Estimation of a generic model for a fleet of machines

Farah Ankoud, Gilles Mourot, Roger Chevalier, Nicolas Paul, José Ragot

Institut National Polytechnique de Lorraine

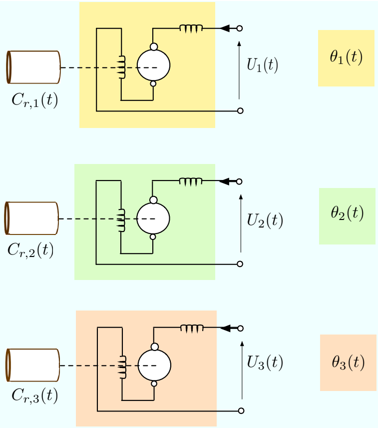
Centre de Recherche en Automatique de Nancy

Electricité de France

International Conference on Communications, Computing and Control Applications. March 3-5, 2011. Hammamet, Tunisia.

Definition and goal

- Fleet of machines : a collection of machines a priori identical
- Estimating a generic model for a fleet of identical machines
- Deduce a generic strategy for the diagnosis of a fleet of machines



Motivations

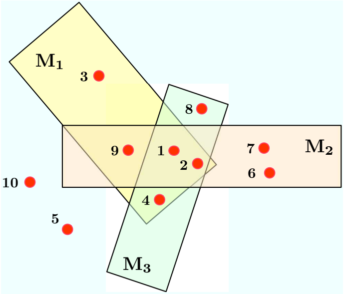
- Reducing the cost of estimating the model of each machine
- Facility to construct the model of a new machine
- Facility to replace a machine by another one
- Reducing the cost of system maintenance

Plan

Introduction

Introduction : generic model

- The problem consists in determining if a generic model representing the normal behavior of each machine of the fleet can be established.
- A generic model is composed of two parts :
  - a common part made up of the variables of the machine itself
  - a distinct part related to the environmental variables.
- The work deals with :
  - classification of the variables : common variables extraction
  - identification of models sharing an estimated common part.
- Exemple : 3 machines M_1, M_2 and M_3 with 10 variables



	1	2	3	4	5	6	7	8	9	10
M_1	×	×	×	×	.	.	.	.	×	.
M_2	×	×	.	.	.	×	×	.	×	.
M_3	×	×	.	×	.	.	.	×	.	.

Variables 1 and 2
Variables 4 and 9
Variables 3, 5, 6, 7, 8

Historical point of view

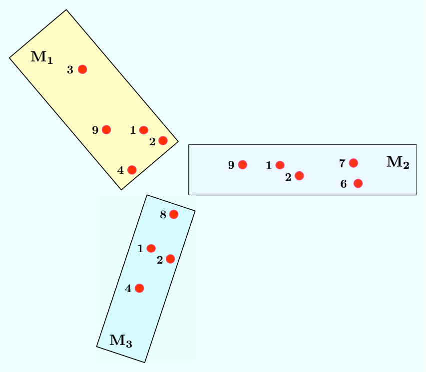
- Multitask learning
  http://books.nips.cc/papers/files/nips19/NIPS2006-0251.pdf
- Fleet Maintenance Systems
  http://www.serco-na.com/Download.aspx?ID=288&Type=Story
- Fleet Inventory Tracking
  http://www.mex.com.au/Products/FleetMEX.aspx
- Patents on fleet of machines
  http://www.freepatentsonline.com/5737215.html

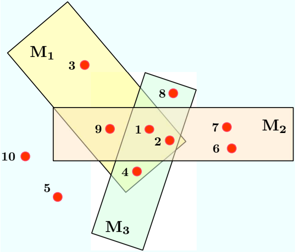
Identifying the models with their common part

Identifying the models with their common part

After finding the (linear) models describing the behavior of each machine independently from the other machines, the method consists in :

1. Identifying the common variables in the models of the different machines
2. identifying the potentially identical coefficients → common part
3. estimating new parameters of the models considering the common part
4. validating the choice of the common part





Identifying the model of one machine

- Consider the kth machine, with :

  - y^{*k} variable to explain
  - W^k variable possibly explaining y^{*k}
  - X^k variable selected for explaining y^{*k}
  - $\hat{\theta}^k$ model parameters

