WWTP diagnosis based on robust principal component analysis

Y. Tharrault * **G. Mourot** * **J. Ragot** *

* *Centre de Recherche en Automatique de Nancy (CRAN)*
Nancy Université, CNRS
2, Avenue de la forêt de Haye. F-54516 Vandoeuvre-lès-Nancy
{*yvon.tharrault, gilles.mourot, jose.ragot*}*@ensem.inpl-nancy.fr*

Abstract: Principal component analysis (PCA) is a powerful fault detection and isolation method. However, the classical PCA which is based on the estimation of the sample mean and variance-covariance matrix of the data is very sensitive to outliers in the training data set. Usually robust principal component analysis was applied to remove the effect of outliers on the PCA model. In this paper, a fast two-step algorithm is proposed. First, the objective was to find a robust PCA. Hence a scale-M estimator is computed using an iterative re-weighted least squares (IRWLS) procedure. This algorithm is initialized from a nearly robust variance-covariance estimate which tends to emphasize the contribution of close observations in comparison with distant observations (outliers). Second, structured residuals are used for multiple fault detection and isolation. These structured residuals are based on the reconstruction principle and the existence condition of such residuals is used to determine the detectable faults and the isolable faults. The proposed scheme avoids the combinatorial explosion of faulty scenarios related to multiple faults to consider. Then, this procedure is successfully applied for sensor fault detection and isolation of the hydraulic part of an activated sludge wastewater treatment plant (WWTP).

Keywords: Diagnosis, Robustness, Fault isolation, Statistical analysis, Environmental engineering

1. INTRODUCTION

Principal component analysis (PCA) has been applied successfully in the monitoring of complex systems [Chiang and Colegrove, 2007, Harkat et al., 2006]. It enables the determination of the redundancy relationships which are then used to detect and isolate faults. It transforms the data to a smaller set of variables which are linear combinations of the original variables while retaining as much information as possible. In the classical approach, the principal components correspond to the directions in which the projected observations have the largest variance. The principal components, correspond to the eigenvectors of the empirical covariance matrix. From a regression point of view, PCA also constructs the optimal orthogonal linear projections (in terms of mean squared error) from the eigenvectors of the data covariance matrix. The performance of PCA model is then based on the accurate estimation of the covariance matrix from the data which is very sensitive to abnormal observations.

In general, the majority of the training data set is associated with normal operating conditions. The remaining data (faulty data, data obtained during shutdown or startup periods or data issued from different operating mode) are referred to as "outliers". They disturb the correlation structure of the "normal data" and then the PCA model does not accurately represent the process. In practice one often tries to detect outliers using diagnostic tools starting from a classical fitting method. However, classical methods can be affected by outliers so strongly that the resulting fitted model does not allow to detect the true outliers (masking and swamping phenomena). To avoid these effects, the goal of robust PCA methods is to obtain principal components that are not influenced much by outliers.

Our presentation is devoted to the problem of sensor fault detection and isolation in data. In this paper, a fast two-step algorithm is proposed. First, a scale-M estimator [Maronna, 2005] is used to determine a robust model. This estimator is computed using an iterative re-weighted least squares (IRWLS) procedure. This algorithm is initialized from a very simple estimate derived from a one-step weighted variance-covariance estimate [Ruiz-Gazen, 1996]. Second, structured residuals are used for multiple fault detection and isolation. These structured residuals are based on the reconstruction principle. The variable reconstruction approach assumes that each set of faulty variables is unknown and suggests to reconstruct these variables using the PCA model from the remaining variables [Dunia and Qin, 1998]. If the faulty variables are reconstructed, the fault effect is eliminated. This property is useful for fault isolation. Moreover instead of considering the isolation of one up to all sensors, we determine the maximum number of faulty scenarios to take into account by evaluating the existence condition of structured residuals. The proposed scheme avoids the combinatorial explosion of faulty scenarios related to multiple faults to consider. Section 2 is a short reminder, on one hand, of the principal component analysis in the traditional case and, on the other hand, of the proposed robust principal component analysis. A detection and isolation procedure for outliers is proposed in section 3. Then, in section 4, this procedure is successfully applied for diagnosis of the hydraulic part of an activated sludge wastewater treatment plant (WWTP).

2. PRINCIPAL COMPONENT ANALYSIS

Let us consider a data matrix $X \in \Re^{N \times n}$, with row vector $x(k)^T$, which gathers N measurements collected on the n system variables.

