Optimization and control in minerals, metals and materials processing

Data reconciliation using **L**inear **M**atrix **I**nequalities approach

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Plan

- What is data reconciliation ?
- Is the classical approach adapted ?
- A new concept
- The linear case : total mass balance
- The non linear case : partial mass balance
- Conclusion

What is data reconciliation ?

Relation between density and solid percent of a pulp

model $$\alpha^* = 1.588\left(1 - \frac{1}{d^*}\right)$$

measurment $$d_m = 1.2$$ $$\alpha_m = .25$$

coherence test $$\alpha_m - 1.588\left(1 - \frac{1}{d_m}\right) = 0.015$$

A general data problem reconciliation

model $$f(x^*) = 0$$

measurement $$x_m = x^* + e$$

reconciliation $$\begin{cases} \min \|\hat{x} - x_m\|_P^2 \\ f(\hat{x}) = 0 \end{cases}$$

Hodouin et Flament, 1989
Ragot et Maquin, 1988

A new formulation. A new concept

Relation between density and solid percent of a pulp

model $\alpha^* = 1.588\left(1 - \frac{1}{d^*}\right)$

measurment $d_m = 1.2$

$\alpha_m = .25$

correction procedure

$$.22 \le \hat{\alpha} \le .26$$

$$1.16 \le \hat{d} \le 1.25$$

$$-0.01 \le \hat{\alpha} - 1.588\left(1 - \frac{1}{\hat{d}}\right) \le 0.01$$

A general problem

Thrue data x^*

Model $f(x^*) = 0$

Measurements $x_m = x^* + e$

Correction procedure

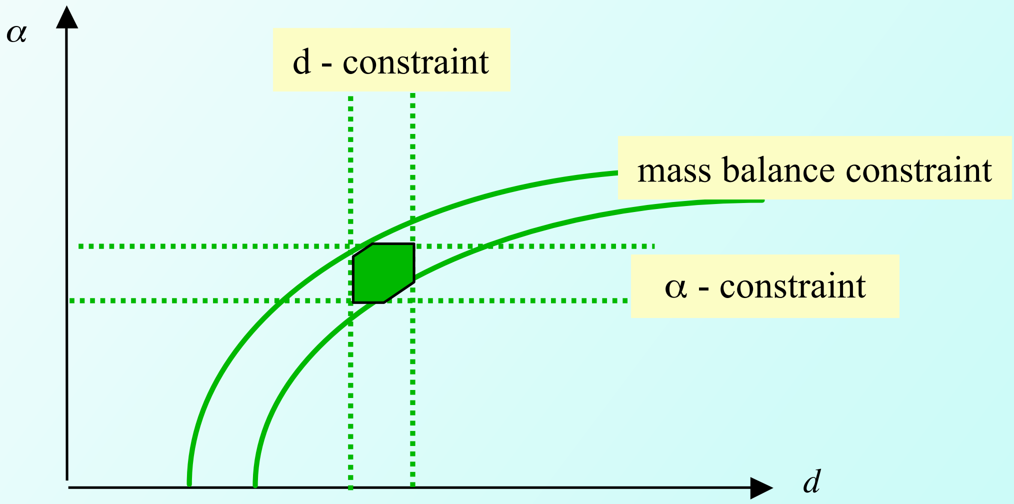
$$\underline{e}_x \le \hat{x} - x_m \le \bar{e}_x$$

$$\underline{r}_x \le f(\hat{x}) \le \bar{r}_x$$

Himmelblau, 1985

A new formulation. A new concept

Geometrical interpretation of data reconciliation



The linear case. Total mass balance

Linear Matrix Inequality (LMI) approach

$$\begin{cases} \min \left\| \hat{x} - x_m \right\|^2 \\ A\hat{x} = 0 \end{cases}$$

$$\begin{cases} \underline{e}_x \le \hat{x} - x_m \le \overline{e}_x \\ \underline{r}_x \le A\hat{x} \le \overline{r}_x \end{cases}$$

$$\begin{cases} -\hat{x} + x_m + \underline{e}_x \le 0 \\ \hat{x} - x_m - \overline{e}_x \le 0 \\ -M\hat{x} + \underline{r}_x \le 0 \\ M\hat{x} - \overline{r}_x \le 0 \end{cases}$$

$$\begin{pmatrix} -I \\ I \\ -A \\ A \end{pmatrix} \hat{x} + \begin{pmatrix} x_m + \underline{e}_x \\ -x_m - \overline{e}_x \\ \underline{r}_x \\ -\overline{r}_x \end{pmatrix} \le \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E\hat{x} + b \le 0$$

