Unknown input estimation with a multiple model. Application to secure communications

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***Abstract*— This paper is dedicated to the synthesis of a multiple observer. The considered system is represented by a (nonlinear) multiple model with unknown inputs. Stability conditions of such observer are expressed in terms of linear matrix inequalities (LMI). A simulation example related to secure communication is given to illustrate the proposed method.**

I. INTRODUCTION

A physical process is often subjected to disturbances which have as origin the noises due to its environment, uncertainty of measurements, fault of sensors and/or actuators. These disturbances have harmful effects on the normal behavior of the process and their estimation can be used to conceive a control strategy able to minimize their effects. The disturbances are called unknown inputs when they affect the input of the process and their presence can make difficult the state estimation.

In the linear system framework, observers can be designed for singular systems, unknown input systems, delay systems and also uncertain system with time-delay perturbations [8]. Several works were also achieved concerning the estimation of the state and the output in the presence of unknown inputs. They can be gathered into two categories. The first one supposes an a priori knowledge of information on these nonmeasurable inputs; in particular, Johnson [12] proposes a polynomial approach and Meditch [16] suggests approximating the unknown inputs by the response of a known dynamic system. The second category proceeds either by estimation of the unknown inputs, or by their complete elimination from the equations of the system.

Among the techniques that do not require the elimination of the unknown inputs, Wang [17] proposes an observer able to entirely reconstruct the state of a linear system in the presence of unknown inputs and in [5], [13], [15], to estimate the state, a model inversion method is used. Using the Walcott and Zak structure observer [17], Edwards et al. [6], [7] have also designed a convergent observer using the Lyapunov approach. Other techniques are based on the elimination of the unknown inputs [9], [14].

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However, the real physical systems are often nonlinear. As it is delicate to synthesize an observer for a nonlinear system, we preferred to represent these systems with a multiple model. The idea of the multiple model approach is to apprehend the total behavior of a system by a set of local models (linear or affine), each local model characterizing the behavior of the system in a particular zone of operation. The local models are then aggregated by means of an interpolation mechanism.

In the case of a nonlinear system affected by unknown inputs and described by a multiple model, a technique for multiple model state estimation by using a multiple observer with sliding mode has already been proposed [1], [4].

In this paper, we consider the state estimation of an uncertain multiple model with unknown input. For that purpose a multiple observer based on convex interpolation of classical Luenberger observers [2] involving additive terms used to overcome the uncertainties is designed. Using a quadratic Lyapunov function, sufficient asymptotic stability conditions are given in LMI formulation [3].

II. STATE AND INPUT ESTIMATION USING A MULTIPLE MODEL

In this work, we consider the estimation of the state vector and the unknown inputs of a nonlinear system represented by a multiple model and subject to the influence of unknown inputs, by using a multiple observer. This multiple observer is based on local Luenberger observers.

A. Multiple model structure

Let us consider a nonlinear system represented by the following discret multiple model (with M local models) subject to unknown inputs:

$$\begin{cases} x(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) \Big(A_i x(t) + B_i u(t) + R_i \bar{u}(t) + D_i \Big) \\ y(t) = Cx(t) + F\bar{u}(t) \end{cases} \tag{1}$$

with :

$$\begin{cases} \sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \\ 0 \leq \mu_i(\xi(t)) \leq 1 \quad \forall\, i \in \{1, .., M\} \end{cases}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ the input vector, $\bar{u}(t) \in \mathbf{R}^q$, $q < n$, the unknown input and $y(t) \in \mathbf{R}^p$ gathers the measured outputs. The i^{th} "local model" uses $A_i \in \mathbf{R}^{n\times n}$ as the state matrix, $B_i \in \mathbf{R}^{n\times m}$ for the input influence matrix, $R_i \in \mathbf{R}^{n\times q}$ for the unknown input influence matrix and $D_i \in \mathbf{R}^{n\times 1}$ is introduced to take into account the functioning point of the system; $C \in \mathbf{R}^{p\times n}$ and $F \in \mathbf{R}^{p\times q}$. At last, $\xi(t)$ is the so-called decision vector which may depend on the known input and/or the measured state variables.