- Model

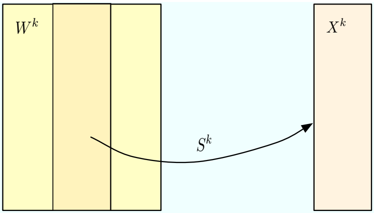
$$\hat{y}^k = X^k \hat{\theta}^k$$

$$X^k = W^k S^k$$

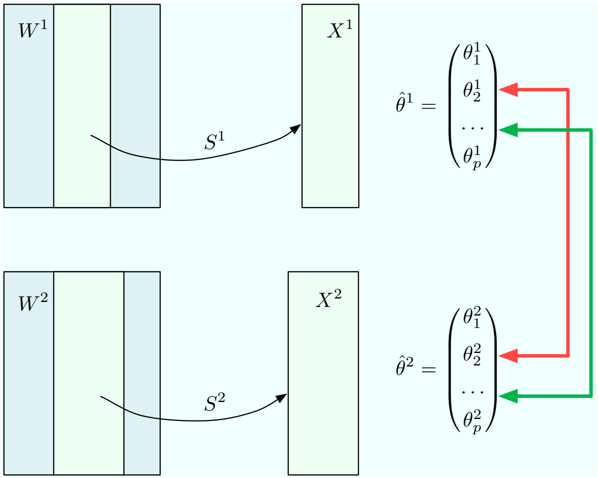
$$\hat{\theta}^k = ({X^k}^T X^k)^{-1} {X^k}^T y^k$$

- S^k is a selection matrix.
  For example, the following matrix permits to select variables 2 and 4 from a set of 5 variables :

$$S^k = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$



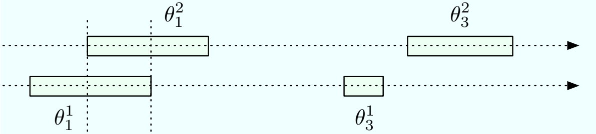
Identifying the models of several machines



- Analyse the proximity of the coefficients of the variables $\hat{\theta}^1$ and $\hat{\theta}^2$
- Decide which coefficients must be forced to be identical

Identifying the identical coefficients of several machines

- Principle :



- Confidence intervals of the parameters

The standard deviation $\hat{\sigma}_i^k$ is the estimated standard deviation calculated from :

$$\tilde{y}^k = y^k - \hat{y}^k, \quad \hat{\sigma}^2 = \frac{1}{n_k - p_k} \parallel \tilde{y}^k \parallel^2$$

$$\hat{\Sigma}_\theta^k = \hat{\sigma}^2 ({X^k}^T X^k)^{-1}$$

$\hat{\sigma}_i^k$ appears on the i^{th} term of the diagonal of $\hat{\Sigma}_\theta^k$.

$$I_i^k = [\hat{\theta}_i^k - 2.32\hat{\sigma}_i^k \ ; \ \hat{\theta}_i^k + 2.32\hat{\sigma}_i^k]$$

- Coefficients $\hat{\theta}_i^k$ $(\forall k)$ are considered identical if a non null intersection exists between their confidence intervals I_i^k :

$$I_i^{k_1} \cap I_i^{k_2} \neq \varnothing \rightarrow \hat{\theta}_i^{k_1} \textit{ and } \hat{\theta}_i^{k_2} \textit{ are identical}$$

Estimating the parameters considering the common part

- Model of machine k

$$y^k = X^k \theta^k$$

The common part of the coefficients is composed of p coefficients α :

$$\theta^k = \begin{pmatrix} \alpha \\ \beta^k \end{pmatrix}, \quad X^k = \begin{pmatrix} U^k \\ V^k \end{pmatrix}$$

The two matrices U^k and V^k are selected, with S_p^k and $S_{\bar{p}}^k$, from X^k :

$$U^k = X^k S_p^k \qquad \text{and} \qquad V^k = X^k S_{\bar{p}}^k$$

- Modèle of the fleet of machines

$$\underbrace{\begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^K \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} U^1 & V^1 & 0 & \cdots & 0 \\ U^2 & 0 & V^2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ U^K & 0 & \cdots & 0 & V^K \end{bmatrix}}_{Z} \underbrace{\begin{bmatrix} \alpha \\ \beta^1 \\ \beta^2 \\ \vdots \\ \beta^K \end{bmatrix}}_{\theta}$$

