Robust input and state estimation for linear discrete-time systems using PI two stage Kalman filter

Karim Khémiri [(1)], Fayçal Ben Hmida [(1)], José Ragot [(2)] , Moncef Gossa [(1)]

[(1)] ESSTT-C3S, 5 av. Taha Hussein BP 56 - 1008 Tunis, Tunisia
kemiri_karim@yahoo.fr ; faycal.benhmida@esstt.rnu.tn ; moncef.gossa@esstt.rnu.tn

[(2)] CRAN-INPL, 2 av. de la Forêt de Haye-54516 Vandoeuvre les Nancy, France
jose.ragot@ensem.inpl-nancy.fr

ABSTRACT. In this paper, we have solved the problem of simultaneously estimating the state and input of linear time varying stochastic systems with uncertain noise covariance. The approach suggested rests on the use of the Proportional-Integral Two Stages Kalman Filter (PI-TSKF). This technique is qualified to be robust against the uncertainty. The filter is tested by an *illustrative example.*

KEYWORDS: Time varying linear system, two stage Kalman filter, Integral action, input estimation.

1. Introduction

The state estimation of the dynamic systems in the presence of unknown input is largely studied in the literature. In the stochastic linear context at discrete time, we can note works which use an approach based on Kalman filtering [1, 2, 4, 6, 8, 9, and 11]. These works represent an extension of Friedland's work which is the first to introduce the technique of TSKF (Two Stage-Kalman Filter). The principal idea, on which this technique rests, consists in using a double transformation to decouple ASKF (Augmented State Kalman Filter) in two sub-filters: The first sub-filter is dedicated to the estimation of the state and the second one to estimation of the unknown input. The filter proposed by Friedland [2] is optimal only in the presence of a constant unknown input. The fact that encouraged Alouani et al. [1] to extend this filter to a dynamic stochastic unknown input; an algebraic constraint is then introduced to ensure the optimality of the TSKF. Hsieh and Chen [4] developed the OTSKF (Optimal Two Stage Kalman Filter) which is applied in a more general context, i.e. when the unknown input is generated by a stochastic dynamic process and jointly affects the state and the measurement equations of the system. Kim et al. [8] developed an adaptive version of TSKF noted ATSKF (Adaptive Two Stage Kalman

Filter) and presented in [9] an analysis of the stability of this filter. Recently, Khémiri et al [10] have used the proposed filter by Hsieh and Chen [4] to solve the fault estimation and isolation problem. The problem disadvantage of this approach is that the filter lost its optimality in the presence of an uncertainty noise covariance.

In this work, we propose to solve this problem by using the PI TSKF (Proportional Integral Two Stage Kalman Filter) to estimate the state and the fault of a stochastic system. The concept of integral action was used in the observer theory [3, 11]. This technique was introduced first in [7] and further developed in [12] and extended in [13] using a fading integral that was capable of coping transitory effect and improve the stability margin. Our aim is to extend the OTSKF by introducing the integral action technique proposed in [13], and solve the problem of state and fault estimation for systems with uncertainty noise covariance.

This paper is organized as follows. In section 2, we present the statement of the problem. In section 3, we recall the OTSKF. In section 4, the design of the PI TSKF is developed. Finally, an illustrative example of the proposed approach technique is presented in section 5.

2. Statement of the problem

Considering the linear time-varying discrete stochastic systems with unknown inputs, is described by:

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + E_k d_k + w_k^x \\ d_{k+1} &= d_k + w_k^d \\ y_k &= H_k x_k + F_k d_k + v_k \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^r$ is the known control input, $y_k \in \mathbb{R}^m$ is the observation vector and $d_k \in \mathbb{R}^p$ is the unknown inputs vector. The matrices A_k, B_k, H_k, E_k and F_k are known and have appropriate dimensions.