At each time, $\mu_i(\xi(t))$ quantifies the relative contribution of each local model to the construct the global model. Choosing the number M of local models of that multiple model may be intuitively achieved when taking into account the number of operating regimes. However, determining the matrices A_i, B_i, R_i and D_i needs the use of specific technics [10]. For a practical point of view, the matrices A_i, B_i, R_i and D_i are those used to describe the local functioning around the i^{th} regime. Indeed, that is exactly the case at the i^{th} regime, where $\mu_i(\xi(t)) = 1$ and $\mu_j(\xi(t)) = 0, j \neq i$. In fact, the values of the functions μ_i are not Boolean and the output of the multiple model is a weighted sum of the output of each local model.

The problem to be solved here is those of the simultaneous reconstruction of the state variable $x(t)$ and the unknown input $\overline{u}(t)$ when only using the information available in the known input u and in the measured output $y(t)$.

B. Multiple observer Design

In this section, we explain how to design the observer. The structure of that observer results of the aggregation of local observers [4] and the obtained analytical form is particularly adapted for studying the stability and the convergence property of the state reconstruction error. The numerical aspects related to the determination of the gains of the observer will be also analyzed. The so-called multiple observer (1) has the following structure:

$$\begin{cases} z(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t))\Big(N_i z(t) + G_{i1}u(t) + G_{i2} + L_i y(t)\Big) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (2)$$

where $N_i \in \mathbf{R}^{n\times n}$, $G_{i1} \in \mathbf{R}^{n\times m}$, $L_i \in \mathbf{R}^{n\times p}$ is the gain of the i^{th} local observer, $G_{i2} \in \mathbf{R}^n$ is a constant vector and E is a transformation matrix. Indeed, the observer only uses known variables u and y, $\bar{u}$ being non measured. This whole set of matrices has to be properly defined, and mainly on a numerical point of view, the objective is to ensure the convergence of the estimated state towards the true state. For that purpose, let us define the state estimation error:

$$e(t) = x(t) - \hat{x}(t) \quad (3)$$

From that definition and using the expression of $\hat{x}(t)$ given by equation (2), the previous error can be written:

$$e(t) = (I + EC)x(t) - z(t) + EF\overline{u}(t) \quad (4)$$

Then, one expresses the time evolution of the state estimation error in order to analyse its convergence towards zero. Thus, at time $t+1$, the state estimation error is expressed:

$$e(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t))\Big(P\big(A_i x(t) + B_i u(t) + R_i\bar{u}(t) + D_i\big) - N_i z(t) - G_{i1}u(t) - G_{i2} - L_i y(t)\Big) + EF\overline{u}(t+1) \quad (5)$$

with :

$$P = I + EC \quad (6)$$

Replacing $y(t)$ and $z(t)$ by their respective expressions given by (1) and (2), the state error takes the form:

$$e(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t))\Big(N_i e(t) + \big(PA_i - N_i P - L_i C\big)x(t) + (PB_i - G_{i1})u(t) + (PD_i - G_{i2}) + \big(PR_i - L_i F)\bar{u}(t)\Big) + EF\bar{u}(t+1) \quad (7)$$

If the following conditions are fulfilled:

$$\begin{cases} P = I + EC \\ N_i P = PA_i - L_i C \\ PR_i = L_i F \\ G_{i1} = PB_i \\ G_{i2} = PD_i \\ EF = 0 \end{cases} \quad (8)$$

equation (7) reduces to :

$$e(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e(t) \quad (9)$$

A simplification that will be further used is proposed. It is straightforward to verify that (8) may be writhen with the help of the matrix K_i:

$$\begin{cases} P = I + EC \\ N_i = PA_i - K_i C \\ K_i = N_i E + L_i \\ PR_i = K_i F \\ G_{i1} = PB_i \\ G_{i2} = PD_i \\ EF = 0 \end{cases} \quad (10)$$

The decay rate of the state estimation error is depending on the matrix $N = \sum_{i=1}^{M} \mu_i(\xi(t))N_i$ and it is important to note that the stability of matrices N_i, $\forall\ i \in \{1, ..., M\}$ does not prove the stability of N. That point will be analyzed in the next section. Thus, the constraints (10) allow to synthesis an observer of a system with unknown inputs. However, for some applications (for example in diagnosis), the estimation of the unknown input $\bar{u}$ has to be performed. That point will be addressed in the section II-D. Moreover, the stability of the matrix N needs to be guaranteed while taking account all the matrix constraints (8); that technical point is the aim of the section II-E.

