14th World Congress
July 5-9,1999 Beijing

# Simultaneous estimation of the state and the parameters of an ARX model. Application to data validation in the field of rainfall processing.

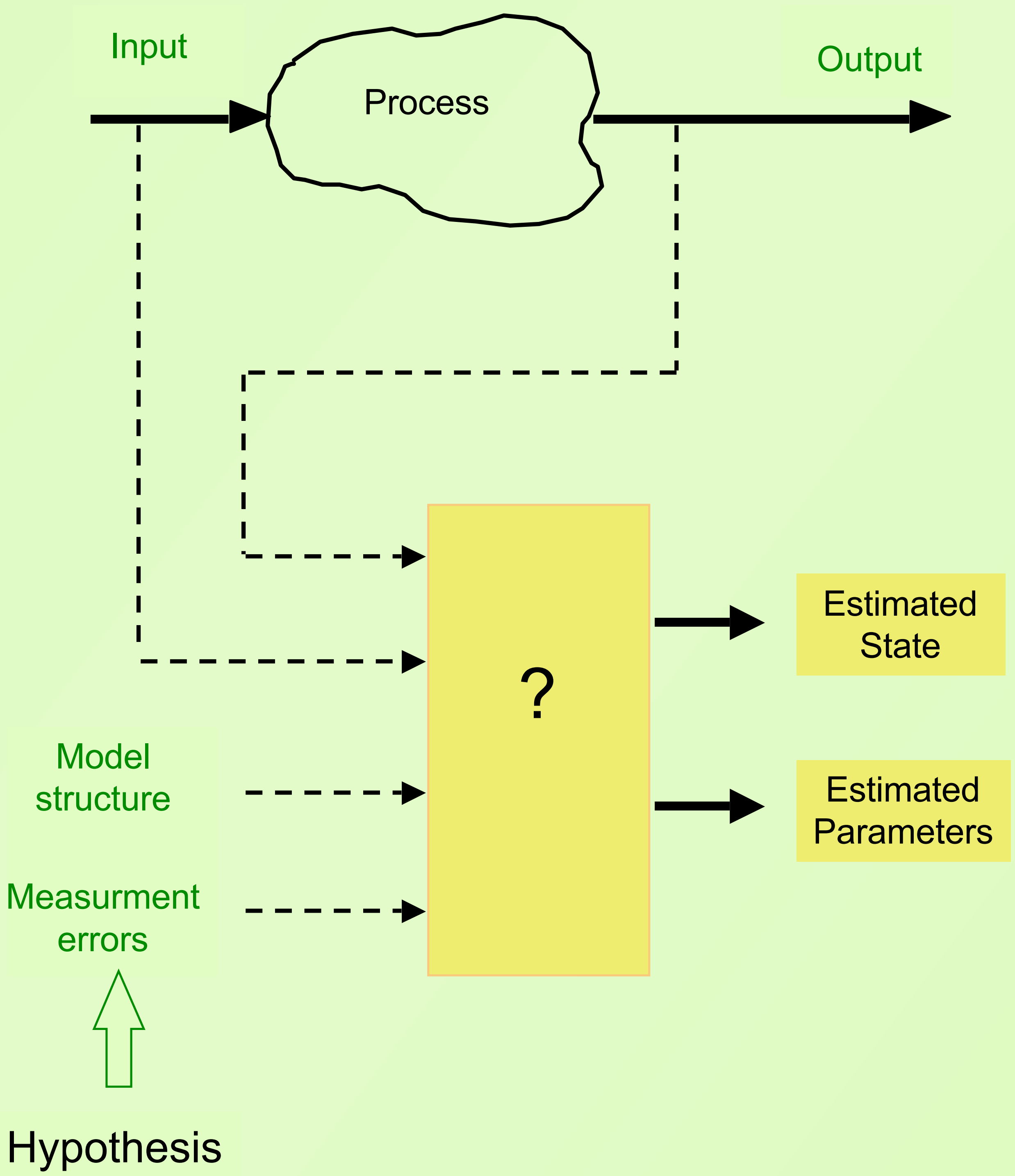
Gilles MOUROT, Didier MAQUIN and José RAGOT

Research Center in Automatic Control. CNRS
National Polytechnic Institute. Nancy, F

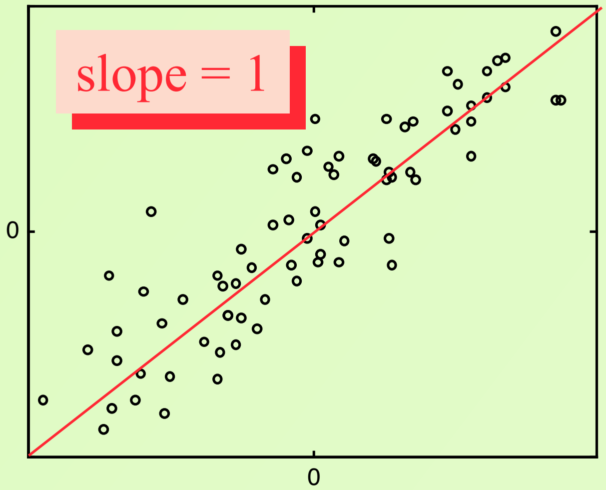
# Content

- Problem statement
- Introductive example
- Parameters and state estimation
- Some results
- Extensions
- Rainfall process application
- Conclusion

