Nonlinear Joint State-Parameter Observer for VAV Damper position Estimation

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Outline

1. *Motivation*
   - Building HVAC Optimization
2. *Problem Formulation*
   - Modeling
   - Joint state and parameter estimation in T-S Models
3. *Results*
   - Algorithm
   - Simulation Results
4. *Concluding Remarks*

Outline

Optimizing Energy Consumption of HVAC

Stages of HVAC optimization

- Commissioning phase of HVAC vs operational phase
- Studies indicate HVAC use 20% more power than designed for

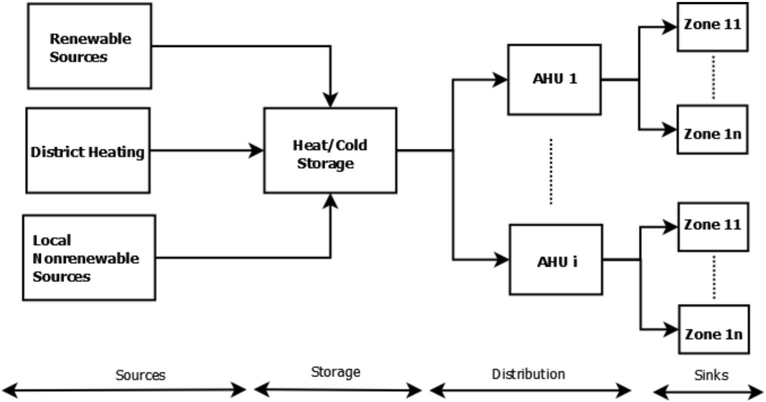
Operational Phase Issues

- Faults from design phase, malfunctioning equipments
- Incorrectly configured control systems (e.g for weather changes)
- Inappropriate operating procedures (e.g. occupancy scheduling)

Energy in Time

- FP7 project: Integrated control systems and methodologies to monitor and improve building energy performance
- Large Scale non-residential buildings' operational phase issues: FDI, FTC, prognosis etc. as focus

Building HVAC Schematic



FDI/FTC in Building HVAC

Challenges

- Difficulty in obtaining model from normal operation data
- Large number of disturbances
- Need to consider fault propagation
- Presence of multiple local control act as a mask of faults
- Nonlinear and bilinear process models

Approaches

- A need for combined model based and data based approach
- Distributed FDI mechanism with fault propagation
- Usage of T-S model based approach for estimation tasks

Relevant Works

Relevant Projects

- OptiControl - ETH Zurich: Stochastic MPC
- Berkeley-Lawrence - UC Berkeley: SQP to solve BMI
- CIESOL - Spain + Brazil: Lagrangian dual method and parallel programming

FDI in Building HVAC

- Major focus in literature is on equipment level fault diagnosis
- Data based approach is prominent

Distributed FDI strategies for Building HVAC

- V.Reppa et.al 2015-VAV, Papadopoulous et.al 2015-FCU
- Strategy for distributed FDI for Sensor fault estimation
- Looking to extend to actuator faults

Takagi-Sugeno Modeling

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t))$$

$$y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t))$$

- Polytopic, Quasi-LPV, Multi-Model, Fuzzy
- The model involves r 'linear sub-models'
- $\xi(t)$: premise variables
- $\mu_i(\xi(t))$: weighting function which follows the convex sum property:

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \text{ and } 0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, 2, ...r\}$$

- Allows extension of results in linear framework to nonlinear systems

Deriving T-S model

Obtaining TS Model

- **Identification**: I/O data from the process to fit model parameters.
- **Linearization**: Around 'appropriate' operating points: I/O data required to improve weighting function to reduce error.
- **Sector Nonlinearity**

Sector Nonlinearity

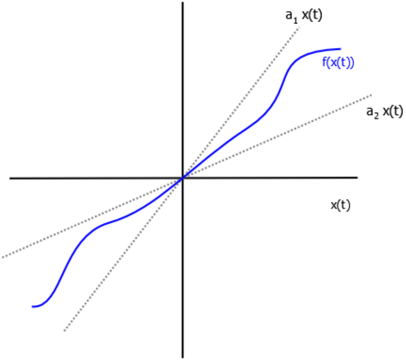
Rewrite function within a compact subspace.

$$\begin{cases} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t), u(t)) \end{cases} \Rightarrow \begin{cases} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