C. Global convergence of the multiple observer

In this part, sufficient conditions of the asymptotic global convergence of the state estimation error are established. As expressed by the model of the state error estimation, (9), the convergence is strongly depending on the matrix $N = \sum_{i=1}^{M} \mu_i (\xi(t)) N_i$.

Theorem [2] : *The state estimation error between the multiple model (1) and the unknown input multiple observer (2) converges towards zero, if all the pairs (A_i, C) are observable, the matrix F is of full column rank and if the following conditions hold $\forall\ i \in \{1, ..., M\}$:*

$$N_i^T X N_i - X < 0 \tag{11a}$$
$$N_i = PA_i - K_i C \tag{11b}$$
$$P = I + EC \tag{11c}$$
$$PR_i = K_i F \tag{11d}$$
$$EF = 0 \tag{11e}$$
$$L_i = K_i - N_i E \tag{11f}$$
$$G_{i1} = PB_i \tag{11g}$$
$$G_{i2} = PD_i \tag{11h}$$

where $X \in \mathbf{R}^{n \times n}$ is a positive definite symmetric matrix.

The proof of that theorem may be found in [2]. Let us just note that the stability condition of N is expressed by the matrix inequalities (11a). The conditions (11b) to (11h) may be seen as an equivalent form of the constraints (10). The system (10) involves bilinear matrix inequalities (11a), that must be solved while taking into account some equality constraints. Let us note that equations (11f), (11g) and (11h) are only used to compute the gains L_i, G_{i1} and G_{i2} since matrices X, N_i, P, K_i and E will be known.

D. Unknown input estimation

We have previously shown that the convergence of the multiple observer (2) is guaranteed if the conditions (10) hold and the pairs (A_i, C) are observable. Under steady state condition, the state estimation error tends towards zero; then substituting the true state x by its estimate $\hat{x}$ in equation (1), the input $\bar{u}$ is replaced by its estimation $\hat{\bar{u}}$:

$$\begin{cases} \hat{x}(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t)) \big(A_i \hat{x}(t) + B_i u(t) + R_i \hat{\bar{u}}(t) + D_i \big) \\ y(t) = C\hat{x}(t) + F\hat{\bar{u}}(t) \end{cases} \tag{12}$$

The unknow input $\bar{u}$ is then estimated by using the whole set of equations (11):

$$\hat{\bar{u}}(t) = (W^T W)^{-1} W^T \times \begin{pmatrix} \hat{x}(t+1) - \sum_{i=1}^{M} \mu_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + D_i \right) \\ y(t) - C\hat{x}(t) \end{pmatrix} \tag{13}$$

assuming that the matrix

$$W = \begin{pmatrix} \sum_{i=1}^{M} \mu_i (\xi(t)) R_i \\ F \end{pmatrix} \tag{14}$$

is of full column rank. Summarizing the estimation procedure, two steps are needed: the first one is dedicated to the state estimation using the observer (2), the second is devoted to the unknown input estimation using the estimated state (12). The condition allowing the expression of the matrices of the observer are linked to the rank of W. However, for the secure communication application (section III), the constraint may be easily fulfilled since we have to design both the observer and the process itself.