2.1 Classical approach

In the classical *PCA* case, data are supposed to be collected on a system being in a normal process operation. *PCA* determines an optimal linear transformation of the data matrix X in terms of capturing the variation in the data:

$$T = XP \qquad \text{and} \qquad X = TP^T \tag{1}$$

with $T \in \Re^{N\times n}$ the principal component matrix and the matrix $P \in \Re^{n\times n}$ contains the principal vectors which are the eigenvectors associated to the eigenvalues λ_i of the covariance matrix (or correlation matrix) Σ of X:

$$\Sigma = P\Lambda P^T \qquad \text{with} \qquad PP^T = P^TP = I_n \tag{2}$$

where $\Lambda = diag(\lambda_1 \dots \lambda_n)$ is a diagonal matrix with diagonal elements in decreasing magnitude order and $I_n \in \Re^{n\times n}$ an identity matrix.

The relations (1) are useful when the dimension of the representation space is reduced. Once the component number ℓ to retain is determined, the data matrix X can be approximated. For that, the eigenvector and eigenvalues matrices are partitioned into the form:

$$P = \begin{pmatrix}\hat{P} & \tilde{P}\end{pmatrix} \qquad \hat{P} \in \Re^{n\times\ell} \tag{3}$$

$$\Lambda = \begin{bmatrix}\hat{\Lambda} & 0\\ 0 & \tilde{\Lambda}\end{bmatrix} \qquad \hat{\Lambda} \in \Re^{\ell\times\ell} \tag{4}$$

From the decomposition (1), $\hat{X}$ is the principal part of the data explained by the ℓ first eigenvectors and the residual part $\tilde{X}$ is explained by the remaining components:

$$\hat{X} = X\hat{P}\hat{P}^T = XC_\ell \tag{5}$$

$$\tilde{X} = X - \hat{X} = X(I - C_\ell) \tag{6}$$

where the matrix $C_\ell = \hat{P}\hat{P}^T$ is not equal to the identity matrix, excepted in the case $\ell = n$.

Hence the residual $r(k)$, for $k = 1..N$, is defined as follows:

$$r(k) = ||\tilde{P}^T x(k) - \tilde{P}^T \mu||^2 \tag{7}$$

where μ correspond to the mean of the data X.

Choosing P as the eigenvectors of the covariance matrix is equivalent as minimizing the function φ of the estimation error with the constraint $P^TP = I$ such as:

$$\varphi = \frac{1}{N}\sum_{k=1}^{N} r(k) \tag{8}$$

The *PCA* model being known according to (5) and (6), a new measurement vector $x(k)$ (which may contains abnormal values) can be decomposed as below:

$$x(k) = \hat{x}(k) + \tilde{x}(k), \quad \hat{x}(k) = C_\ell\, x(k), \quad \tilde{x}(k) = (I - C_\ell)\, x(k) \tag{9}$$

where $\hat{x}(k)$ and $\tilde{x}(k)$ are respectively the projections of $x(k)$ onto the principal space and the residual space.

2.2 Robust approach

Our approach consists in carrying out *PCA* directly on the data possibly contaminated by outliers. For that, a simple robust estimator, called scale-M estimator, is used. However, this estimator is computed by an iterative procedure. Then, good initialization parameters are needed to avoid local minimum. To initialize this scale-M estimator a robust covariance matrix is first calculated with a low computational cost.

Robust covariance Ruiz-Gazen (1996) define a "local" matrix of variance in the sense that the suggested form tends to emphasize the contribution of close observations in comparison with distant observations (outliers). The matrix is defined in the following way:

$$T = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w(i,j)(x(i)-x(j))(x(i)-x(j))^T}{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w(i,j)} \tag{10}$$

where the weights $w_{i,j}$ themselves are defined by:

$$w(i,j) = \exp\left(-\frac{\beta}{2}(x(i)-x(j))^T\Sigma^{-1}(x(i)-x(j))\right) \tag{11}$$

β being a tuning parameter to reduce the influence of the observations faraway, the authors recommend a value close to 2. For $\beta = 0$, the robust covariance matrix T is equal to 2Σ. And for a high value of β, only the closest observations are taken into account in the robust covariance matrix T.