Assumptions :

- **A1:** the noises w_k^x and v_k are zero-mean white noise sequences with the following covariance's : $E\left\{w_k^x \left(w_j^x\right)^T\right\} = Q_k^x\, \delta_{kj}$, $E\left\{v_k v_j^T\right\} = R_k\, \delta_{kj}$ and $E\left\{w_k^x v_j^T\right\} = 0$, where ^{T} denotes transpose and δ_{kj} denotes the Kronecker delta function.
- **A2 :** the noises w_k^d is zero-mean white noise sequence with the following covariance : $E\left\{w_k^d \left(w_j^d\right)^T\right\} = Q_k^d\, \delta_{kj}$ and $E\left\{w_k^x \left(w_j^d\right)^T\right\} = Q_k^{xd}\, \delta_{kj}$

- **A3 :** the initial state x_0 is uncorrelated with the white noises processes w_k^x, w_k^d and v_k and is a gaussian random variable with $E\{x_0\} = \bar{x}_0$ and $E\left\{\left(x_0 - \bar{x}_0\right)\left(x_0 - \bar{x}_0\right)^T\right\} = P_0^x$.
- **A4 :** the initial fault d_0 is satisfied the following

$$E\{d_0\} = \bar{d}_0,\ E\left\{\left(d_0 - \bar{d}_0\right)\left(d_0 - \bar{d}_0\right)^T\right\} = P_0^d,\ E\left\{\left(x_0 - \bar{d}_0\right)\left(x_0 - \bar{d}_0\right)^T\right\} = P_0^{xd}$$

- **A5 :** the conditions on matrix ranks :
  - $rank\left(H_k\right) = m\left(\geq p\right)$: to avoid the redundancy of measurement
  - $rank\left(F_k\right) = p$: to guarantee the independence of the unknown input directions

We know that, if the model parameters bias from the reality, the estimating precision would become worse. Our problem consists of designing a filter that gives a robust state and unknown inputs estimation for linear time-varying stochastic systems, such that, all covariance matrices are not perfectly known.

3. Optimal Two Stage Kalman filter

The OTSKF (Optimal Two Stage Kalman Filter) is derived by Hsieh and Chen [4] where it is equivalent to the ASKF (Augmented State Kalman Filter). This technique is developed to solve the problem of optimal estimation for linear systems with unknown inputs. Recently, Khemiri et al. [10] have used this filter to solve the problem of FDI. This latter permits an optimal estimation of the state and the unknown inputs. The following OTSKF proposed by Hsieh and Chen [4], may be used:

Initialization

$$V_0 = P_0^{xd}\left(P_{0/0}^d\right)^{-1},\ \bar{x}_{0/0} = \bar{x}_0 - V_0\bar{d}_0,\ \bar{d}_{0/0} = \bar{d}_0,$$

$$\bar{P}_{0/0} = P_0^x - V_0 P_0^d V_0^T,\ \bar{P}_{0/0}^d = P_0^d,\ k = 0$$

State sub-filter

$$\bar{x}_{k+1/k} = A_k\bar{x}_k + B_k u_k + \bar{u}_k \tag{2}$$

$$\bar{P}_{k+1/k}^x = A_k\bar{P}_{k/k}^x A_k^T + \bar{Q}_k^x \tag{3}$$

$$\bar{x}_{k+1/k+1} = \bar{x}_{k+1/k} + \bar{K}_{k+1}^x\left(y_{k+1} - H_{k+1}\bar{x}_{k+1/k}\right) \tag{4}$$

$$\bar{K}_{k+1}^x = \bar{P}_{k+1/k}^x H_{k+1}^T (H_{k+1}\bar{P}_{k+1/k}^x H_{k+1}^T + R_{k+1})^{-1} \tag{5}$$

$$\bar{P}_{k+1/k+1}^x = \left(I - \bar{K}_{k+1}^x H_{k+1}\right)\bar{P}_{k+1/k}^x \tag{6}$$

Unknown input sub-filter

$$\bar{d}_{k+1/k} = \bar{d}_{k/k} \tag{7}$$

$$\bar{P}^d_{k+1/k} = \bar{P}^d_{k/k} + Q^d_k \tag{8}$$

$$\bar{d}_{k+1/k+1} = \bar{d}_{k+1/k} + \bar{K}^d_{k+1} \times \left(y_{k+1} - H_{k+1}\bar{x}_{k+1/k} - S_{k+1}\bar{d}_{k+1/k} \right) \tag{9}$$

$$\bar{K}^d_{k+1} = \bar{P}^d_{k+1/k} S^T_{k+1} \times \left(S_{k+1}\bar{P}^d_{k+1/k} S^T_{k+1} + H_{k+1}\bar{P}^x_{k+1/k} H^T_{k+1} + R_{k+1} \right)^{-1} \tag{10}$$

$$\bar{P}^d_{k+1/k+1} = \left(I - \bar{K}^d_{k+1} S_{k+1} \right) \bar{P}^d_{k+1/k} \tag{11}$$