Particular case

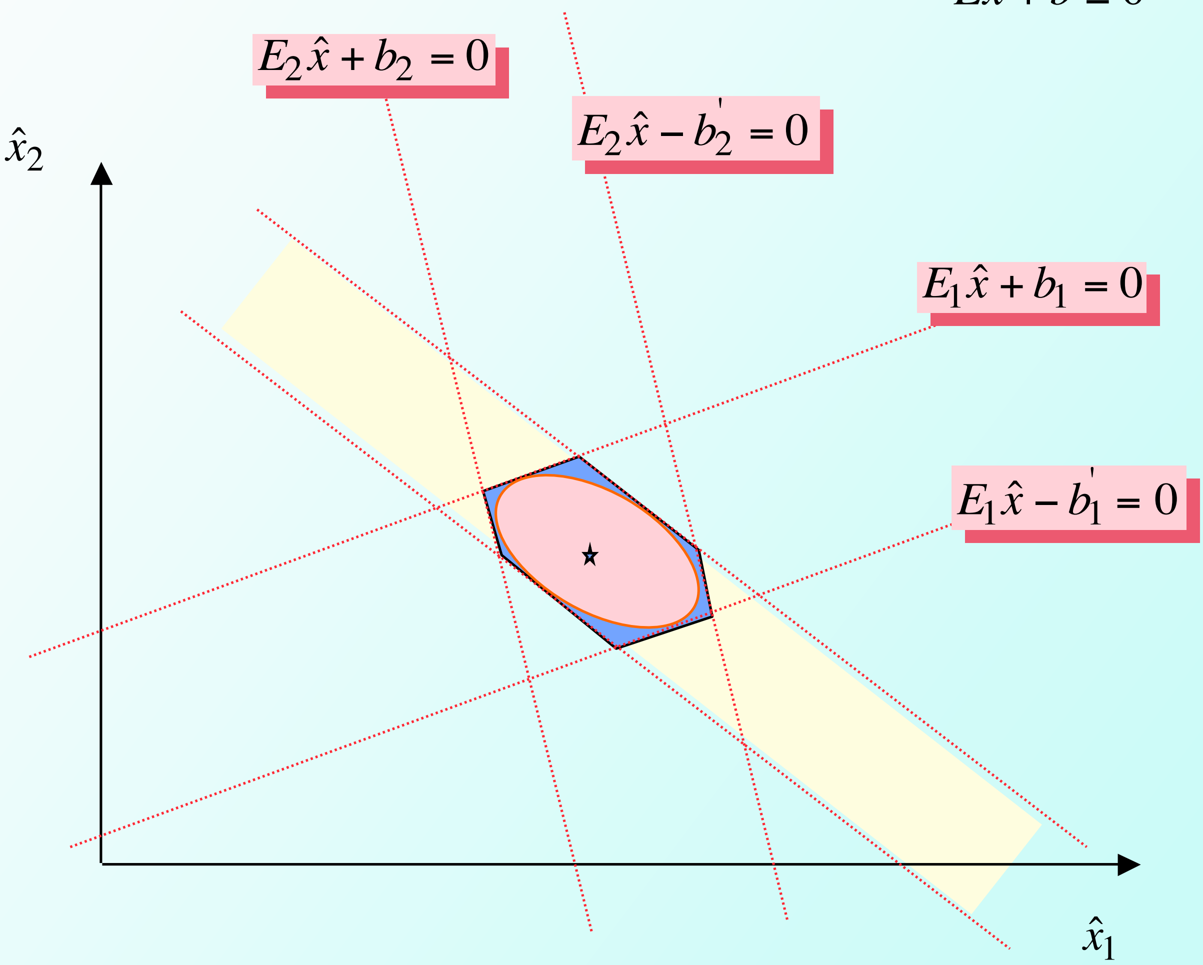
$$\underline{r}_x = \overline{r}_x = 0$$