E. Resolution method for determining the observer matrices

When analyzing the different constraints, (11e) completely determine the matrix E of the observer. Noting $F^{(-)}$ a generalized inverse of F, E may be deduced:

$$E = I - FF^{(-)} \tag{15}$$

As a consequence, the matrix P may be deduced from (11c). Then, the matrix inequalities (11a) have to be solved after substituting the matrix N_i by its value derived from (11b).

$$N_i^T X N_i - X = (PA_i - K_i C)^T X (PA_i - K_i C) - X < 0 \tag{16}$$

which is equivalent, using the Schur complement to:

$$\begin{pmatrix} X & (PA_i - K_i C)^T X \\ X(PA_i - K_i C) & X \end{pmatrix} > 0 \tag{17}$$

With the following change of variables:

$$W_i = XK_i \tag{18}$$

the constraint (11d) is rewritten:

$$XPR_i = XK_i F = W_i F \tag{19}$$

It is then necessary to solve the LMI (17) subject to the constraint (19)

$$\begin{pmatrix} X & A_i^T P X - C^T W_i^T \\ XPA_i - W_i C & X \end{pmatrix} > 0 \tag{20a}$$
$$XPR_i = W_i F \tag{20b}$$

The system being linear in respect to the unknown matrices X and W_i, conventional LMI tools (LMI MATLAB Toolbox for example) may be extensively used for that resolution.

The other matrices defining the observer are then deduced knowing E, P, X and W_i:

$$G_{i1} = PB_i \tag{21a}$$

$$G_{i2} = PD_i \tag{21b}$$

$$K_i = X^{-1}W_i \tag{21c}$$

$$N_i = PA_i - K_iC \tag{21d}$$

$$L_i = K_i - N_iE \tag{21e}$$

III. Application to communication

Let us consider a discrete SISO multiple model resulting of the aggregation of two local models:

$$\begin{cases} x(t+1) = \sum_{i=1}^{2} \mu_i(\xi(t))\big(A_i x(t) + R_i \bar{u}(t)\big) \\ y(t) = Cx(t) + F\bar{u}(t) \end{cases} \tag{22}$$

The system (22) has the particularity to be controlled by the unique input $\bar{u}(t)$ and its output $y(t)$ is the input of the observer. The activation functions are expressed with exponential fonctions and only depend on the multiple model output ($\xi(t) = y(t)$):

$$\begin{cases} \xi(t) = y(t) \\ \mu_1(\xi(t)) = \frac{1}{2}(1 - \tanh(\xi(t))) \\ \mu_2(\xi(t)) = 1 - \mu_1(\xi(t)) \end{cases} \tag{23}$$

Applying results given in section II-C, the observer is defined by:

$$\hat{x}(t+1) = \sum_{i=1}^{M} \mu_i(\xi(t))\big(N_i\hat{x}(t) + K_i y(t)\big) \tag{24}$$

with the definitions:

$$E = 0 \tag{25a}$$

$$P = I \tag{25b}$$

$$R_i = K_iF \tag{25c}$$

$$N_i = A_i - K_iC \tag{25d}$$

$$L_i = K_i \tag{25e}$$

The numerical values of matrices are as follows:

$$A_1 = \begin{bmatrix} 0 & 0.4 & 1 \\ -1.12 & 0.4 & 0 \\ -0.8 & 0 & 0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0.4 & 1 \\ 1 & 0.4 & 0 \\ -0.8 & 0 & 0.9 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.15 & 0 & 0 \end{bmatrix}, \quad F = 50$$

The figure 1 shows the signal y transmitted to the observer and the message contained in y. The figure 2 compares the true and the estimated states of the system. The figure 3 depicts the trajectory of the system; as there are 3 states, the trajectory is drawn in the plans $\{x_1(t), x_2(t)\}$, $\{x_2(t), x_3(t)\}$ and $\{x_3(t), x_1(t)\}$; thus it is possible to appreciate the "chaotic" behavior of the system. The figure 4 presents the estimated message, the true message and the activation function μ. Excepted around the time origine (due to unappropriate initial conditions), the estimated message fully agree with the true one.