# Problem under consideration



# Introductive example




$$\Phi_1 = \sum_{k=1}^{N} \left( y_k - a x_k \right)^2$$

$$\Phi_2 = \sum_{k=1}^{N} \left( x_k - \frac{1}{a} y_k \right)^2$$

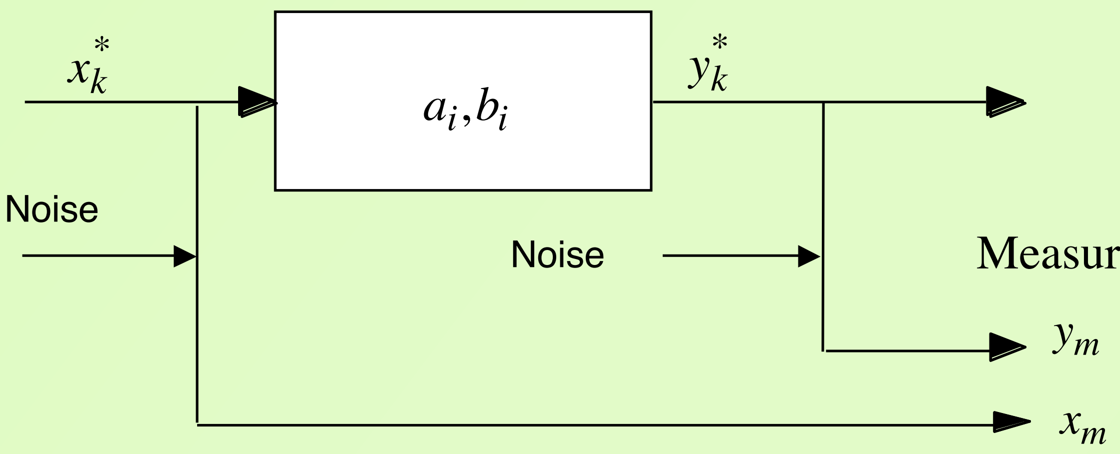
$$\Phi_3 = \sum_{k=1}^{N} \left( \begin{pmatrix} x_k & -y_k \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \right)^2$$

## Parameter estimation

| Run | Reg y/x | Reg x/y | Ort.Reg |
|---|---|---|---|
| 1 | 0.905 | 1.101 | 1.000 |
| 2 | 0.910 | 1.103 | 1.004 |
| 3 | 0.903 | 1.103 | 1.002 |
| | | | |
| 9 | 0.901 | 1.096 | 0.994 |
| 10 | 0.908 | 1.105 | 1.003 |

Conclusion : there is a need to take into account
the distribution of the error
both on the input and the output

# State and parameters estimation




- Estimation Principle

$$\hat{y}(k) = \sum_{i=1}^{n} a_i \hat{y}(k-i) + \sum_{i=1}^{m} b_i \hat{x}(k-i)$$

$$\Phi = \sum_{k=1}^{N} \left(\hat{y}(k) - y_m(k)\right)^2 + \sum_{k=1}^{N-1} \left(\hat{x}(k) - x_m(k)\right)^2$$

$\Rightarrow$ $\hat{x}, \hat{y}, a, b$

- Practical resolution
  - Linearisation
  - A two level hierachical algorithm

# State and parameters estimation

- Model of the process

$$Z = \left(y(1)\ \ x(1)\ \ y(2)\ \ldots x(N-1)\ \ y(N)\right)^T$$

$$\hat{Z} = \left(\hat{y}(1)\ \ \hat{x}(1)\ \ \hat{y}(2)\ \ldots \hat{x}(N-1)\ \ \hat{y}(N)\right)^T$$

$$\theta = \left(a_1\ \ldots a_m\ \ b_1\ \ldots\ \ b_n\right)^T$$

$$M(\theta)\hat{Z} = 0$$

- Optimality equations

$$L = \frac{1}{2}\left\|\hat{Z} - Z\right\|^2 + \lambda^T M(\theta)\hat{Z}$$

$$\frac{\partial L}{\partial \hat{Z}} = \hat{Z} - Z + M^T(\theta)\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = M(\theta)\hat{Z}$$

$$\frac{\partial L}{\partial \hat{Z}} = \left(I \otimes \lambda^T\right)\frac{\partial M(\theta)}{\partial \theta}\hat{Z} = 0$$