Correction

$$\hat{x}_{k+1/k+1} = \bar{x}_{k+1/k+1} + V_{k+1}\, \bar{d}_{k+1/k+1} \tag{12}$$

$$\hat{d}_{k+1/k+1} = \bar{d}_{k+1/k+1} \tag{13}$$

where

$$\bar{U}_{k+1} = A_k V_k + E_k \tag{14}$$

$$U_{k+1} = \bar{U}_{k+1} + \left(Q^{xd}_k - \bar{U}_{k+1} Q^d_k \right) \left(\bar{P}^d_{k+1/k} \right)^{-1} \tag{15}$$

$$\bar{u}_{k+1} = \left(\bar{U}_{k+1} - U_{k+1} \right) \bar{d}_{k/k} \tag{16}$$

$$\bar{Q}^x_k = Q^x_k - Q^{xd}_k \bar{U}^T_{k+1} - U_{k+1} \times \left(Q^{xd}_k - \bar{U}_{k+1} Q^d_k \right)^T \tag{17}$$

$$V_{k+1} = U_{k+1} - \bar{K}^x_{k+1} S_{k+1} \tag{18}$$

$$S_{k+1} = H_{k+1} U_{k+1} + E_{k+1} \tag{19}$$

However, application of OTSKF requires an accurate model of the process and if there are any errors in the process model, the OTSKF may lead to poor performance. So, an uncertainty noise covariance can lead to large error estimation and a loss of optimality of OTSKF [10]. Our contribution consists of extending the last technique by modifying the state and the fault sub-filters with integral actions. The robust PI TSKF (Proportional Integral Two Stage Kalman Filter) will be developed in the next section.

4. Robust PI Two Stage Kalman filter

This section is devoted to the PI TSKF design. So, we present the modified state and unknown input sub-filters in section 4.1 and 4.2. The filter is summarised in section 4.3.

4.1. The modified state sub-filter

In this section, we will modify the state sub-filter, which is described in section 3 by equations (2) – (6). We will introduce the integral action [13] so the modified sub-filter can have the following form

$$\bar{x}_{k+1/k} = A_k \bar{x}_{k/k} + B_k u_k + \bar{u}_k \tag{20}$$

$$f_{k+1}^x = f_k^x + G_{k+1}^{xi}\left(y_{k+1} - H_{k+1}\bar{x}_{k+1/k}\right) \tag{21}$$

$$\bar{x}_{k+1/k+1} = \bar{x}_{k+1/k} + f_{k+1}^x + G_{k+1}^{xp}\left(y_{k+1} - H_{k+1}\bar{x}_{k+1/k}\right) \tag{22}$$

where f_k^x represents the integral term of error estimation and G_{k+1}^{xp} and G_{k+1}^{xi} are the proportional and integral gain matrices of the state sub-filter.

In this section we introduce two arbitrary matrices [13] : the fading constant and the integral effect coefficient are both incorporated in the above system. Therefore, the resulting update state sub-filter is given in an augmented system as

$$\begin{bmatrix} \bar{x}_{k+1/k+1} \\ f_{k+1}^x \end{bmatrix} = \begin{bmatrix} A_k & D_i^x \\ 0 & D_f^x \end{bmatrix} \begin{bmatrix} \bar{x}_{k/k} \\ f_k^x \end{bmatrix} + \begin{bmatrix} B_k \\ 0 \end{bmatrix} u_k + \begin{bmatrix} I \\ 0 \end{bmatrix} \bar{u}_k + \begin{bmatrix} G_{k+1}^{xp} \\ G_{k+1}^{xi} \end{bmatrix} \left(y_{k+1} - H_k \bar{x}_{k+1/k}\right) \tag{23}$$