The linear case. Total mass balance

Geometrical interpretation

$$\begin{pmatrix} -I \\ I \\ -A \\ A \end{pmatrix} \hat{x} + \begin{pmatrix} x_m + \underline{e}_x \\ -x_m - \bar{e}_x \\ \underline{r}_x \\ -\bar{r}_x \end{pmatrix} \leq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E\hat{x} + b \leq 0$$



Resolution

ellipsoid algorithm, Nemirovskii, 1970
interior point methods, Nesterov, 1988
Toolboxes MATLAB : LMI, GBT

The non linear case. Partial mass balance

$$\begin{cases} \min \left\{ \|\hat{x} - x\|^2 + \|\hat{y} - y\|^2 \right\} \\ A\hat{x} = 0 \\ A\hat{x} \bullet \hat{y} = 0 \end{cases}$$

$$\begin{cases} x \ : \ \textit{flowrate} \\ y \ : \ \textit{concentration} \end{cases}$$

- Mass balance problem

Total balance

$$\begin{cases} \underline{e}_x \le \hat{x} - x_m \le \bar{e}_x \\ \underline{r}_x \le A\hat{x} \le \bar{r}_x \end{cases} \quad \Rightarrow \quad \begin{cases} -\hat{x} + x_m + \underline{e}_x \le 0 \\ \hat{x} - x_m - \bar{e}_x \le 0 \\ -M\hat{x} + \underline{r}_x \le 0 \\ M\hat{x} - \bar{r}_x \le 0 \end{cases}$$

Partial balance

$$\begin{cases} \underline{e}_y \le \hat{y} - y_m \le \bar{e}_y \\ \underline{r}_y \le A\hat{x} \bullet \hat{y} \le \bar{r}_y \end{cases} \quad \Rightarrow \quad \begin{cases} -\hat{y} + y_m + \underline{e}_y \le 0 \\ \hat{y} - x_m - \bar{e}_y \le 0 \\ -M\hat{x} \bullet \hat{y} + \underline{r}_y \le 0 \\ M\hat{x} \bullet \hat{y} - \bar{r}_y \le 0 \end{cases}$$

The non linear case. Partial mass balance

$$\begin{cases} -\hat{x} + x_m + \underline{e}_x \le 0 \\ \hat{x} - x_m - \overline{e}_x \le 0 \\ -M\hat{x} + \underline{r}_x \le 0 \\ M\hat{x} - \overline{r}_x \le 0 \end{cases}$$

$$\begin{cases} -\hat{y} + y_m + \underline{e}_y \le 0 \\ \hat{y} - x_m - \overline{e}_y \le 0 \\ -M\hat{x} \bullet \hat{y} + \underline{r}_y \le 0 \\ M\hat{x} \bullet \hat{y} - \overline{r}_y \le 0 \end{cases}$$

- Resolution

  Linearization of the bilinear mass balances around the measurement

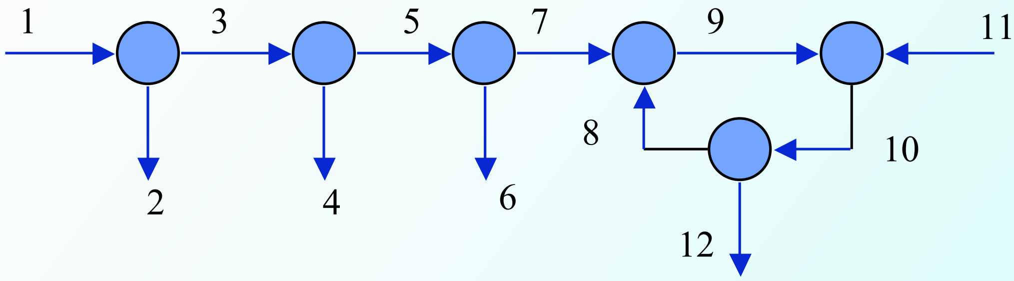
  Successive linearization of the bilinear mass balances