- Resolution

$$\hat{Z} = \left(I - M^T(MM^T)^{-1}M\right)Z$$

$$M = M(\theta_i)$$

$$\theta_{i+1} = \theta_i - \Delta\left(\left(\frac{\partial^2 L}{\partial\theta\partial\theta^T}\right)^{-1}\frac{\partial L}{\partial \theta}\right)_i$$

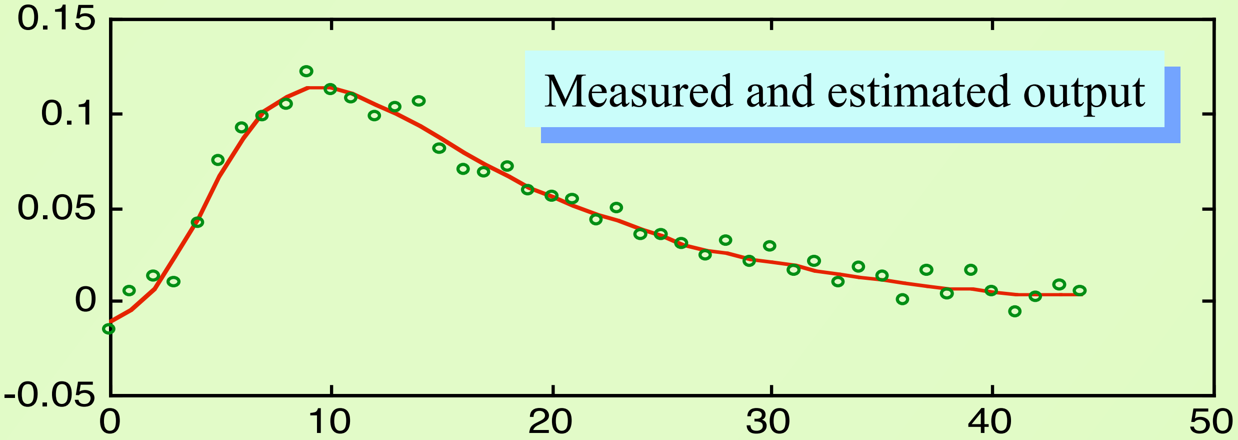
# Numerical results

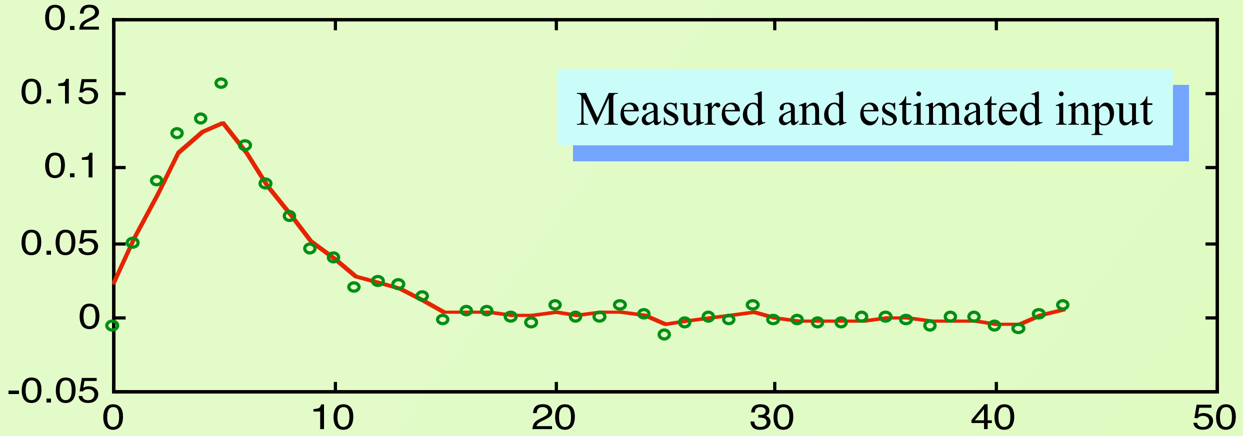
- Simulated process

  First order process

$$y(k+1) = 0.8y(k) + 0.2x(k)$$
$$y_m(k) = y(k) + e_y(k)$$
$$x_m(k) = x(k) + e_x(k)$$