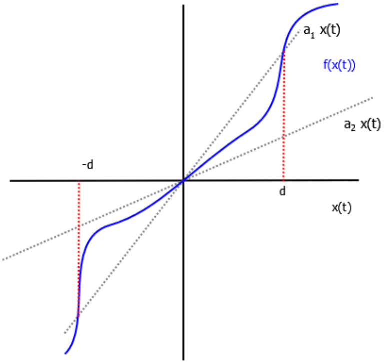
for $f(x(t)) \in [a_1 \, a_2] x(t)$

Sector Nonlinearity Idea

Global Sector



Local Sector



- Physical systems: bounded $\Rightarrow$ TS representation feasible.
- T-S Models 'exactly' represents the nonlinear system within the sector

Outline

Objectives

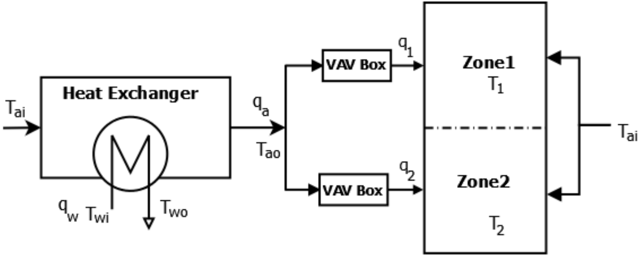
Overall

- A distributed FDI strategy for large building HVAC systems
- A nonlinear model based fault estimation strategy
- Use of joint state and parameter estimation

Present work

- Develop a model of AHU-VAV-Zones with unknown time varying parameter
- Derive T-S equivalent models for joing state and parameter estimation
- Illustrate feasibility of nonlinear joint state and parameter estimation using T-S method by customizing existing literature results
- Outline the direction of integrating such strategy in the overall framework

System under consideration



- The VAV box has a local control loop whose set point is given by a central controller
- Energy balance models are used for modeling the heat exchanger and zones
  - Heat exchanger: lumped nonlinear model of a counter flow heat exchanger
  - Room modeled as energy balance with interaction between zones and zone and external environment

Models

Heat Exchanger

$$\frac{dT_{ao}(t)}{dt} = \frac{q_a(t)}{M_a}(T_{ai}(t) - T_{ao}(t)) + \frac{U_A(t)}{2C_{pa}M_a}\Delta T(t)$$
$$\frac{dT_{wo}(t)}{dt} = \frac{q_w(t)}{M_w}(T_{wi}(t) - T_{wo}(t)) - \frac{U_A(t)}{2C_{pw}M_w}\Delta T(t)$$

$\Delta T \triangleq T_{wo} + T_{wi} - T_{ao} - T_{ai}$

Zones

$$C_1\frac{dT_1(t)}{dt} = q_1(t)C_{pa}(T_{ao}(t) - T_1(t)) + K_{12}(T_2(t) - T_1(t)) + K_{1amb}(T_{ai}(t) - T_1(t)) + K_{d_2}d_2(t)$$
$$C_2\frac{dT_2(t)}{dt} = q_2(t)C_{pa}(T_{ao}(t) - T_2(t)) + K_{21}(T_1(t) - T_2(t)) + K_{2amb}(T_{ai}(t) - T_2(t)) + K_{d_3}d_3(t)$$

VAV

$$q_1(t) = \beta_1 q_a(t)$$
$$q_2(t) = \beta_2 q_a(t)$$
$\beta_1 + \beta_2 = 1$

State space representation

State space model

$$\begin{aligned}
\dot{x}_1 &= \alpha_1 q_a(d_1 - x_1) + \alpha_{2au}(T_{wi} + x_2 - d_1 - x_1) \\
\dot{x}_2 &= \alpha_3 u(T_{wi} - x_2) - \alpha_{2wu}(T_{wi} + x_2 - d_1 - x_1) \\
\dot{x}_3 &= \alpha_4 \beta_1 q_a(x_1 - x_3) + \alpha_5(x_4 - x_3) + \alpha_6(d_1 - x_3) + \alpha_7 d_2 \\
\dot{x}_4 &= \alpha_8(1 - \beta_1) q_a(x_1 - x_4) + \alpha_9(x_3 - x_4) + \alpha_{10}(d_1 - x_4) + \alpha_{11} d_3
\end{aligned}$$

States and Inputs:

$$x_1 \triangleq T_{ao},\ x_2 \triangleq T_{wo},\ x_3 \triangleq T_1,\ x_4 \triangleq T_2\ u \triangleq q_w,\ d_1 \triangleq T_{ai}$$