where D_i^x , D_f^x are constant matrices determined according to performance specifications for the system.. The optimal design of the gain matrices can be determined using the augmented covariance matrix as follow:

$$\bar{P}_{k/k}^{xa} = \begin{bmatrix} P_{k/k}^{x11} & P_{k/k}^{x12} \\ P_{k/k}^{x21} & P_{k/k}^{x22} \end{bmatrix} = \bar{P}_{k/k-1}^{xa} - \begin{bmatrix} G_k^{xp} \\ G_k^{xi} \end{bmatrix} H_k^a \bar{P}_{k/k-1}^{xa} \tag{24}$$

$$\bar{P}_{k/k-1}^{xa} = A_{k-1}^a \bar{P}_{k-1/k-1}^{xa} A_{k-1}^{aT} + Q_{k-1}^{xa} \tag{25}$$

where

$$A_k^a = \begin{bmatrix} A_k & D_i^x \\ 0 & D_f^x \end{bmatrix}, \quad Q_k^{xa} = \begin{bmatrix} \bar{Q}_k^x & 0 \\ 0 & 0 \end{bmatrix} \text{ and } H_k^a = \begin{bmatrix} H_k & 0 \end{bmatrix} \tag{26}$$

By introducing equation (25) into (24) and using (26) the proportional and the integral gain matrices have the following forms:

$$\begin{aligned} G_{k+1}^{xp} = &\left(A_k P_{k/k}^{x11} A_k^T H_k^T + D_i^x P_{k/k}^{x21} A_k^T H_k^T + A_k P_{k/k}^{x12} D_i^{xT} H_k^T + D_i^x P_{k/k}^{x22} D_i^{xT} H_k^T + Q_k^x H_k^T\right) \times \\ &\left(H_k A_k P_{k/k}^{x11} A_k^T H_k^T + H_k D_i^x P_{k/k}^{x21} A_k^T H_k^T + H_k A_k P_{k/k}^{x12} D_i^{xT} H_k^T + \right. \\ &\left. H_k D_i^x P_{k/k}^{x22} D_i^{xT} H_k^T + R_k + H_k Q_k^x H_k^T\right)^{-1} \end{aligned} \tag{27}$$

$$G_{k+1}^{xi} = \left(D_f^x P_{k/k}^{x21} A_k^T H_k^T + D_f^x P_{k/k}^{x22} D_f^{xT} H_k^T \right) \times \left(H_k A_k P_{k/k}^{x11} A_k^T H_k^T + H_k D_i^x P_{k/k}^{x21} A_k^T H_k^T + \right.$$
$$\left. H_k A_k P_{k/k}^{x12} D_i^{xT} H_k^T + H_k D_i^x P_{k/k}^{x22} D_i^{xT} H_k^T + R_k + H_k Q_k^x H_k^T \right)^{-1} \quad (28)$$

- D_i^x as a full rank constant matrix presents the extent of the effect of the integral term on the state estimation.
- D_f^x is the fading constant matrix for the state estimation, where $D_f^x = \alpha I$, with $0 < \alpha \leq 1$

4.2. The modified unknown input sub-filter

In this section, we follow the same manner for the last section to introduce the integral action in the unknown input sub-filter. The modified unknown input sub-filter will have the following form:

$$\bar{d}_{k+1/k} = \bar{d}_{k/k} \quad (29)$$

$$f_{k+1}^d = f_k^d + G_{k+1}^{di} \left(y_{k+1} - H_{k+1} \bar{x}_{k+1/k} - S_{k+1} \bar{d}_{k+1/k} \right) \quad (30)$$

$$\bar{d}_{k+1/k+1} = \bar{d}_{k+1/k} + f_{k+1}^d + G_{K+1}^{dp} \left(y_{k+1} - H_{k+1} \bar{x}_{k+1/k} - S_{k+1} \bar{d}_{k+1/k} \right) \quad (31)$$

where, f_k^d represents the integral term and G_{k+1}^{dp} and G_{k+1}^{di} are the proportional and integral gain matrices of the unknown input sub-filter..