Scale-M estimator Two M-estimators are used, one for estimation of the objective function φ (8) and another one for the estimation of the robust residual scale. The general scale-M estimator minimizes the following objective function with the constraint $\tilde{P}^T\tilde{P} = I_{n-\ell}$ [Maronna, 2005]:

$$\frac{1}{N}\sum_{k=1}^{N}\rho\left(\frac{r(k)}{\hat{\sigma}}\right) \tag{12}$$

with $r(k)$ the residual defined by equation (7), $\hat{\sigma}$ the robust scale of the residual $r(k)$ and the function ρ:$\Re^+ \to [0,1]$ is nondecreasing, with $\rho(0) = 0$ and $\rho(\infty) = 1$, and differentiable. $\tilde{P}$ is the eigenvector matrix of the robust covariance matrix C (14) corresponding to its $n-\ell$ smallest eigenvalues. Then the weighted mean μ and the covariance C are defined as follows:

$$\mu = \frac{\sum_{k=1}^{N} w(k)x(k)}{\sum_{k=1}^{N} w(k)} \qquad \text{with } w(k) = \dot{\rho}\left(\frac{r(k)}{\hat{\sigma}}\right) \tag{13}$$

$$C = \sum_{k=1}^{N} w(k)(x(k)-\mu)(x(k)-\mu)^T \tag{14}$$

Then the scale factor $\hat{\sigma}$ is defined as the solution of:

$$\frac{1}{N}\sum_{k=1}^{N}\rho\left(\frac{r(k)}{\hat{\sigma}}\right) = \delta \tag{15}$$

with $\delta \in (0,1)$.

Then an iterative algorithm is necessary to determine all these parameters. To avoid local minimum a good initialization is needed. Here the robust covariance matrix T (10) is used to determined the values of the initial parameters.
However, this method is only robust to fault with a projection into the residual space. Then to be robust to all possible fault, a similar approach in the principal space is used, in that case the scale-M estimator maximizes the following objective function with the constraint $\hat{P}^T\hat{P} = I_\ell$:

$$\frac{1}{N}\sum_{k=1}^{N}\rho\left(\frac{||\hat{P}^T x(k) - \hat{P}^T\mu||^2}{\hat{\sigma}}\right) \tag{16}$$

To ensure the elimination of fault disturbing the residual space when the scale-M estimator in the principal space is used, the minimum between the weight determine with the first scale-M estimator (robust to fault with a projection into the residual

space) and the weight obtained with the second scale-M estimator is used (step 9f of the algorithm). Finally, to improve the estimation of the covariance matrix, and thus the PCA model, a last weighting step is done using the Mahalanobis distance to eliminate outliers.

The algorithm is described as follows:

(1) $it = 1$ and $\sigma_0 = \infty$
(2) Compute $\tilde{P}$ the eigenvector matrix of the robust covariance matrix T corresponding to its $n-\ell$ smallest eigenvalues.
(3) Compute $a = median(X\tilde{P})$
(4) Do until $it = N_1$ or $\Delta \leq tol$
  (a) Compute $r(k) = ||\tilde{P}x(k) - a||^2$ for $k = 1...N$
  (b) Compute $\hat{\sigma}$ from (15)
  (c) If $it > 1$, set $\Delta = 1 - \hat{\sigma}/\sigma_0$
  (d) Set $\sigma_0 = \hat{\sigma}$
  (e) Compute the $w(k) = \dot{\rho}\,(r(k)/\hat{\sigma})$ for $k = 1...N$
  (f) Compute μ from (13)
  (g) Compute C from (14)
  (h) $\tilde{P}$ the eigenvector matrix of the covariance matrix C corresponding to its $n-\ell$ smallest eigenvalues.
  (i) Compute $a = \tilde{P}^T\mu$
  (j) Set $it = it + 1$
(5) End do.
(6) Set $w_{res} = w$, $it = 1$ and $\sigma_0 = \infty$
(7) Compute $\hat{P}$ the eigenvector matrix of the robust covariance matrix C corresponding to its ℓ largest eigenvalues.
(8) Compute $a = median(X\hat{P})$
(9) Do until $it = N_1$ or $\Delta \leq tol$
  (a) Compute $r(k) = ||\hat{P}x(k) - a||^2$ for $k = 1...N$
  (b) Compute $\hat{\sigma}$ from (15)
  (c) If $it > 1$, set $\Delta = 1 - \hat{\sigma}/\sigma_0$
  (d) Set $\sigma_0 = \hat{\sigma}$
  (e) Compute the $w(k) = \dot{\rho}\,(r(k)/\hat{\sigma})$ for $k = 1...N$
  (f) Set $w = min(w, w_{res})$
  (g) Compute μ from (13)
  (h) Compute C from (14)
  (i) $\hat{P}$ the eigenvector matrix of the covariance matrix C corresponding to its ℓ largest eigenvalues.
  (j) Compute $a = \hat{P}^T\mu$
  (k) Set $it = it + 1$
(10) End do.
(11) Compute