Constants:

$$\alpha_1 \triangleq \frac{1}{M_a}, \alpha_3 \triangleq \frac{1}{M_w}, \alpha_{2au} \triangleq \frac{U_A}{2C_{pa}M_a}, \alpha_{2wu} \triangleq \frac{U_A}{2C_{pw}M_w}, \alpha_4 \triangleq \frac{C_{pa}}{C_1},\ \alpha_5 \triangleq \frac{K_{12}}{C_1}$$

$$\alpha_6 \triangleq \frac{K_{1amb}}{C_1},\ \alpha_7 \triangleq \frac{K_{d1}}{C_1} \alpha_8 \triangleq \frac{C_{pa}}{C_2},\ \alpha_9 \triangleq \frac{K_{21}}{C_2},\ \alpha_{10} \triangleq \frac{K_{2amb}}{C_2},\ \alpha_{11} \triangleq \frac{K_{d3}}{C_2}$$

T-S equivalent model

First step is to obtain a model of the form

$$\dot{x}(t) = \sum_{i=1}^{2^p} \mu_i(z(t))(A_i(\theta(t))x(t) + B_i^u u(t) + B^T T_{wi} + B_i^d d(t))$$
$$y(t) = Cx(t) + H\nu(t)$$

where, $\theta = \beta_1$ and the premise variables are: $z_1 \triangleq q_a$ (measured) and $z_2 \triangleq T_{wo} = x_2$ (unmeasured state)

$$A_i(\theta(t)) = \begin{bmatrix} -\alpha_1 z_1^i - \alpha_{2au} & \alpha_{2au} & 0 & 0 \\ \alpha_{2wu} & -\alpha_{2wu} & 0 & 0 \\ \alpha_4\theta(t)z_1^i & 0 & -\alpha_4\theta(t)z_1^i - \alpha_5 - \alpha_6 & \alpha_5 \\ \alpha_8(1-\theta(t))z_1^i & 0 & \alpha_9 & -\alpha_8(1-\theta(t))z_1^i - \alpha_9 - \alpha_{10} \end{bmatrix}$$

$$B_i^u = \begin{bmatrix} 0 \\ \alpha_3(T_{wi} - z_2^i) \\ 0 \\ 0 \end{bmatrix}, \; B^T = \begin{bmatrix} \alpha_{2au} \\ -\alpha_{2wu} \\ 0 \\ 0 \end{bmatrix}, \; B_i^d = \begin{bmatrix} \alpha_1 z_1^i - \alpha_{2au} & 0 & 0 \\ \alpha_{2wu} & 0 & 0 \\ \alpha_6 & \alpha_7 & 0 \\ \alpha_{10} & 0 & \alpha_{11} \end{bmatrix}$$

where i subscript refers to the boundary value of the premise variable corresponding to the submodel.

Models for Joint State and Parameter Estimation

Model derivation

- T-S model with time varying A-matrix (i.e., SNL applied to system considering premise variables),

$$\dot{x}(t) = \sum_{i=1}^{2^p} \mu_i^z(z)(A_i(\theta(t))x(t) + B_i(\theta(t))u(t))$$

- To apply SNL again to take care of unknown time varying parameter (where θ is the unknown time varying parameter with the bounds of $[\theta^1, \theta^2]$):

$$A_i(\theta(t)) = \breve{A}_i + \sum_{j=1}^{n_\theta} \sum_{k=1}^{2} \mu_j^\theta(\theta(t))\theta_j^k \bar{A}_j \qquad B_i(\theta(t)) = \breve{B}_i + \sum_{j=1}^{n_\theta} \sum_{k=1}^{2} \mu_j^\theta(\theta(t))\theta_j^k \bar{B}_j$$

- This can be simplified as,

$$\dot{x}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z(t))\mu_j^\theta(\theta(t))(A_{ij}x(t) + B_{ij}u(t))$$

$$y(t) = Cx(t)$$

- The constant system matrices are split as

$$A_{ij} = \breve{A}_i + \sum_{j=1}^{n_\theta} \theta_j^k \bar{A}_j \qquad B_{ij} = \breve{B}_i + \sum_{j=1}^{n_\theta} \theta_j^k \bar{B}_j$$