In this section we introduce two arbitrary matrices: the fading constant and the integral effect coefficient are both incorporated in the above system. Therefore, the resulting update unknown input sub-filter is given in an augmented system as

$$\begin{bmatrix} \bar{d}_{k+1/k+1} \\ f_{k+1}^d \end{bmatrix} = \begin{bmatrix} I & D_i^d \\ 0 & D_f^d \end{bmatrix} \begin{bmatrix} \bar{d}_{k/k} \\ f_k^d \end{bmatrix} + \begin{bmatrix} G_{k+1}^{dp} \\ G_{k+1}^{di} \end{bmatrix} \left(y_{k+1} - H_k \bar{x}_{k+1/k} - S_k \bar{d}_{k+1/k} \right) \quad (32)$$

where D_i^d, D_f^d are constant matrices determined according to performance specifications for the system. The optimal design of the gain matrices can be determined using the augmented covariance matrix as follow:

$$\bar{P}_{k/k}^{da} = \begin{bmatrix} P_{k/k}^{d11} & P_{k/k}^{d12} \\ P_{k/k}^{d21} & P_{k/k}^{d22} \end{bmatrix} = \bar{P}_{k/k-1}^{da} - \begin{bmatrix} G_p^d \\ G_i^d \end{bmatrix} S_k^a \bar{P}_{k/k-1}^{da} \quad (33)$$

$$\bar{P}_{k/k-1}^{da} = A_{k-1}^{d}\bar{P}_{k-1/k-1}^{da}A_{k-1}^{dT} + Q_{k-1}^{da} \tag{34}$$

where

$$A_k^d = \begin{bmatrix} I & D_i^d \\ 0 & D_f^d \end{bmatrix}, \quad Q_k^{da} = \begin{bmatrix} Q_k^d & 0 \\ 0 & 0 \end{bmatrix} \text{ and } S_k^a = \begin{bmatrix} S_k & 0 \end{bmatrix} \tag{35}$$

By introducing equation (32) into (31) and using (33) the proportional and the integral gain matrices have the following forms:

$$\begin{aligned} G_{k+1}^{dp} = &\left(A_k^d P_{k/k}^{d11} A_k^{dT} S_k^T + D_i^d P_{k/k}^{d21} A_k^{dT} S_k^T + A_k^d P_{k/k}^{d12} D_i^{dT} S_k^T + D_i^d P_{k/k}^{d22} D_i^{dT} S_k^T + Q_k^d S_k^T \right) \times \\ &\Big(H_k A_k P_{k/k}^{x11} A_k^T H_k^T + H_k D_i^x P_{k/k}^{x21} A_k^T H_k^T + H_k A_k P_{k/k}^{x12} D_i^{xT} H_k^T + H_k D_i^x P_{k/k}^{x22} D_i^{xT} H_k^T + \\ &S_k A_k^d P_{k/k}^{d11} A_k^{dT} S_k^T + S_k D_i^d P_{k/k}^{d21} A_k^{dT} S_k^T + S_k A_k^d P_{k/k}^{d12} D_i^{dT} S_k^T + \\ &S_k D_i^d P_{k/k}^{d22} D_i^{dT} S_k^T + R_k + H_k \bar{Q}_k^x H_k^T + S_k Q_k^d S_k^T \Big)^{-1} \end{aligned} \tag{36}$$

$$\begin{aligned} G_{k+1}^{di} = &\left(D_f^d P_{k/k}^{d21} A_k^{dT} S_k^T + D_f^d P_{k/k}^{d22} D_f^{dT} S_k^T \right) \times \\ &\Big(H_k A_k P_{k/k}^{x11} A_k^T H_k^T + H_k D_i^x P_{k/k}^{x21} A_k^T H_k^T + H_k A_k P_{k/k}^{x12} D_i^{xT} H_k^T + H_k D_i^x P_{k/k}^{x22} D_i^{xT} H_k^T + \\ &S_k A_k^d P_{k/k}^{f11} A_k^{dT} S_k^T + S_k D_i^d P_{k/k}^{d21} A_k^{dT} S_k^T + S_k A_k^d P_{k/k}^{d12} D_i^{dT} S_k^T + \\ &S_k D_i^d P_{k/k}^{d22} D_i^{dT} S_k^T + R_k + H_k \bar{Q}_k^x H_k^T + S_k Q_k^d S_k^T \Big)^{-1} \end{aligned} \tag{37}$$