$$\mu = \frac{\sum_{k=1}^{N} w(k)x(k)}{\sum_{k=1}^{N} w(k)} \tag{17}$$

$$S = \left(\sum_{k=1}^{N} w(k)(x(k)-\mu)(x(k)-\mu)^T\right) \Big/ \left(\sum_{k=1}^{N} w(k) - 1\right) \tag{18}$$

with

$$\begin{cases} w(k) = 1 & \text{if} \quad D(k) \leq \chi^2_{n,0.975} \\ \quad = 0 & \text{else} \end{cases} \tag{19}$$

where $D(k)$ is the Mahalanobis distance defined as follows:

$$D(k) = x(k)^T P \Lambda^{-1} P^T x(k) \tag{20}$$

In the experiments of this article, ρ, defined by equation (12), was chosen as the bisquare function (r represents the squared distances).

$$\rho(r) = min\{1, 1-(1-r)^3\} \tag{21}$$

The constant δ in equation (15) is chosen as defined by Maronna (2005):

$$\delta = \frac{N-n+\ell-1}{2N} \tag{22}$$

However, this algorithm needs the number of principal components. Hence, a robust method to find the number of principal components is introduced.

3. ROBUST DETERMINATION OF THE NUMBER OF PRINCIPAL COMPONENTS

The number of principal components to choose is obtained by minimizing the normalized VRE (variance of reconstruction error) with respect to the number ℓ [Qin and Dunia, 2000], the criterion is then :

$$J(\ell) = \sum_{j=1}^{n} \frac{\xi_j^{\mathrm{T}}\,(I_n - C_\ell)\,S\,(I_n - C_\ell)\,\xi_j}{\left(\xi_j^T (I_n - C_\ell)\xi_j\right)^2} \tag{23}$$

with $\ell = 1, \ldots, n-1$, $I_n \in \Re^{n\times n}$ an identity matrix, S the robust covariance matrix and ξ_j the reconstruction direction ($\xi_j = [0 \ldots 1 \ldots 0]^T$ where value 1 is at the j$^{\text{th}}$ position)

Qin and Dunia [2000] show that this criterion may present a minimum in the interval $[1, n]$.

From this new model, detection and isolation of outliers are carried out using the Mahalanobis distance (20) and the reconstruction principle.

4. FAULT DETECTION AND ISOLATION

The variable reconstruction approach assumes that a group of variables may be faulty and suggests to reconstruct the assumed faulty variables using the PCA model from the remaining variables [Dunia and Qin, 1998]. This approach is used for fault isolation.

4.1 Data reconstruction

The reconstruction $\hat{x}_R(k)$ of an observation $x(k)$ is obtained by minimizing the influence of fault. It is defined as follows:

$$\hat{x}_R(k) = x(k) - \Xi_R f_R \tag{24}$$

with f_R the fault magnitude (unknown) and the matrix Ξ_R indicates the reconstruction directions. This matrix is orthonormal with dimension ($n \times$ r), with r the number of component to reconstruct, and is built with 0 and 1, where 1 indicates the reconstructed variables from the other variables (with 0). For example, to reconstruct the set of variables $R = \{2, 4\}$ among 5 variables, matrix Ξ_R is formed as follows:

$$\Xi_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

The estimation of the fault magnitude f_R is obtained by solving the following optimization problem:

$$\hat{f}_R = \arg\min_{f_R} \{D_R(k)\} \tag{25}$$

where $D_R(k)$ is the fault detection indicator (Mahalanobis distance) and is given by:

$$D_R(k) = \hat{x}_R^T(k)\Phi\hat{x}_R(k) \tag{26}$$

with

$$\Phi = P\Lambda^{-1}P^T \tag{27}$$

The system is considered normal if:

$$D_R(k) \leq \gamma^2 \tag{28}$$

where γ^2 is the detection threshold. The expression for the reconstruction $\hat{x}_R(k)$ of the vector $x(k)$ is given by:

$$\hat{x}_R(k) = G_R\, x(k) \tag{29}$$

with $G_R = \left(I - \Xi_R(\Xi_R^T \Phi \Xi_R)^{-1}\Xi_R^T \Phi\right)$

Condition of reconstruction :
To reconstruct a fault, it must be at least projected into the principal space ($r \leq \ell$) or into the principal space ($r \leq n-\ell$). This implies that the number of reconstructed variables r must respect the following inequality (30):

$$r \leq \max(n-\ell, \ell) \tag{30}$$

4.2 Structured residual generation

In a diagnosis objective, residuals are generated for fault detection and isolation. Considering a measurement $x(k)$ composed with the true value $x^*(k)$, a noise $\varepsilon(k)$ with zero mean and one fault with amplitude d and direction Ξ_F, where F is a subset containing the indices of the fault directions:

$$x(k) = x^*(k) + \varepsilon(k) + \Xi_F d \tag{31}$$

Considering all possible reconstruction direction:

- If the reconstruction directions Ξ_R are the same as the fault directions, i.e. if $R = F$, then $D_R(k)$ is under the detection threshold γ^2, indeed:
  $D_R(k) = x^{*T}(k)\Phi x^*(k) - x^*(k)^T \Phi \Xi_R (\Xi_R^T \Phi \Xi_R)^{-1} \Xi_R^T \Phi x^*(k)$
  where $x^{*T}(k)\Phi x^*(k) \leq \gamma^2$
  and $x^{*T}(k)\Phi\Xi_R(\Xi_R^T\Phi\Xi_R)^{-1}\Xi_R^T\Phi x^*(k) > 0$
  then $D_R < \gamma^2$

- If the reconstruction directions Ξ_R are different from the fault directions, then $D_R(k)$ is higher than the detection threshold γ^2 if the projection of the reconstruction directions are not collinear to the fault projection into the residual space and into the principal space.

For the faulty observations k, the faulty variables $\hat{R}$ are determine as follows:

$$\hat{R} = \underset{R \in \Im}{\arg} D_R(k) < \gamma^2 \tag{32}$$

with $\Im$ all combinations of possible reconstruction directions.

4.3 Fault isolation

All the directions of reconstruction Ξ_R have to be explored for fault isolation. The maximum reconstruction number can be calculated as follows:

$$\sum_{r=1}^{\max(n-\ell,\ell)-1} \mathbb{C}_n^r \tag{33}$$

with $\mathbb{C}_n^r$ denotes the combination of r from n.
This number takes only into account the number of reconstructions. However, collinear projections have the same fault signature. Then we will analyze the angles between the different projection of reconstruction directions. The largest primary angle θ between two sub-spaces of the same size is linked to the concept of distance between these two sub-spaces [Golub and Van Loan, 1996].

This distance is defined in the principal space $d(R1,R2)$ and in the residual space $\tilde{d}(R1,R2)$ as follows:

$$d(R1,R2) = ||\hat{\Xi}_{R_1}(\hat{\Xi}_{R_1}^T\hat{\Xi}_{R_1})^{-1}\hat{\Xi}_{R_1}^T - \hat{\Xi}_{R_2}(\hat{\Xi}_{R_2}^T\hat{\Xi}_{R_2})^{-1}\hat{\Xi}_{R_2}^T||_2 \tag{34}$$

$$\tilde{d}(R1,R2) = ||\tilde{\Xi}_{R_1}(\tilde{\Xi}_{R_1}^T\tilde{\Xi}_{R_1})^{-1}\tilde{\Xi}_{R_1}^T - \tilde{\Xi}_{R_2}(\tilde{\Xi}_{R_2}^T\tilde{\Xi}_{R_2})^{-1}\tilde{\Xi}_{R_2}^T||_2 \tag{35}$$

with $\hat{\Xi}_{R_1} = \hat{\Lambda}^{-1/2}\hat{P}^T\Xi_{R_1}$, $\tilde{\Xi}_{R_1} = \tilde{\Lambda}^{-1/2}\tilde{P}^T\Xi_{R_1}$ and R_1 and R_2 correspond to sets of variable reconstruction.