T-S joint state and parameter models

- The bound for unknown parameter $\theta \triangleq \beta_1$ is $[0,\ 1]$
- Only the A matrix depends on the unknown parameter. Hence $B^u_{ij} = B^u_i$ and so on.

$$A_{ij} = \begin{bmatrix} -\alpha_1 z_1^i - \alpha_{2au} & \alpha_{2au} & 0 & 0 \\ \alpha_{2wu} & -\alpha_{2wu} & 0 & 0 \\ \alpha_4 \theta^j z_1^i & 0 & -\alpha_4 \theta^j z_1^i - \alpha_5 - \alpha_6 & \alpha_5 \\ \alpha_8 (1-\theta^j) z_1^i & 0 & \alpha_9 & -\alpha_8 (1-\theta^j) z_1^i - \alpha_9 - \alpha_{10} \end{bmatrix}$$

And since the two zone temperatures are the measured variables, we have,

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T-S Parameter Estimation [Bezzaoucha et.al 2013]

Model structure

- System Model structure

$$\dot{x}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z(t))\mu_j^\theta(\theta(t))(A_{ij}x(t) + B_{ij}u(t))$$
$$y(t) = Cx(t) \quad (1)$$

- Observer Structure

$$\dot{\hat{x}}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[A_{ij}\hat{x}(t) + B_{ij}u(t) + L_{ij}(y(t) - \hat{y}(t))]$$
$$\dot{\hat{\theta}}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[K_{ij}(y(t) - \hat{y}(t)) - \eta_{ij}\hat{\theta}(t)]$$
$$\hat{y}(t) = C\hat{x}(t) \quad (2)$$

- Original system in uncertain-like form for comparison

$$\dot{x}(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[(A_{ij} + \Delta A(t))x(t) + (B_{ij} + \Delta B(t))u(t)]$$
$$y(t) = Cx(t)$$

T-S Parameter Estimation [Bezzaoucha et.al 2013]

Error Dynamics

- This representation helps to obtain the observer error dynamics of the form:

$$\dot{e}_a(t) = \sum_{i=1}^{2^p} \sum_{j=1}^{2^{n_\theta}} \mu_i^z(z)\mu_j^\theta(\hat{\theta})[\Phi_{ij} e_a(t) + \Psi_{ij}(t)\tilde{u}(t)]$$

- $e_a(t) \triangleq [e_x(t)\ e_\theta(t)]^T$, $\tilde{u} \triangleq [x(t)\ \theta(t)\ \dot{\theta}(t)\ u(t)\ \nu(t)]^T$.
- Two objectives: stabilize Φ_{ij} and reduce impact of $\Psi_{ij}(t)$ by bounding it.
- Stabilization: Bounded Real Lemma applied considering quadratic Lyapunov function $(V(e(t)) = e^T(t)Pe(t))$ with a derivative limit given by: $\dot{V}(t) + e_a^T(t)Pe_a(t) - \tilde{u}^T(t)\Gamma_2\tilde{u}(t) < 0$
- To bound $\Psi_{ij}(t)$, the convexity property of weighting functions and known matrix properties are used.

T-S Parameter Estimation [Bezzaoucha et.al 2013]

Theorem

There exists a robust state and parameter observer for the linear time varying parameter system with a bounded $\mathcal{L}_2$ gain β of the transfer from $\tilde{u}(t)$ to $e_a(t)$ ($\beta > 0$) if there exists $P_0 = P_0^T > 0$, $P_1 = P_1^T > 0$, $\beta > 0$, λ_1, λ_2, Γ_2^0, Γ_2^1, Γ_2^2, Γ_2^3, η_1, η_2, F_1, F_2, R_1 and R_2 (for $i = 1, ..2^p$, $j = 1, .., 2^{n_\theta}$):

$$\underset{P_0, P_1, R_{ij}, F_{ij}, \eta_{ij}, \lambda_1, \lambda_2, \Gamma_2^0, \Gamma_2^1, \Gamma_2^2, \Gamma_2^3}{\text{minimize}} \quad \beta$$

$$\Gamma_2^k < \beta I \text{ for } k = 0, 1, 2, 3$$

under the LMI constraints

$$\begin{bmatrix} T_{11} & -C^T F_{ij}^T & 0 & 0 & 0 & 0 & P_0\mathcal{A} & P_0\mathcal{B} \\ * & -\bar{\eta}_{ij} - \bar{\eta}_{ij}^T + I & 0 & \bar{\eta}_{ij} & P_1 & 0 & 0 & 0 \\ * & * & -\Gamma_2^0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\Gamma_2^1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Gamma_2^2 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma_2^3 & 0 & 0 \\ * & * & * & * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & * & * & 0 & -\lambda_2 I \end{bmatrix} < 0$$