- D_i^d as full rank constant matrix presents the extent of the effect of the integral term on the unknown input estimation.
- D_f^d is the fading constant matrix for the unknown input estimation, where $D_f^d = \alpha I$ with $0 < \alpha \leq 1$

4.3. Summary of filter equations

Finally, the PI TSKF filter is summarized in the Table 1. We suppose to know the following :

- The known input u_k
- Matrices A_k, B_k, H_k, E_k and F_k
- Matrices Q_k^x, R_k and Q_k^d
- Initial values $\bar{x}_0$, $\bar{d}_0$, P_0^x and P_0^d

Algorithm : state and unknown input estimation by PI TSKF

- Step 0: Initialization

  $V_0 = P_0^{xd}\left(P_{0/0}^d\right)^{-1}$, $\bar{x}_{0/0} = \bar{x}_0 - V_0\bar{d}_0$, $\bar{d}_{0/0} = \bar{d}_0$, $\bar{P}_{0/0} = P_0^x - V_0 P_0^d V_0^T$,

  $\bar{P}_{0/0}^d = P_0^d$, $k = 0$

- Step 1: state sub-filter

  To calculate $\bar{U}_{k+1}$, U_{k+1}, $\bar{u}_{k+1}$, $\bar{Q}_{k+1}^x$ and $\bar{P}_{k+1/k}^d$ respectively from (14), (15), (16), (17) and (11)

  To calculate $\bar{x}_{k+1/k}$, f_{k+1}^x, $\bar{x}_{k+1/k+1}$, $\bar{P}_{k+1/k}^{xa}$, $\bar{P}_{k+1/k+1}^{xa}$, G_{k+1}^{xp} and G_{k+1}^{xi} respectively from (20), (21), (22), (24), (25), (27) and (28)

- Step 2: unknown input sub-filter

  To calculate S_{k+1} from (19)

  To calculate $\bar{d}_{k+1/k}$, f_{k+1}^d, $\bar{d}_{k+1/k+1}$, $\bar{P}_{k+1/k}^{da}$, $\bar{P}_{k+1/k+1}^{da}$, G_{k+1}^{dp} and G_{k+1}^{di} respectively from (29), (30), (31), (33), (34), (36) and (37)

- Step 3: correction

  Update $V_k = U_{k+1} - G_{k+1}^{xp} S_{k+1}$

  To calculate $\hat{x}_{k+1/k+1}$ and $\hat{d}_{k+1/k+1}$ from (12) and (13)

- Step4: $k = k+1$ go to Step1

Table 1. PI TSKF algorithm

The obtained PI TSKF is qualified to be robust against the uncertainty noise covariance of the state, the unknown inputs and the measurement. In the next section, the filter will be tested by an illustrative example.

5. Illustrative example

In this section, we present the application of the proposed filters. The parameters of the system (1) are given by:

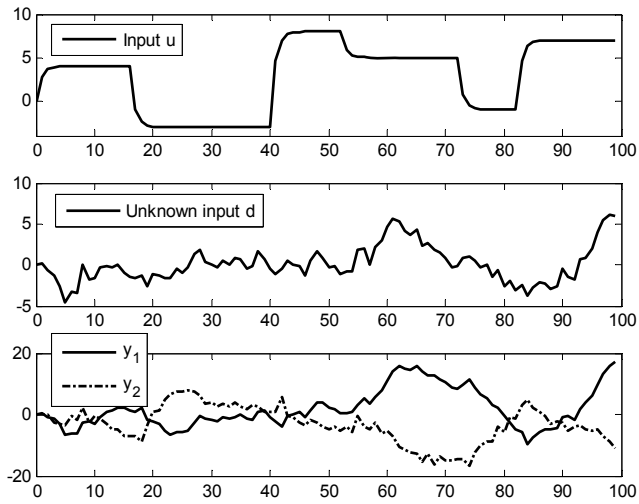
$$x_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \end{bmatrix}, A_k = \begin{bmatrix} a_k & 0.5 & 0.08 \\ 0.6 & 0.01 & 0.04 \\ 0.1 & 0.7 & 0.05 \end{bmatrix}, a_k = 0.2 + 0.2\sin(0.1k), \; B_k = \begin{bmatrix} 2 \\ -1.5 \\ 0.5 \end{bmatrix},$$