- Estimation results $a = 0.808 \quad b = 0.198$



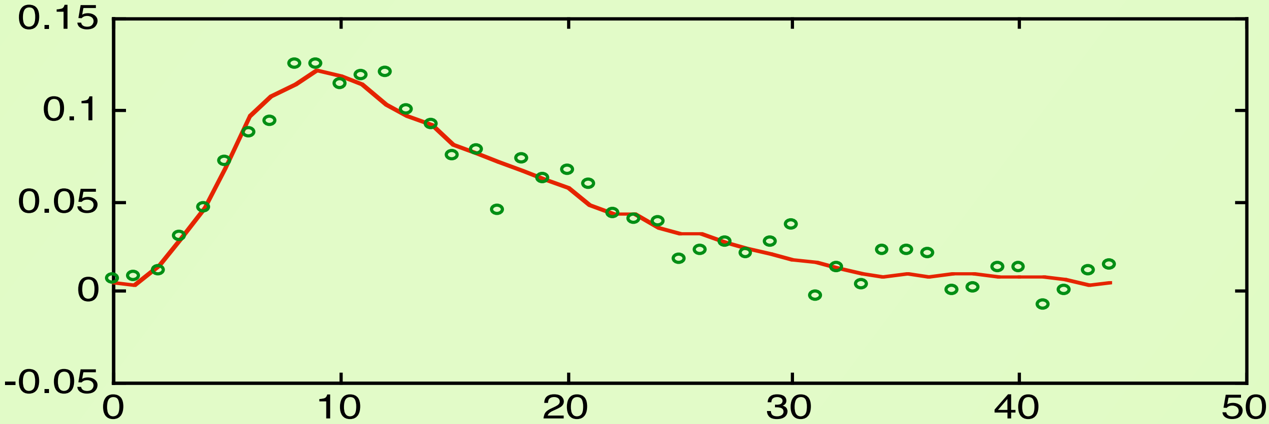
# Noisy measurements. Effect on input estimation

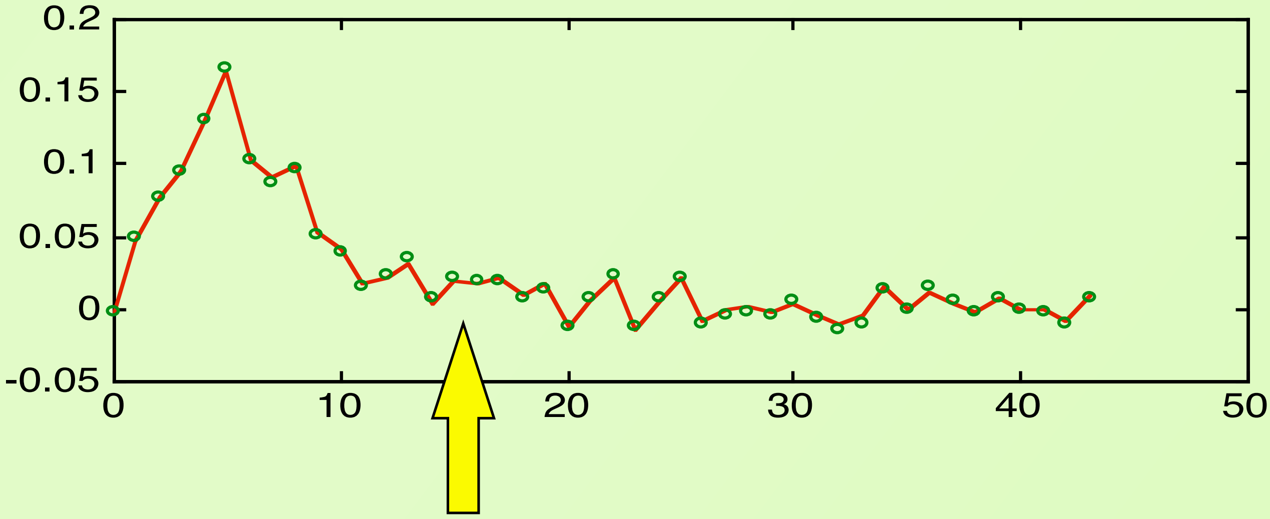
- Simulated process

  First order process $y(k+1) = 0.8y(k) + 0.2x(k)$

$$y_m(k) = y(k) + 3 * e_y(k)$$

$$x_m(k) = x(k) + 3 * e_x(k)$$




No filtering effect on the input

# Extension 1 : filtering

- Constraints on the estimation

$$\varphi_x = \sum_{k=1}^{N-1} \left(\hat{x}(k+1) - \hat{x}(k)\right)^2$$

$$\varphi_u = \sum_{k=1}^{N-1} \left(\hat{u}(k) - \hat{u}(k-1)\right)^2$$

$$C = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 & \dots & & & \\ & \dots & 0 & 1 & 0 & -1 & 0 & \dots & \\ \dots & & & & & & & & \\ & & \dots & 0 & 1 & 0 & -1 & 0 & \end{pmatrix}$$

$$\varphi = \left\| C\hat{Z} \right\|_P^2$$

- Optimality equation

$$L = \frac{1}{2}\left\| \hat{Z} - Z \right\|^2 + \lambda^T M(\theta)\hat{Z} + \left\| C\hat{Z} \right\|_P^2$$