with $T_{11} = P_0 A_{ij} + A_{ij}^T P_0 - R_{ij} C - C^T R_{ij}^T + I$

Outline

Algorithm

Implementation steps

- Choose η such that its eigenvalues are comparable to Φ_{ij}. Needs iterations. Diagonal with requisite eigenvalues.
- Choose Γ_2^k values to reduce the number of LMI variables.
- Enforce $P_0 > P_{0init}$ and $P_1 > P_{1init}$ for sufficiently large P_{0init} and P_{1init}. This would ensure that P_0^{-1} and P_1^{-1} are not close to singular and make the computation of the observer gains K_{ij} and L_{ij} unreliable.
- To ensure that there is a balance between the gains K_{ij} and η, an additional LMI constraint is considered as,

$$F_{ij} > \rho P_1 \eta$$

Updated LMI conditions

Theorem

There exists a robust state and parameter observer (2) for the Takagi-Sugeno time varying parameter system (1) with a bounded gain of $\gamma = [\gamma_x \; \gamma_\theta \; \gamma_{\dot\theta} \; \gamma_u \; \gamma_\nu]^T$ from $\tilde{u}(t)$ to $e_a(t)$, if there exists $P_0 = P_0^T > 0$, $P_1 = P_1^T > 0$, $\lambda_1, \lambda_2 > 0$, F_{ij}, R_{ij} such that (for $i = 1.., 2^p$ and $j = 1.., n_\theta$), the following LMIs are satisfied

$$\begin{bmatrix} T_{11} & T_{12} & 0 & 0 & 0 & 0 & -R_{ij}I_\nu & P_0\mathcal{A} & P_0\mathcal{B} \\ * & T_{22} & 0 & \eta_0 P_1 & P_1 & 0 & -F_{ij}I_\nu & 0 & 0 \\ * & * & T_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma_\theta I_{n_\theta} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma_{\dot\theta} I_{n_\theta} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & T_{55} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma_\nu I_\nu & 0 & 0 \\ * & * & * & * & * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & * & * & * & * & -\lambda_2 I \end{bmatrix} < 0$$

$$F_{ij} > P_1 \rho \eta_0 I_{n_\theta}$$

where, $T_{11} = P_0 A_{ij} + A_{ij}^T P_0 - R_{ij} C - C^T R_{ij}^T + I_{n_x}$, $T_{12} = -C^T F_{ij}^T$, $T_{22} = -2\eta_0 P_1 + 1$, $T_{33} = -\gamma_x I_{n_x} + \lambda_1 E_A^T E_A$ and $T_{55} = -\gamma_u I_{n_u} + \lambda_2 E_B^T E_B$.
The observer gains are given by:

$$\eta_{ij} = \eta_0, \quad L_{ij} = P_0^{-1} R_{ij} \text{ and } K_{ij} = P_1^{-1} F_{ij}$$

Summary of Simulation results

Model parameters (sector min/max)