- Estimate of the new coefficients

$$\hat{\theta} = \left(Z^T Z\right)^{-1} Z^T Y \quad \Longrightarrow \quad \hat{\alpha}, \hat{\beta}^1, \ldots, \hat{\beta}^K$$

Validating the choice of the identical coefficients

▷ First estimated $\hat{y}^k$ of y^k without coupling effect

$$\tilde{y}^k = y^k - \hat{y}^k, \qquad \hat{y}^k = X^k \hat{\theta}^k, \qquad \boxed{\Phi_1 = \sum_{k=1}^{K} \| \tilde{y}^k \|^2}$$

▷ Second estimates $\hat{\hat{y}}^k$ of y^k with coupling effect :

$$\tilde{\tilde{y}}^k = y^k - \hat{\hat{y}}^k, \qquad \hat{\hat{y}}^k = \begin{bmatrix} U^k & V^k \end{bmatrix} \begin{bmatrix} \hat{\alpha} \\ \hat{\beta}^k \end{bmatrix}, \qquad \boxed{\Phi_2 = \sum_{k=1}^{K} \| \tilde{\tilde{y}}^k \|^2}$$

▷ Comparison of the two sums of squares

$$\boxed{\tau = \frac{N-P}{(K-1)p} \cdot \frac{\Phi_2 - \Phi_1}{\Phi_1}} \qquad \begin{cases} N = \sum_{k=1}^{K} n_k & \textit{Number of data} \\ P = \sum_{k=1}^{K} p_k & \textit{Number of parameters} \end{cases}$$

$$\boxed{\textit{If } \tau \leq \mathcal{F}_a((K-1)p, N-P) \rightarrow \textit{no significative loss of information}}$$

Remark : a direct formulation ?

Previous formulation

- Construct the machine models one by one without any interaction

$$\Phi_i = \| y_i - X_i\theta_i \|^2, \qquad i = 1, \dots, K$$

- Determine the common parts between the different models
- Analyse the structure of the models
- Construct a global model taking identity coefficient constraints into account

A one step formulation

- Global objective

$$\Phi = \sum_{i=1}^{K} \frac{1}{2} \| y_i - X_i\theta_i \|^2 + \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \frac{1}{2} \gamma_{i,j} (\theta_i - \theta_j)^T W_{i,j}^2 (\theta_i - \theta_j)$$

- Estimate simultaneoulsy

$$\theta_i, W_{i,j}$$

- The values of the weights directly point out the link between the parameters

Numerical exemple

Numerical exemple

Three models and three databases of 250 observations each are generated according to :

$$y^1 = x_1^1 + 5x_3^1 + 5.5x_5^1 - 10 + \varepsilon^1$$

$$y^2 = x_2^2 + 5x_3^2 + 0.6x_4^2 + 5.68x_5^2 - 12 + \varepsilon^2$$

$$y^3 = 0.5x_1^3 + 1.2x_2^3 + 5.1x_3^3 + 0.7x_4^3 + 5.3x_5^3 - 14 + \varepsilon^3$$

where ε^k is a zero-mean signal with a variance proportional to the range of y^k.

Step 1 : Identifying the model in each database

The estimate of y^k in each database independently from the others is given by :

$$\hat{y}^1 = 0.89x_1^1 + 5.09x_3^1 + 5.66{x_5}^1 - 10.29$$

$$\hat{y}^2 = 1.15x_2^2 + 4.89x_3^2 + 0.54x_4^2 + 5.40x_5^2 - 11.52$$

$$\hat{y}^3 = 0.58x_1^3 + 1.18x_2^3 + 4.98x_3^3 + 0.66x_4^3 + 5.31x_5^3 - 13.84$$

Numerical exemple

Step 2 : Finding the identical coefficients

Coef.	Interval
$\hat{\theta}_0^1$	[-11.13 ; -9.46]
$\hat{\theta}_1^1$	[0.62 ; 1.15]
$\hat{\theta}_3^1$	[4.86 ; 5.34]
$\hat{\theta}_5^1$	[5.41 ; 5.91]