  Resolution using LMI techniques

- Extension : missing data

$$\begin{cases} -H\hat{x} + x_m + \underline{e}_x \le 0 \\ H\hat{x} - x_m - \overline{e}_x \le 0 \\ -M\hat{x} + \underline{r}_x \le 0 \\ M\hat{x} - \overline{r}_x \le 0 \end{cases}$$

Numerical example



Total mass balance

$$\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_3 - x_4 - x_5 = 0 \\ x_5 - x_6 - x_7 = 0 \\ x_7 + x_8 - x_9 = 0 \\ -x_8 + x_{10} - x_{12} = 0 \\ x_9 - x_{10} - x_{11} = 0 \end{cases}$$

Partial mass balance

$$\begin{cases} x_1 y_1 - x_2 y_2 - x_3 y_3 = 0 \\ x_3 y_3 - x_4 y_4 - x_5 y_5 = 0 \\ x_5 y_5 - x_6 y_6 - x_7 y_7 = 0 \\ x_7 y_7 + x_8 y_8 - x_9 y_9 = 0 \\ -x_8 y_8 + x_{10} y_{10} - x_{12} y_{12} = 0 \\ x_9 y_9 - x_{10} y_{10} - x_{11} y_{11} = 0 \end{cases}$$

	Flowrate		Concentration	
Stream	Measure	Precision	Measure	Precision
1	250.5	15	0.46	5
2	21.6	5	0.49	2
3			0.52	5
4	36.5	5	1.71	2
5				
6	17.6	5	0.64	2
7	144.9	5		
8			0.20	2
9			0.14	2
10				
11	90.5	20	0.08	2
12	47.7	5	0.18	2

Numerical example

Flowrates and concentrations estimated

$$\begin{cases} \underline{r}_x = -1 & \overline{r}_x = 1 \\ \underline{r}_y = -0.5 & \overline{r}_y = 0.5 \end{cases}$$

Stream	Flowrate min	Flowrate	Flowrate max	Conc. min	Conc.	Conc. max
1	212.9	219.42	288.1	0.414	0.475	.506
2	20.5	21.68	22.7	0.470	0.489	.509
3	0	197.74	500	0.468	0.473	.572
4	34.7	37.40	38.3	1.641	1.729	1.778
5	0	160.34	500	0	0.178	5
6	16	17.66	18.5	0.614	0.642	0.665
7	137.6	143.42	152.1	0	0.118	5
8	0	54.65	500	0.192	0.200	0.208
9	0	197.96	500	0.134	0.140	0.145
10	0	102.62	500	0	0.192	5
11	72.4	93.25	108.6	0.077	0.081	0.083
12	45.3	47.86	50.1	0.173	0.181	0.187

Mass balance residual

Total mass balance	0.000	0.000	-0.742	0.105	0.100	0.093
Partial mass balance	-0.048	0.343	0.256	0.205	0.189	0.188