Parameter	Min	Max
z_1	0.16 kg/s	1.6 kg/s
z_2	293 K	368 K
β_1	0	100

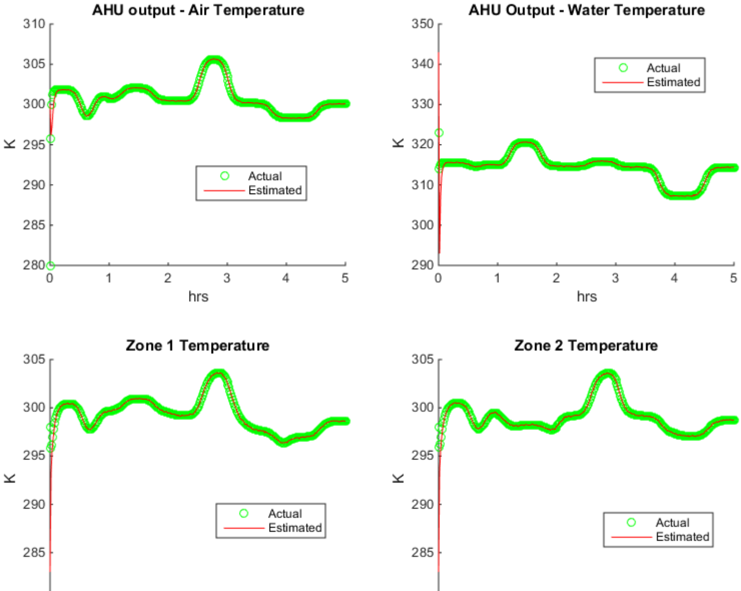
Simulation parameters

Parameters	Values
η	10^{-4}
ρ	10^5
Γ_2^{θ}	0.1
$\Gamma_2^{\dot\theta}$	0.1
Γ_2^{x}	$0.1 I_4$
Γ_2^{u}	0.1
Γ_2^{ν}	0.1

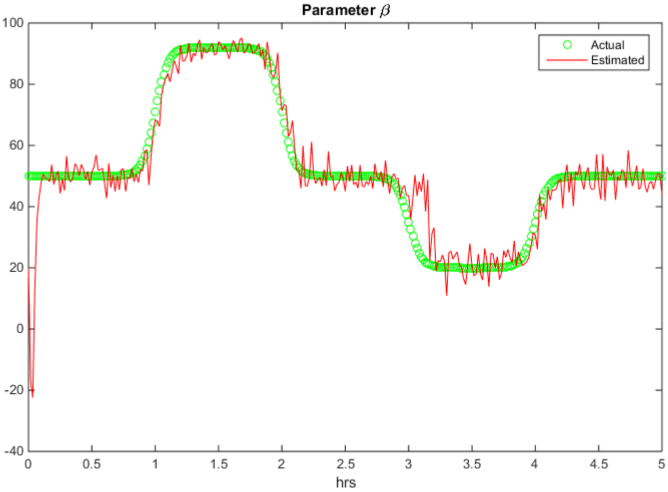
Summary of Simulation results

Error	Mean (%)	Standard Deviation (%)
$\lvert e_{x_1} \rvert$	0.04	0.4
$\lvert e_{x_2} \rvert$	0.07	0.55
$\lvert e_{x_3} \rvert$	0.03	0.3
$\lvert e_{x_4} \rvert$	0.03	0.3
$\lvert e_{\theta} \rvert$	10.9	18.34

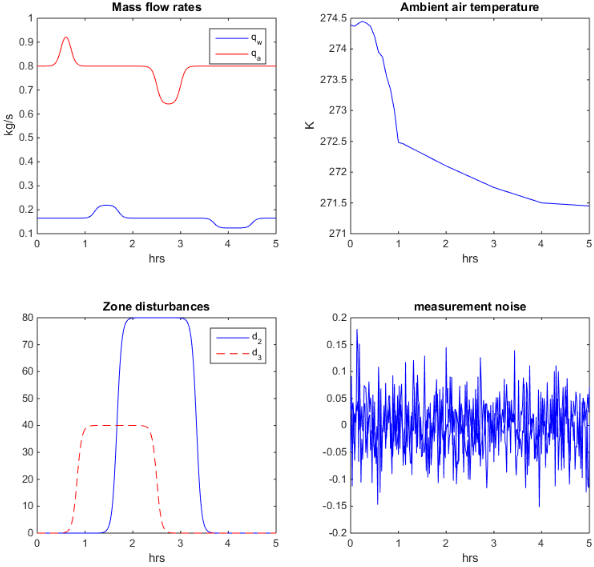
State Estimation



Parameter Estimation

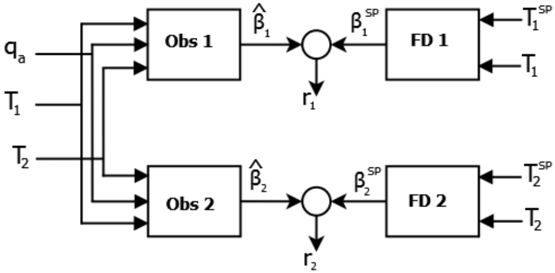


Input used to generate results



Outline

FDI using the estimation



Future Improvements

- A vanishing estimation error instead of an $\mathcal{L}_2$ gain.
- Extending results to discrete-time TS models
- Exploring approaches to cases where the unknown time varying parameter takes only discrete values (VAV damper position estimation with ON/OFF position)
- Using the T-S approach results in a distributed estimation framework

Thank you