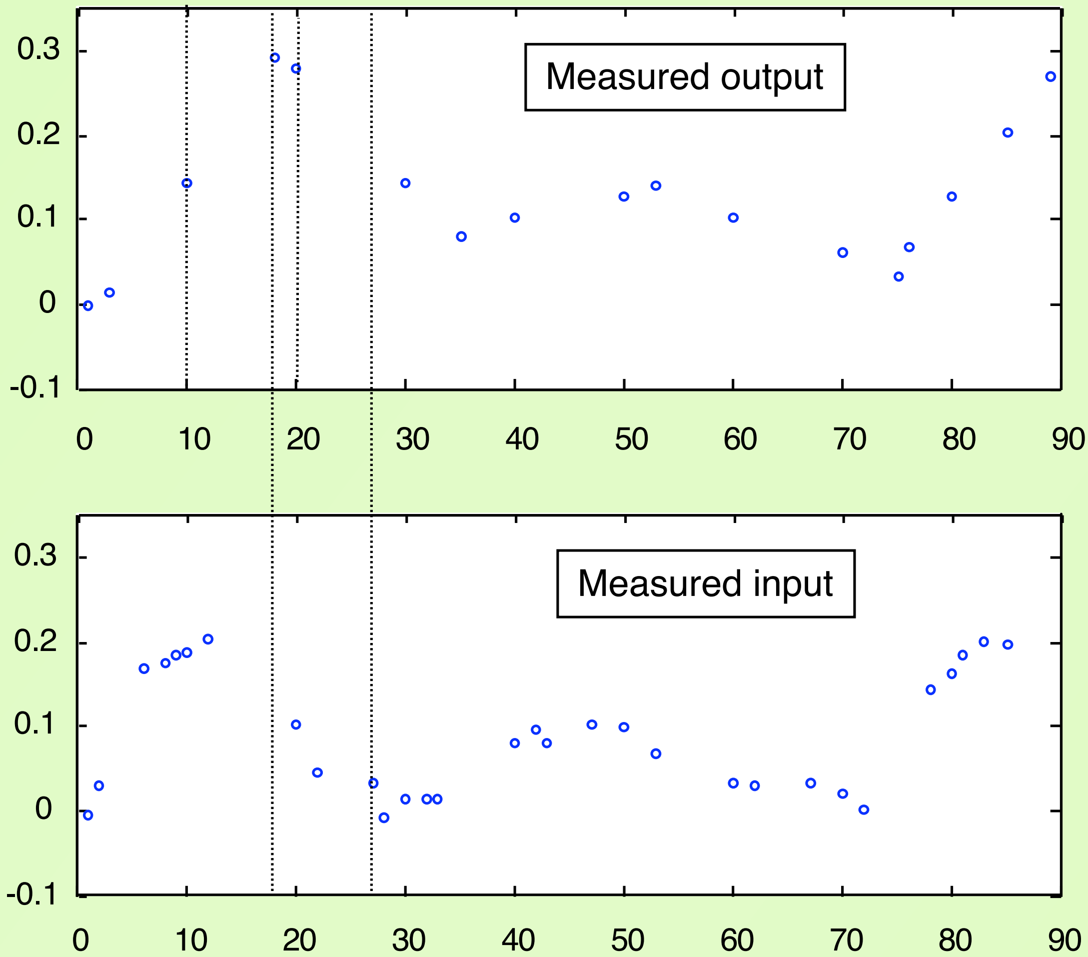
- Resolution

$$\hat{Z} = Q\left(I_{2N-1} - M^T(\theta)(M(\theta)QM^T(\theta))^{-1} M(\theta)Q\right)Z$$