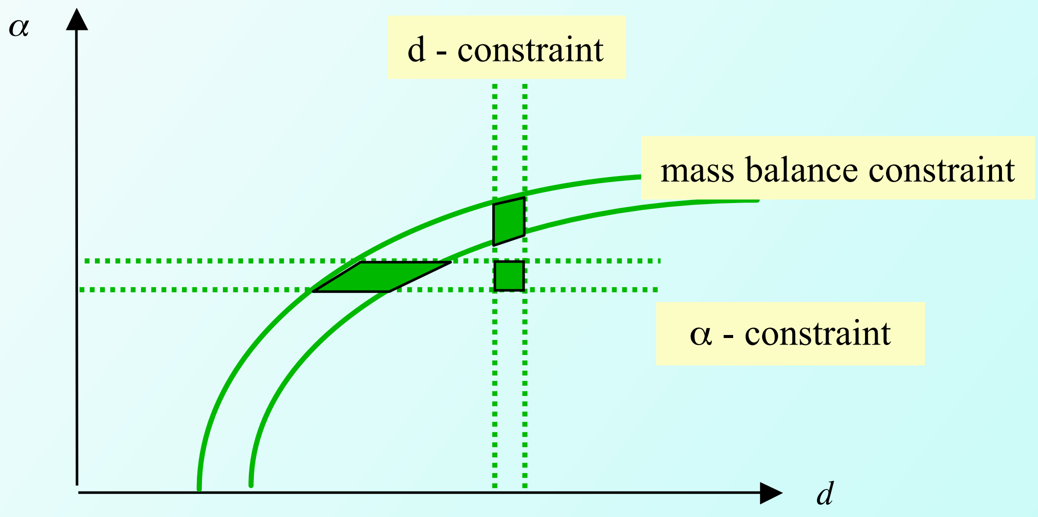
The same process with outliers

Stream	Flow-rate min	Flow-rate	Flow-rate max	Conc. min	Conc.	Conc. max
1	212.9	239.68	288.1	.414	.457	.506
2	20.5	21.63	22.7	.470	.490	.509
3	219	218.05	267	.468	.453	.572
4	34	39.14	38.3	1.641	1.722	1.778
5	0	178.09	500	.136	.177	.204
6	16.7	19.29	18	.614	.645	.665
7	137.6	152.97	152.1	.088	.124	.132
8	0	52.10	500	.192	.208	.208
9	172	203.71	211	.134	.144	.145
10	0	101.58	500	.152	.191	.228
11	72.4	100.78	108.6	.077	.098	.083
12	45.3	48.13	50.1	.173	.188	.187

suspicious measurements :
flowrate 3, 4, 6 and 7
concentration 3, 11 and 12

A new formulation. A new concept

Geometrical interpretation of gross error



Gross errors detection and localisation

A procedure for detection and localization
of gross errors : a bank of estimators + a logical test

- Reference estimator : all data are used

$$P \ : \ \begin{cases} -H\hat{x} + x_m + \underline{e}_x \leq 0 \\ x_m - H\hat{x} - \overline{e}_x \leq 0 \\ -M\hat{x} + \underline{r}_x \leq 0 \\ M\hat{x} - \overline{r}_x \leq 0 \end{cases}$$