Analyzing these distances, then the isolable fault can be determine. Hence, a global indicator k is built.

$$k(R_1,R_2) = \max\{(d(R_1,R_2), \tilde{d}(R_1,R_2)\} \tag{36}$$

Thus if $k(R_1,R_2)$ is close to zero, it means that the projection of the set of reconstructed variables R_1 and R_2 are collinear into the residual space and into the principal space. It means that a fault for the sets of reconstructed variables R_1 or R_2 are not isolable. The process to detect useful directions of reconstruction can be summarized as follows:

(1) $r = 1$
(2) Calculate for all available directions ($R_1 \in \Im$ and $R_2 \in \Im$) the indicator $k(R_1,R_2)$ (36). Smaller the value of this indicator is, higher the magnitude of the fault has to be important to ensure fault isolation. And if this indicator is equal to zero, then only a set of variables potentially faulty may be determined, i.e. the faulty variables are associated to the indices R_1 or R_2 or R_1 and R_2. Thus, it is only required to determine one direction, for example R_1.
(3) $r = r + 1$
(4) While $r < \max(\ell, n-\ell)$ do to the step 2

This analysis of the structure of the model allows to determine the isolable faults. The number of useful reconstructions can then be reduced.

5. APPLICATION TO HYDRAULIC PART OF A WASTEWATER TREATMENT PLANT

Figure 1 describes the hydraulic part of a real industrial plant and the position of different sensors. The measures with a gray background (2, 5, 7, 8, 9) correspond to the different command of the station with the location of actuators, other numbers (1, 3, 4, 6, 10) represent sensors for which information has to be validated.

The raw water first goes through bar screen to remove any solids which are larger than their openings. The water then comes in a sump, the level of water is measured in this sump. Then a pumping station brings up the raw water to flow by gravity in the rest of the station. On leaving the pumping station, there is a flow measurement. Wastewater are then treated by activated sludge. There is a recirculation and extraction circuit (sludge and excess sludge surface) of sludge. Then a sensor measures the level of the overflow after the clarifier, this overflow can restrict the flow in the second in biology. This measure allows to estimate the flow directly rejected in the river named "Sûre". For the different recirculation and extraction flows of sludge as well as for pumping station, commands pumps and nominal flows are known. A measure of each variable is recorded every 15 minutes.

First to apply PCA, a data matrix must be constructed.

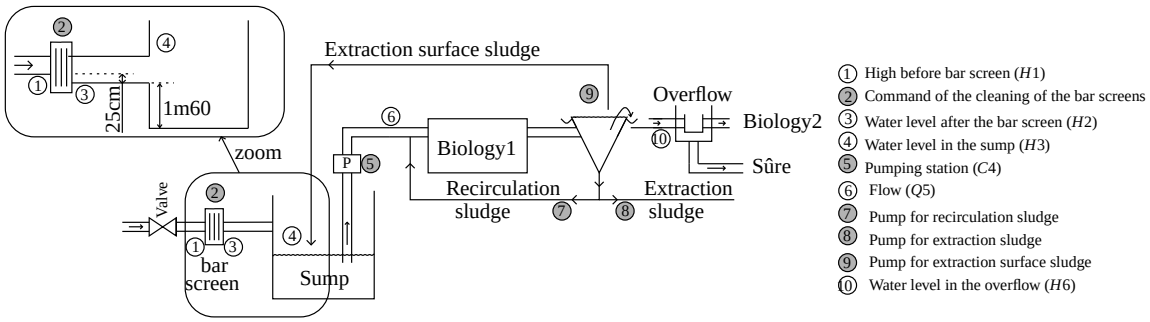

Fig. 1. Description of the station

5.1 Construction of the data matrix

To take into account dynamic process with PCA, the data matrix has to be composed of data with temporal lag. Moreover, transformed variables can be added to take into account non linear process. To determine the different temporal lag and non linear transformation, a simple linear modeling step is used. Then a new variable $\tanh((Q5(k-1)-550)./150)$ and $H1(k-1)$, $H6(k-1)$ are used in the data matrix.

Two sets of data are used, one for model construction and second for fault detection and isolation. A vector $x(k)$ is defined as follows:

$$x(k) = [\, H1(k)\ H2(k)\ H3(k)\ Q5(k)\ H6(k) \\ \tanh((Q5(k-1)-550)./150)\ H1(k-1) \\ H6(k-1)\ C4(k)\,]^{\mathrm{T}} \qquad (37)$$

The data matrix X is constituted of N observations of the vector $x(k)$. After having constructed the data matrix X, PCA can be applied. The first step is to determine the number of principal components.

5.2 Number of principal components

To determine the number of principal components, the robust approach using the VRE, proposed in the section 3, is used. Four principal components are selected. The robust model is then built.