$$Q = \left(I + C^T PC\right)^{-1}$$

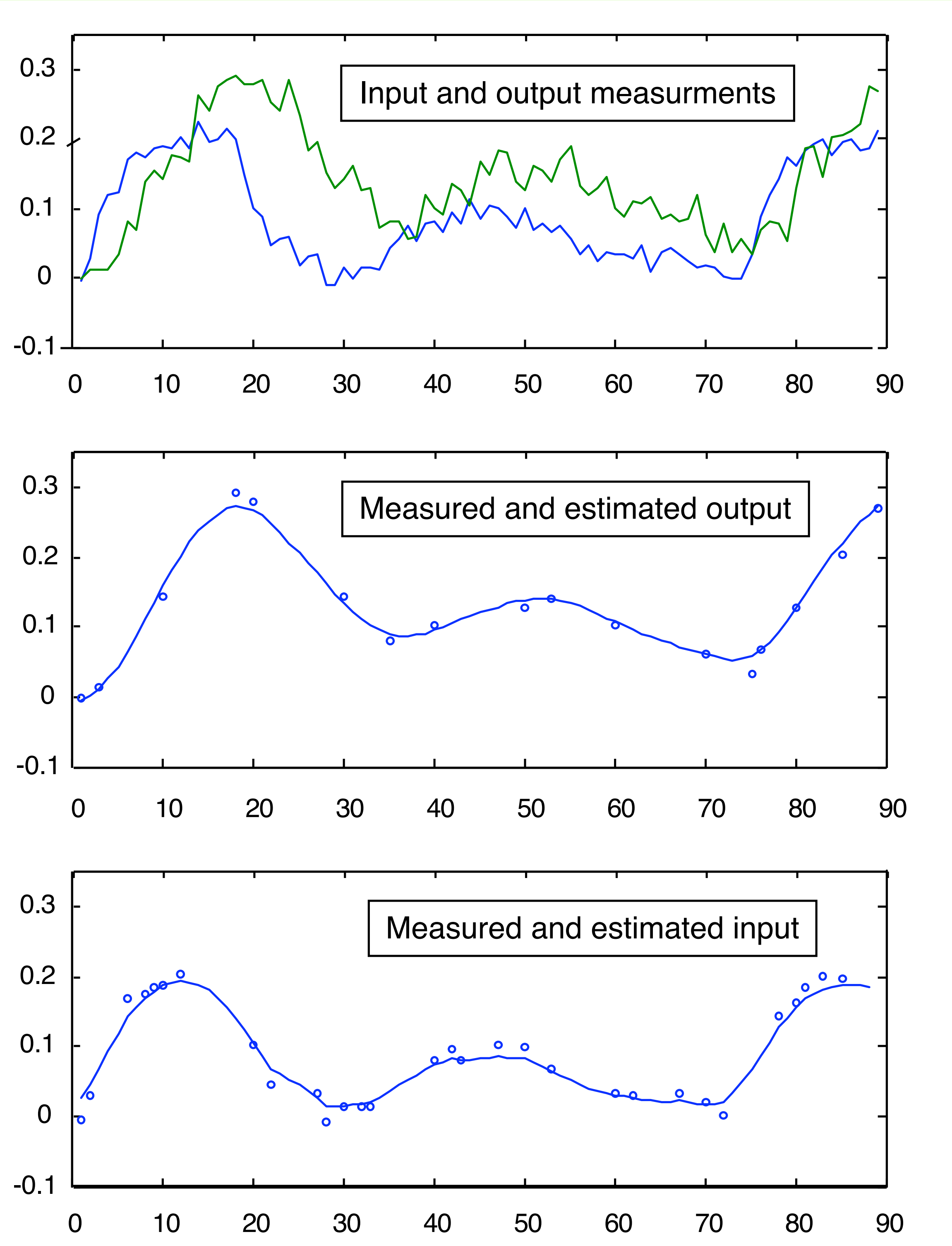
$$\theta_{i+1} = \theta_i - \Delta\left(\lambda^T \frac{\partial M(\theta)}{\partial \theta} \hat{Z}\right)_i$$