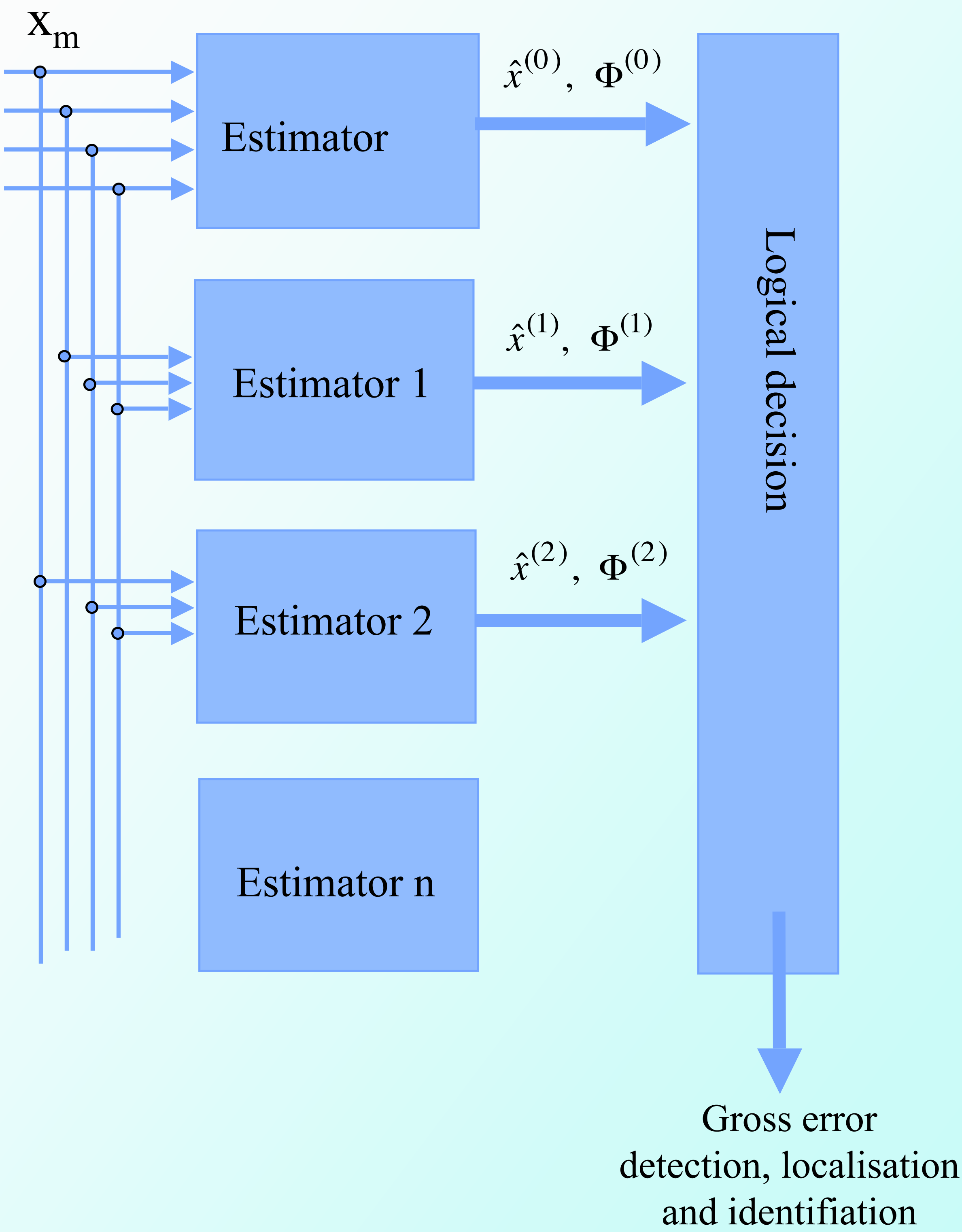
- Secondary estimators : all data are used, exept the ith

$$P_i \ : \ \begin{cases} -H_i\hat{x} + x_m + \underline{e}_x \leq 0 \\ x_m - H_i\hat{x} - \overline{e}_x \leq 0 \\ -M\hat{x} + \underline{r}_x \leq 0 \\ M\hat{x} - \overline{r}_x \leq 0 \end{cases} \qquad H_i = \begin{pmatrix} h_1 \\ \vdots \\ h_{i-1} \\ h_{i+1} \\ \vdots \\ h_p \end{pmatrix}$$

repeat the estimation procedure for i=1, 2, ...

- Compare the differents estimations

Gross errors detection and localisation



Gross errors detection and localisation

Reconciliated data after gross error suppression

Stream	Flowrate min	Flowrate	Flowrate max	Conc. min	Conc.	Conc. max
1	212.9	219.42	288.1	.414	.475	.506
2	20.5	21.68	22.7	.470	.489	.509
3	0	197.70	500	.468	.473	.572
4	34.7	37.40	38.3	1.641	1.729	1.778
5	0	160.34	500	.136	.178	.204
6	16.7	17.66	18.5	.614	.642	.665
7	137.6	143.42	152.1	.088	.118	.132
8	0	54.65	500	.192	.200	.208
9	172	197.96	211	.134	.140	.145
10	0	102.62	500	.152	.193	.228
11	72.4	95.25	108.6	.077	.081	.083
12	45.3	47.86	50.1	.173	.181	.187

Conclusion

- Advantages

    a self understanding formulation
    no statistical hypothesis
    linear mass balance reconciliation
    non linear mass balance reconciliation
    multi-component mass balance reconciliation
    gross error detection and localization
    missing measurements may be considered

- Further extensions

    dynamical systems
    observability conditions
    adding fuzzy constraints