$$H_k = \begin{bmatrix} 0.2 & 0.4 & 0 \\ -0.5 & -0.8 & 0.4 \end{bmatrix}, \; E_k = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \; F_k = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$D_i^x = I_{3\times3}, \; D_f^x = 0.3 I_{3\times3}, \; D_i^d = 1.5, \; D_f^d = 0.3$$

$$Q_k^x = 0.1 I_{3\times3}, \; R_k = 0.01 I_{2\times2}, \; Q_k^{xd} = \begin{bmatrix} 0.1 & 0.2 & 0.2 \end{bmatrix}^T, \; Q_k^d = 0.2$$

Fig. 2 presents the input/output sequence of the system. The sampling time is N = 100.



.

Figure 2. System input/output

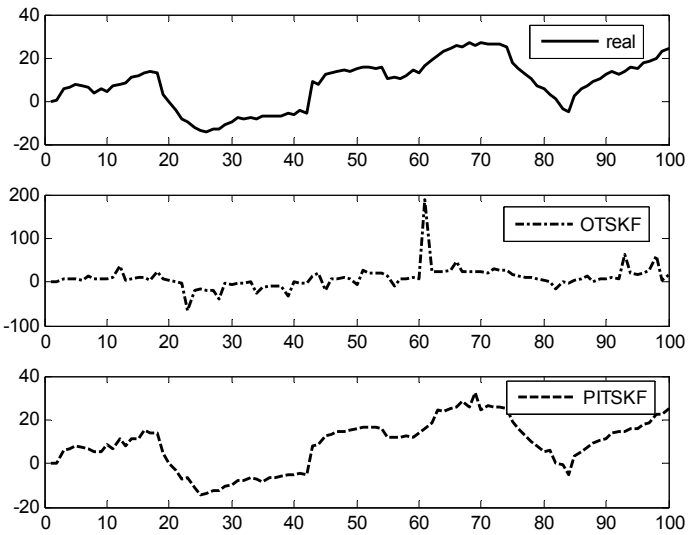
In the simulation, we introduce a perturbation on the noise covariance matrices of the state, the measurement and the unknown input as follow:

$$Q_k^{x'} = \begin{cases} Q_k^x & if\ k \le 50 \\ (1+\alpha_1) Q_k^x & if\ k > 50 \end{cases}$$

$$R_k^{'} = \begin{cases} R_k & if\ k \le 50 \\ (1+\alpha_2) R_k & if\ k > 50 \end{cases}$$

$$Q_k^{d'} = \begin{cases} Q_k^d & if\ k \le 50 \\ (1+\alpha_3) Q_k^d & if\ k > 50 \end{cases}$$

where $\alpha_1 = \alpha_2 = \alpha_3 = 1$



.

Figure 3. State estimation

Fig. 3 and Fig. 4 represent the states, the unknown inputs and their estimates. We can show that, with the perturbation in the covariance matrices, the PI TSKF have the better performances then the OTSKF. The last leads to a large error estimation and loss her optimality.

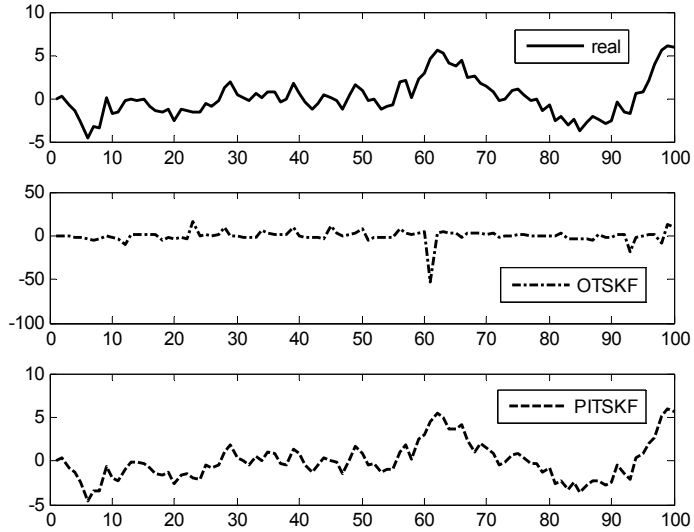


Figure 4. Unknown input estimation

The simulation results in Table 2 show the average root mean square errors (RMSE) in the estimated states and unknown input. For example, the RMSE of the first component of state vector $x_{1,k}$ is calculated by :

$$RMSE(x_{1,k}) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(x_{1,k} - \hat{x}_{1,k/k}\right)^2}$$