Figure 2 shows a part of the measure $H1$ with its estimation obtained with the classic PCA model and the robust PCA model and the associated residual (measure - estimate).

We noticed that in estimating $H1$, a fault is visible on the robust residual around the observation 1550 while with the classical PCA, it is not visible. This shows the advantage of using a robust approach.

5.3 Analysis of the reconstruction directions

From the size of the residual space and of the principal space, we cannot reconstruct more than five variables simultaneously. The maximum number of reconstructions is then equal to 255 (33). Table 1 shows the values of the global indicator k (36) with $r = 1$, i.e. only one variable is reconstructed. r represents the number of variables simultaneously faulty. Sets R_1 and R_2 contain the indices of the reconstructed variables. Smaller the value of this indicator k is, higher the magnitude of the fault has to be important to ensure fault isolation. All the values of k are not null, then all faults on one variable are isolable.

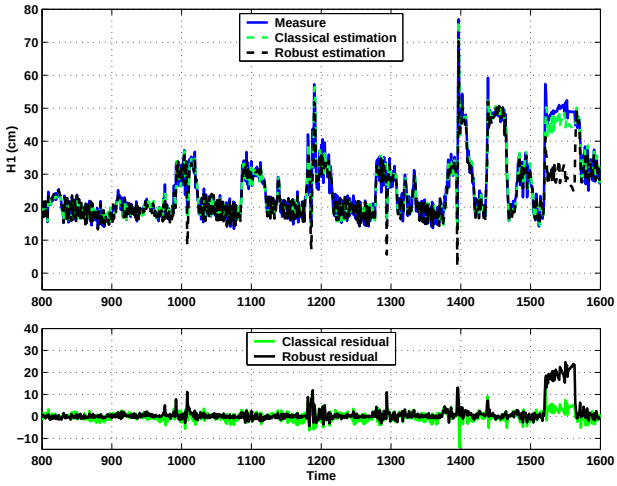

Fig. 2. Measure and estimation of $H1$

k		R_1								
		1	2	3	4	5	6	7	8	9
	1	0	0.94	0.99	1.00	0.99	0.98	0.97	0.99	1.00
	2		0	0.52	1.00	0.98	1.00	1.00	1.00	1.00
	3			0	0.99	1.00	1.00	1.00	1.00	0.99
R_2	4				0	1.00	0.99	1.00	1.00	0.33
	5					0	0.82	0.96	0.90	0.99
	6						0	0.80	1.00	1.00
	7							0	1.00	0.97
	8								0	0.97

Table 1. Indicator k for $r = 1$

For all the directions of reconstruction ($r = 2, 3, 4, 5$) this indicator is calculated. A case where k is close to zero is detected between $D_{1,2,6}$ and $D_{1,3,6}$. Then the fault signatures of these two directions are identical ($D_{1,2,6} = D_{1,3,6}$). Therefore only one indicator is useful to detect a fault, for example $D_{1,2,6}$. Moreover, we concluded that the signatures of reconstruction directions taking into account these sets are identical ($D_{1,2,6,7} = D_{1,3,6,7}$, $D_{1,2,6,8} = D_{1,3,6,8}$, ...). The number of useful reconstruction can be reduced to 202.

5.4 Fault detection and isolation

For fault detection, Mahalanobis distance is used. Figure 3 shows the Mahalanobis distance divided by its detection threshold, i.e. a fault is detected if the normalised Mahalanobis distance is greater than one. The detection threshold was empirically chosen. Then 22 faults are detected, on this figure, the different faults are numbered in order to isolate them in the next section.

To isolate fault, all useful reconstruction directions are calculated. Figure 4 shows some values of the indicator D for differ-

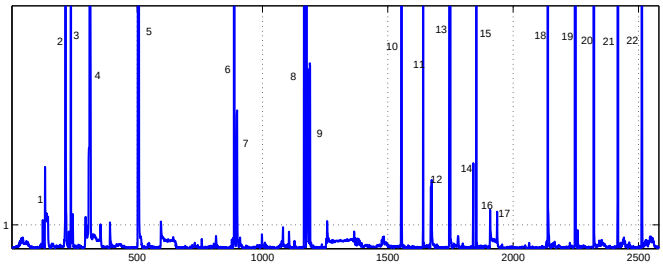

Fig. 3. Fault detection with Mahalanobis distance

ent reconstruction directions. Table 2 summarizes the isolation approach.