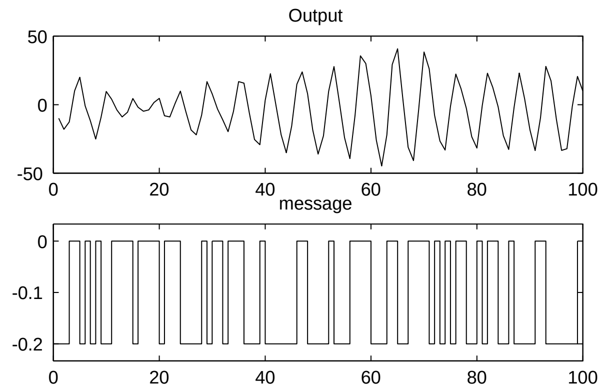

Fig. 1. Output $y(t)$ and message $\bar{u}(t)$

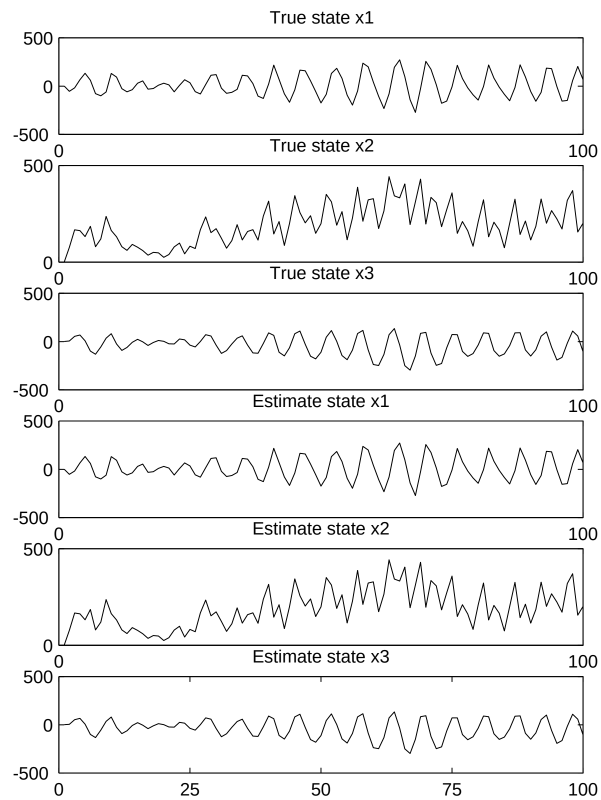

Fig. 2. True and estimated states and estimated state

IV. Conclusion

In this communication, we propose a method for estimating the state of a non linear discrete system; this system is modelized by a multiple model in which some input are unknown. The calculation of the gain of the global observer reduces to the calculation of the gains of the local observers; the stability of the whole requires taking

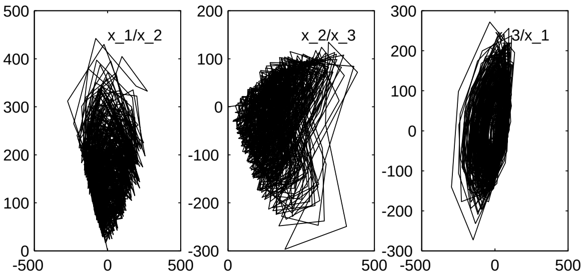

Fig. 3. Phase plan of the system

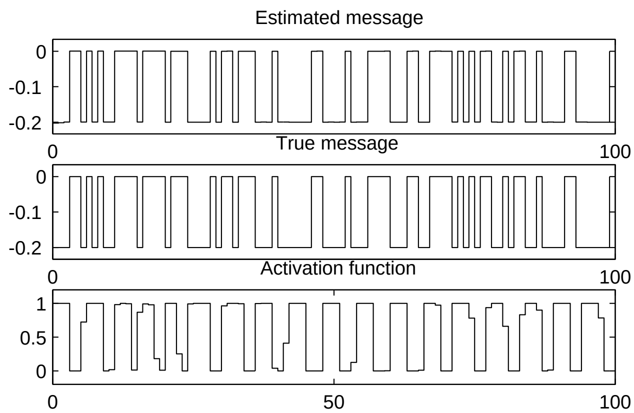

Fig. 4. True and estimated states and activation function

into account the coupling constraints between the local observers, which leads to the resolution of a LMI (Linear Matrix Inequality) problem.

A particular application of the proposed method deals with decryption communication; the objective is to recover a message imbedded in a signal generated by a dynamical nonlinear system. As future works, we aim to construct multiple model and associated multiple observer to ensure a chaotic time evolution of the system in such a way that the decryption of the transmitted signal will be impossible without knowing the model.

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