Coef.	Interval
$\hat{\theta}_0^2$	[-12.38 ; -10.66]
$\hat{\theta}_2^2$	[0.89 ; 1.39]
$\hat{\theta}_3^2$	[4.66 ; 5.12]
$\hat{\theta}_4^2$	[0.31 ; 0.78]
$\hat{\theta}_5^2$	[5.19 ; 5.61]

Coef.	Interval
$\hat{\theta}_0^3$	[-14.87 ; -12.81]
$\hat{\theta}_1^3$	[0.34 ; 0.81]
$\hat{\theta}_2^3$	[0.87 ; 1.49]
$\hat{\theta}_3^3$	[4.73 ; 5.22]
$\hat{\theta}_4^3$	[0.39 ; 0.93]
$\hat{\theta}_5^3$	[5.09 ; 5.53]

The coefficients of variables x_3 and x_5 are unique over all the databases : a non null intersection of [4.86 ; 5.12] and [5.41 ; 5.53], respectively, exists between their confidence intervals.

Numerical exemple

Step 3 : Validating the choice of identical coefficients

The expressions of the new estimates of y^k are :

$$\tilde{y}^1 = 0.94x_1^1 + 4.98x_3^1 + 5.44x_5^1 - 9.72$$

$$\tilde{y}^2 = 1.13x_2^2 + 4.98x_3^2 + 0.54x_4^2 + 5.44x_5^2 - 11.75$$

$$\tilde{y}^3 = 0.58x_1^3 + 1.16x_2^3 + 4.98x_3^3 + 0.66x_4^3 + 5.44x_5^3 - 14.09$$

Before coupling effect : $\Phi_1 = 127.95$, after coupling effect : $\Phi_2 = 129.31$

$$\boxed{\tau = \frac{N-P}{(K-1)p} \cdot \frac{\Phi_2 - \Phi_1}{\Phi_1} = 1.95}$$

with : $N = 750$, $P = 15$, $K = 3$, $p = 2$ and for a confidence level of 99%

$$\boxed{F_a = 3.34}$$

Then $\tau \leq F_a$ and thus the coupling effect may be considered without a significative loss of information.

Numerical exemple

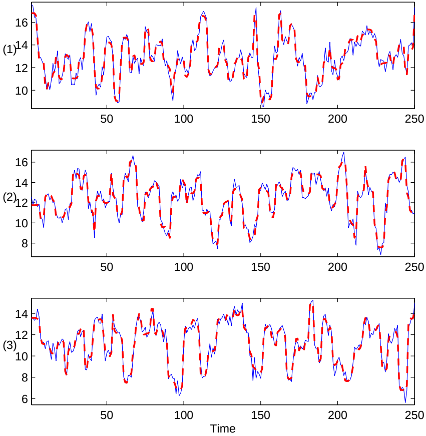


FIGURE: y^k (blue solid line) and $\hat{y}^k$ (red dashed line) in each database

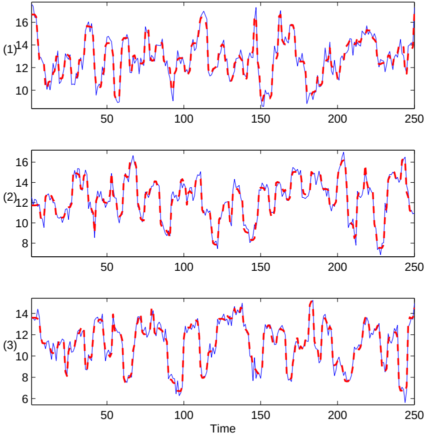


FIGURE: y^k (blue solid line) and $\tilde{y}^k$ (red dashed line) in each database

Interpretation :

Conclusions and perspectives

Conclusions and perspectives

Conclusions

- A method for identifying the common part of a fleet of machines based on data collected from this fleet.
- A generic model describing the normal behavior of the machines is estimated taking into account the common part.

Perspectives

- The method will be completed in order to identify the part of the model associated to the environment.
- When a new machine is added, how to establish its model while taking into account the generic model ?
- The method will be tested on real data collected from different pumps working in several nuclear power plants of EDF.

Thank you for your attention