Fault index	Reconstruction direction under the detection threshold
16, 17	D_1
3, 10, 11, 14, 15, 19, 20, 21	D_3
7, 12	$D_{1,6}$
6, 8, 13, 18, 22	$D_{3,9}$
1, 2, 5	$D_{1,3,4}$
4	$D_{3,7,9}$
9	$D_{1,2,3,7}$

Table 2. Summary of fault isolation

When the indicator D_3 is calculated, the faults 3, 10, 11, 14, 15, 19, 20 and 21 are close to zero, then one concludes that during these periods variable $H3$ is faulty. Physically all these faults are due to a change in the relationships between different measures when the value of $H3$ is less than $1m85$. Indeed, upper left corner of the figure 1 details the physical relation between the bar screen and the sump. We can infer that when the water level in the sump drops below about $1m85$, the relationship between the water level before, after the bar screen and the level in the sump changes.

For faults 16, 17, the residual is close to zero when the first variable ($H1(k)$) is reconstructed. For faults 7, 12 residual is close to zero when the first ($H1(k)$) and the sixth ($tanh((Q5(k-1)) - 550/150)$) variables are reconstructed. These fault are probably due to an element that disrupts the flow in the bar screen.

For fault 4, the residual is close to zero when the third ($H3(k)$), seventh ($H1(k-1)$) and ninth ($C4(k)$) variables are reconstructed. For faults 6, 8, 13, 18, 22, the residual is close to zero when the third ($H3(k)$) and ninth ($C4(k)$) variables are reconstructed. For faults 1, 2, 5, the residual are close to zero when the first ($H1(k)$), third ($H3(k)$) and fourth ($Q5(k)$) variables are reconstructed. An explanation for all these faults may be that the water level in the sump $H3$ is so low that a pump of the pumping station does not work properly. Indeed it is possible that a pump, because of a lack of water, does not work.

For fault 9, the residual is close to zero when the first ($H1(k)$), second ($H2(k)$), third ($H3(k)$) and seventh ($H1(k-1)$) variables are reconstructed. It concludes that an element probably disrupts the flow in the bar screen and that the $H3$ is less than $1m85$.

6. CONCLUSION

A new robust PCA, based on a scale-M estimator, has been presented in this paper. Then, structured residuals are used for multiple faults detection and isolation. To avoid the combinatorial explosion of faulty scenarios related to multiple faults to consider, the existence condition of structured residuals is

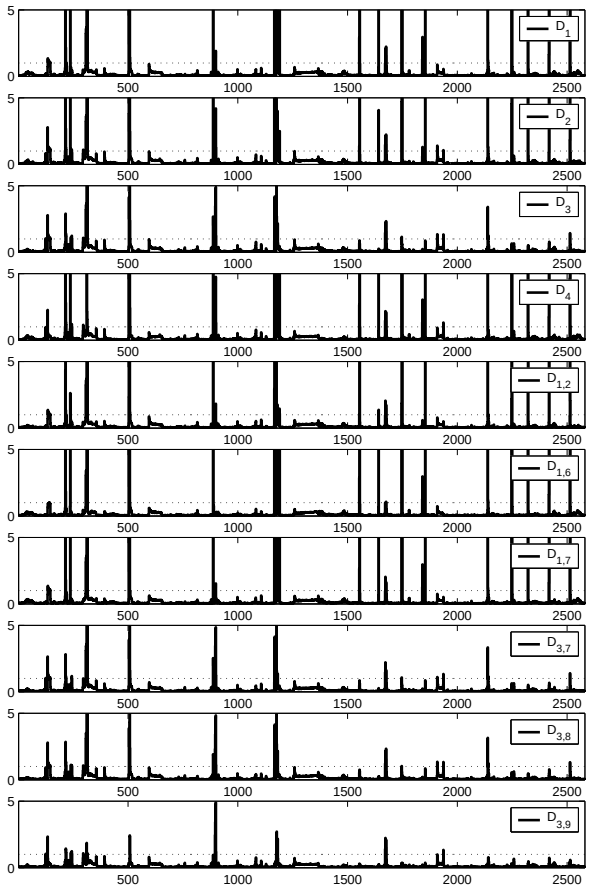

Fig. 4. Fault isolation

evaluated. This procedure is finally applied for diagnosis of the hydraulic part of an activated sludge WWTP. Hence, the proposed method allows to determine simultaneous faulty sensors or system fault for whose fault signature is associated to several errors on signals from sensors.

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