RMSE	$x_{1,k}$	$x_{2,k}$	$x_{3,k}$	d_k
OTSKF	**21.15**	**8.32**	**54.59**	**7.08**
PI TSKF	**1.47**	**1.32**	**2.77**	**0.31**

Table 2. RMSE values

6. Conclusion

In this paper, a robust estimation for linear stochastic discrete-time varying systems with uncertainty noise covariance is proposed by using a PI TSKF. The filter is obtained by a

modification of the state and the unknown inputs sub-filters of the OTSKF with integral actions. This recursive method is tested by an illustrative example. Indeed, the PI TSKF remains powerful in spite of the errors made on the covariance matrices characterizing the noises of the state Q_k^x , the measurement R_k and the unknown inputs Q_k^d .

7. References

[1] A. T. Alouani, T. R. Rice, et W. D. Blair, "A two-stage filter for state estimation in the presence of dynamical stochastic bias", in Proceeding American Control Conference, Chicago, IL, pp. 1784-1788, 1992.

[2] B. Friedland, "Treatment of bias in recursive filtering", IEEE Transactions on Automatic Control, vol. 14, pp. 359-367, 1969.

[3] B. Shafai, S. Beale, H. H. Niemann, and J. L. Stroustrup, "LTR design of discrete-time proportional-integral observers," IEEE Transactions on Automatic Control, vol. 5, no. 7, pp. 1056-1062, 1996.

[4] C. S. Hsieh et F. C. Chen, "Optimal solution of the two-stage Kalman estimator", IEEE Transactions on Automatic Control, vol. 44, pp.194-199, 1999.

[5] C. S. Hsieh, "Robust Two-Stage Kalman filters for systems with unknown inputs", IEEE Transactions on Automatic Control, vol. 45, pp 2374-2378, 2000.

[6] J. Y. Keller, L. Summerer et M. Darouach, "Two-Stage Kalman estimator with unknown exogenous inputs", Automatica, vol. 35, pp. 339-342, 1999.

[7] K. Kim, B. Shafai, and E. Kappos, "proportional integral estimator," in Proceeding of SPIE Signal and Data Processing of Small Targets, Orlando. FL, vol. 1096, pp. 187-208, 1989.

[8] K. H. Kim, J. G. Lee and C. G. Park, "Adaptative two-stage Kalman filter in the presence of unknown random bias," International Journal of Adaptative Control and Signal Processing, vol. 20, pp. 305-319, 2006.

[9] K. H. Kim, J. G. Lee and C. G. Park, "The stability analyses of the adaptative two-stage Kalman filter," International Journal of Adaptative Control and Signal Processing, vol. 21, pp. 856-870, 2007.

[10] K. Khémiri, F. Ben Hmida, J. Ragot and M. Gossa, "Filtre de Kalman a deux étages pour l'estimation d'état et de défauts de systèmes stochastiques linéaires" , 5th international conference (JTEA2008), Hammamet, TUNISIA, pp. 1609–1617, 2008.

[11] R. Orjuela, B. Marx, J. Ragot and D. Maquin, " Conception d'un observateur PI pour un multimodèle découplé," 8th International Conference (STA'2007), Monastir, Tunisia, November 2007.

[12] S. P. Linder and B. Shafai, "Robust PFI Kalman filters," in Proceedings of the 1998 American Control Conference, Philadelphia, PA , pp. 3163-3164, 1998.

[13] O. Y. Bas, B. Shafai and S. P. Linder, "Design of Optimal Gains for the Proportional Integral Kalman Filter with application to Single Particle Tracking," Proceedings of the 38th Conference on Decision and Control Phoenix, Arizona USA, December 1999.