# Extension 2 : iregular sampling

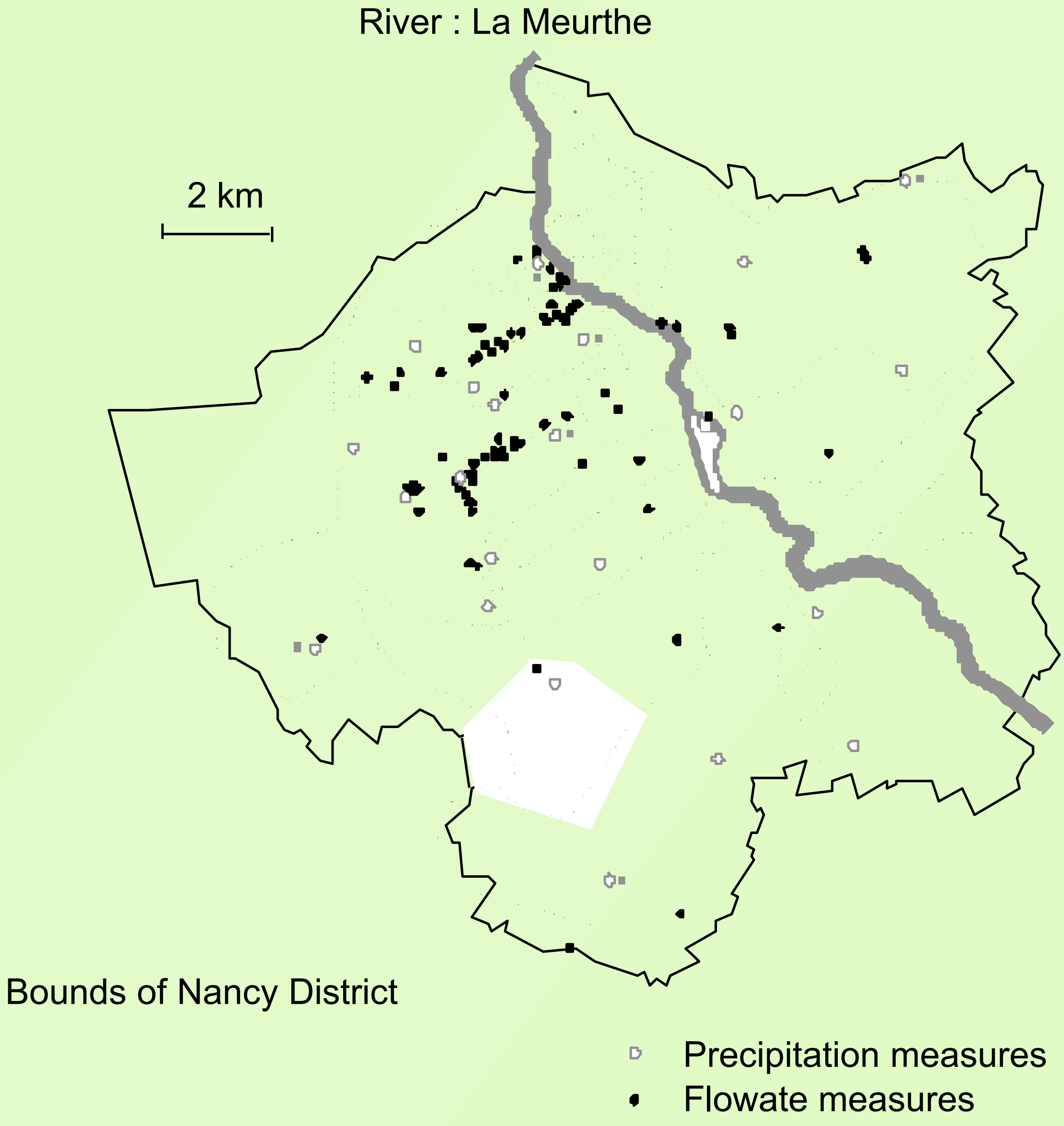



$$L = \frac{1}{2}\left\|H\hat{Z} - Z\right\|^2 + \lambda^T M(\theta)\hat{Z} + \left\|C\hat{Z}\right\|_P^2$$

# Extension 2 : iregular sampling



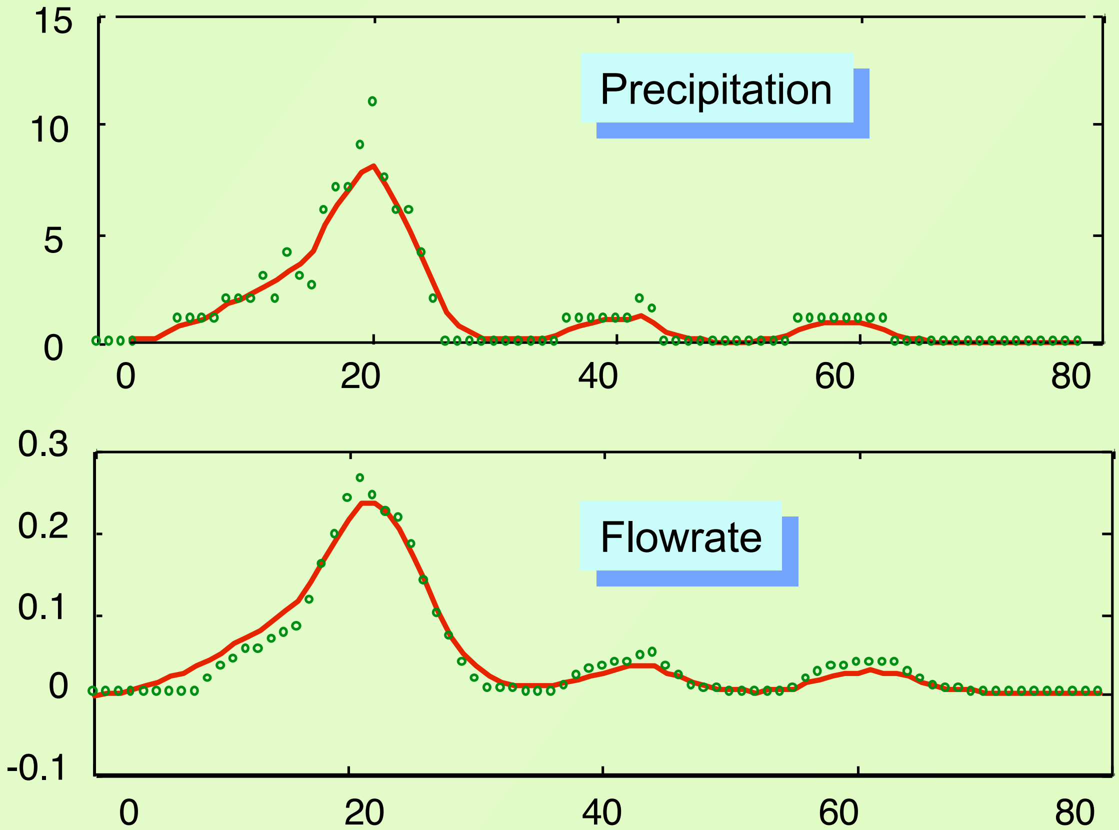
# Rainfall process. Urban District of Nancy (F)



