x}_a(t) \;,$$

Estimation error

$$
\begin{aligned}
e_a(t) &= x_a(t) - \hat{x}_a(t) \ , \\
\dot{e}_a(t) &= \sum_{i=1}^{L} \mu_i(t)(\bar{A}_i - K_P \tilde{C}_i \bar{C}_1^T - K_I \bar{C}_2^T) e_a(t) - (\bar{C}_2 W - K_P W) w(t) \ ,
\end{aligned}
$$

Estimation error

$$
\begin{aligned}
e_a(t) &= x_a(t) - \hat{x}_a(t) \ , \\
\dot{e}_a(t) &= \sum_{i=1}^{L} \mu_i(t)(\bar{A}_i - K_P \tilde{C}_i \bar{C}_1^T - K_I \bar{C}_2^T) e_a(t) - (\bar{C}_2 W - K_P W) w(t) \ , \\
&= A_{obs}(t) e_a(t) - (\bar{C}_2 W - K_P W) \tilde{w}(t) \ .
\end{aligned}
$$

Estimation error

$$
\begin{aligned}
e_a(t) &= x_a(t) - \hat{x}_a(t) \ , \\
\dot{e_a}(t) &= \sum_{i=1}^{L} \mu_i(t)(\bar{A}_i - K_P\tilde{C}_i\bar{C}_1^T - K_I\bar{C}_2^T)e_a(t) - (\bar{C}_2 W - K_P W)w(t) \ , \\
&= A_{obs}(t)e_a(t) - (\bar{C}_2 W - K_P W)\tilde{w}(t) \ .
\end{aligned}
$$

Comments

- The impact of $w(t)$ on $e_a(t)$ is directly modified by K_P
- Dynamics performances may be adjusted via K_I
- Goal: finding the gain matrices K_P and K_I such that the influence of $w(t)$ on $e_a(t)$ is attenuated.

Theorem

The optimal PIO for the decoupled multiple model is obtained if there exist a matrix $P = P^T > 0$ and matrices L_P and L_I minimizing $\bar{\gamma} > 0$ under the following LMIs

$$\begin{bmatrix} \mathbb{S}(P\bar{A}_i - L_P\tilde{C}_i\bar{C}_1^T - L_I\bar{C}_2^T) + \mathrm{I} & P\bar{C}_2W - L_PW \\ (P\bar{C}_2W - L_PW)^T & -\bar{\gamma}\mathrm{I} \end{bmatrix} < 0, \quad i = 1...L$$

where $\mathbb{S}(M) = M + M^T$.
The observer gains are given by

$$K_P = P^{-1}L_P \quad \text{and} \quad K_I = P^{-1}L_I$$

the L_2 gain from $\omega(t)$ to $e_a(t)$ is given by

$$\gamma = \sqrt{\bar{\gamma}}$$

.

Example

Example

Consider the following decoupled multiple model with $L = 2$ submodels, the parameters are given by:

$$A_1 = \begin{bmatrix} -2.0 & 0.5 & 0.6 \\ -0.3 & -0.9 & -0.5 \\ -1.0 & 0.6 & -0.8 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.8 & -0.4 \\ 0.1 & -1.0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.0 & 0.8 & 0.5 \end{bmatrix}^T, \qquad B_2 = \begin{bmatrix} -0.5 & 0.8 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.9 & -0.8 & -0.5 \\ -0.4 & 0.6 & 0.7 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.8 & 0.6 \\ 0.4 & -0.7 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.3 \end{bmatrix}.$$

The weighting functions are normalised Gaussian functions

$$\mu_i(\xi(t)) = \omega_i(\xi(t)) / \sum_{j=1}^{L} \omega_j(\xi(t)) \quad \text{with} \quad \omega_i(\xi(t)) = \exp\left(-(\xi(t) - c_i)^2/\sigma^2\right),$$

with $\sigma = 0.5$, $c_1 = 0.25$ et $c_2 = 0.75$. The decision variable is $\xi(t) = u(t)$.

Proportional observer

$$\tilde{K} = \begin{bmatrix} 0.193 & -0.030 & 0.092 & -0.032 & -0.014 \\ 0.224 & 0.113 & 0.220 & -0.016 & -0.027 \end{bmatrix}^T,$$

with an attenuation index $\gamma^2 = 2$.

Proportional Integral observer

$$K_P = \begin{bmatrix} 0.004 & -0.035 & -0.007 & -0.017 & -0.003 & 0.934 & -0.032 \\ 0.017 & 0.027 & 0.030 & 0.007 & -0.005 & 0.002 & 0.941 \end{bmatrix}^T,$$

$$K_I = \begin{bmatrix} 0.142 & 0.568 & 0.131 & 0.050 & 0.008 & 2.626 & 0.017 \\ 0.139 & 0.519 & 0.156 & -0.010 & 0.008 & 0.017 & 2.496 \end{bmatrix}^T,$$

with an attenuation index $\gamma^2 = 0.1$.

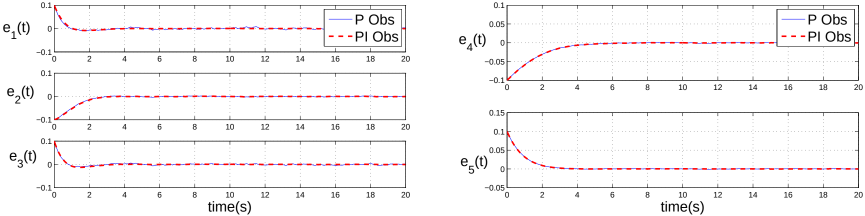


Figure: State estimation errors

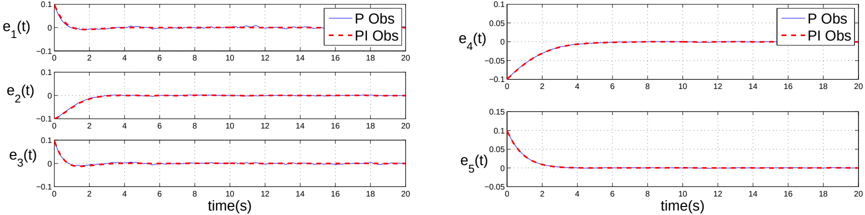


Figure: State estimation errors

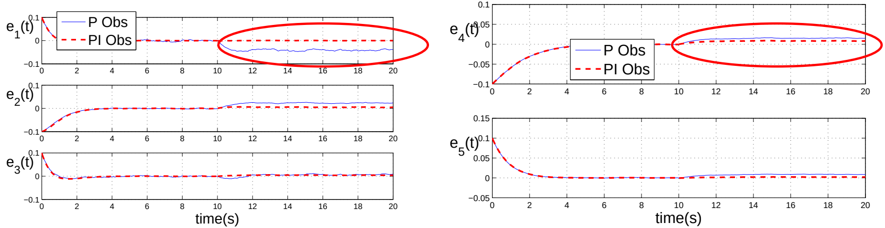


Figure: State estimation errors with an output perturbation

Conclusions

- State estimation based on a multiple model representation of a nonlinear system is investigated
- In the proposed multiple model, the dimension of each submodel may be different (flexibility in a black box modelling stage can be provided)
- Conception of a Proportional and a Proportional-Integral observer is proposed using the Lyapunov theory
- The Proportional-Integral observer offers more degrees of freedom consequently state estimation robustness is introduced

Thank you!