# Rainfall process. Urban District of Nancy (F)

State and parameters estimation

$$F(k+1)=0.6155\,F(k)+0.0126\,P(k)$$

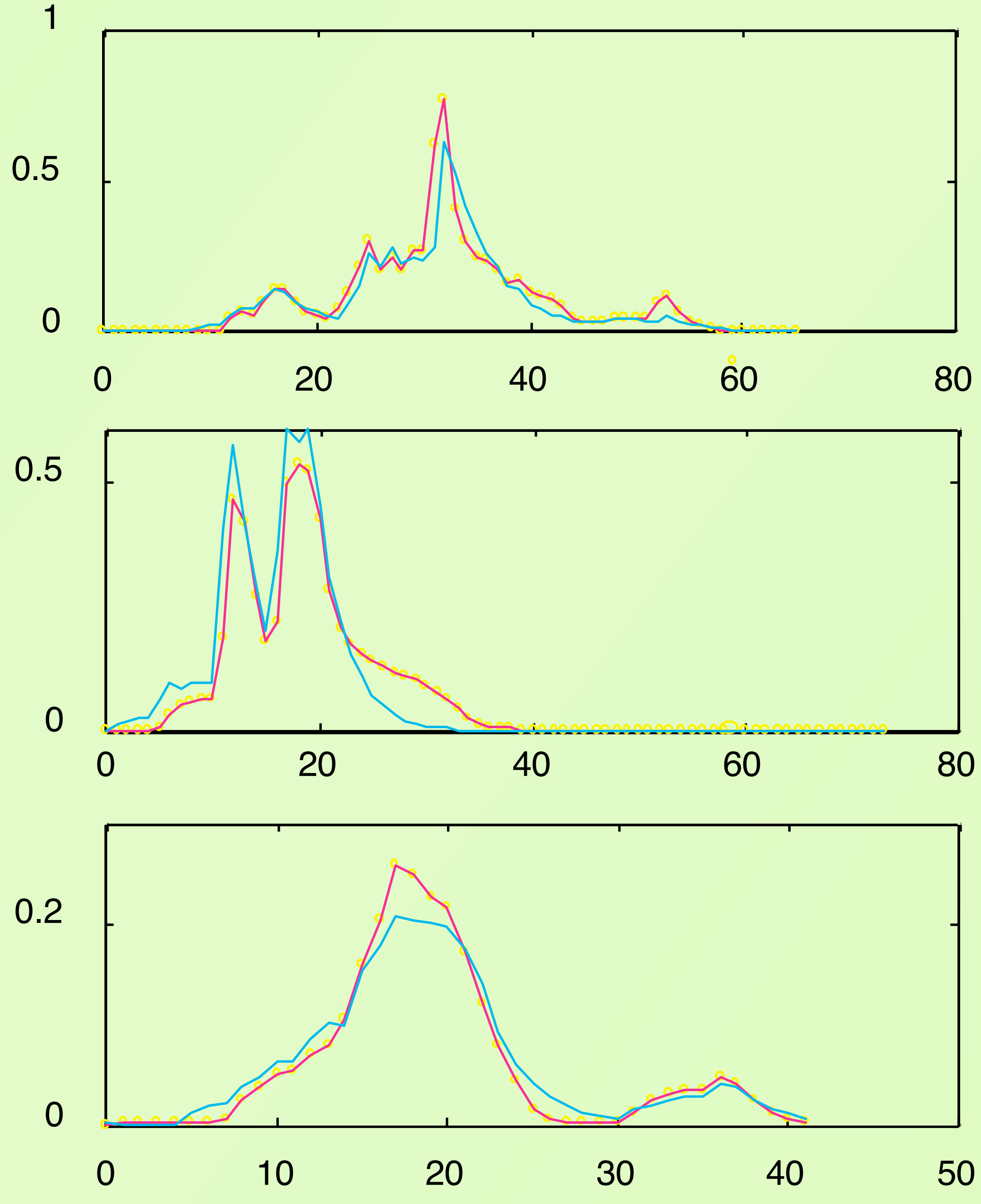


# Rainfall process. Urban District of Nancy (F)

F : flowrate

P : precipitation

$$F(k+1) = 0.6651\ F(k) + 0.0109\ P(k)$$



# Conclusion

Contribution to :

- ✓ simultaneous estimation of states and parameters of a process
- ✓ filtering the measurement of the input and the output of the process
- ✓ irregular sampling may be taken into account
- ✓ application to a rainfall process