State and unknown input estimation for nonlinear systems described by Takagi-Sugeno models with unmeasurable premise variables

Dalil Ichalal, Benoît Marx, José Ragot and Didier Maquin

Centre de Recherche en Automatique de Nancy (CRAN)
Nancy-Université, CNRS

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Objective

Estimate the state and the unknown inputs of nonlinear systems described by a Takagi-Sugeno model.

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Difficulties

The premise variables intervening in the weighting functions are unmeasurable.

Outline

1. Takagi-Sugeno approach for modeling
   - Takagi-Sugeno principle
   - Takagi-Sugeno model
2. Observer design
3. State and unknown input estimation
4. Numerical example
5. Conclusions

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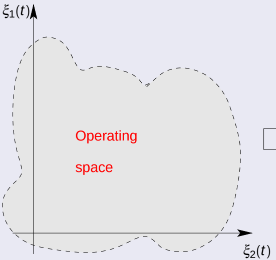
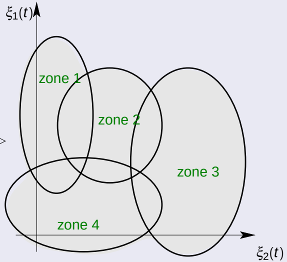
Takagi-Sugeno approach for modeling

- Operating range decomposition into several local zones.
- A local model represents the behavior of the system in each zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system



Multiple Model representation

Interests of Takagi-Sugeno approach

- Simple structure for modeling complex nonlinear systems.
- The specific study of the nonlinearities is not required.
- Possible extension of the theoretical LTI tools to nonlinear systems.

The considered model is described by the following equations

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

- Interpolation mechanism : $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1,...,r\}$
- The premise variable $\xi(t)$ can be measurable (input $u(t)$,output $y(t)$,...) or unmeasurable (state $x(t)$,...).

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Interests of T-S model with unmeasurable premise variables

- Exact representation of the model $\dot{x}(t) = f(x(t), u(t))$.

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- Exact representation of the model $\dot{x}(t) = f(x(t), u(t))$.
- Only one T-S model for FDI with observer banks.
- Application example: security improvement in cryptography systems.

Observer design

Let us consider the T-S model represented by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(C_i x(t) + D_i u(t)\right) \end{cases}$$

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Let us denote the estimated state by $\hat{x}$. By adding and subtracting the term

$$\sum_{i=1}^{r} \left(\mu_i(x) - \mu_i(\hat{x})\right)\left(A_i x + B_i u\right)$$

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The following equivalent system is obtained

$$\dot{x} = \sum_{i=1}^{r} \mu_i(\hat{x})\,(A_i x + B_i u) + \sum_{i=1}^{r} (\mu_i(x) - \mu_i(\hat{x}))\,(A_i x + B_i u)$$

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$$\dot{x} = \sum_{i=1}^{r} \mu_i(\hat{x})\left(A_i x + B_i u\right) + \sum_{i=1}^{r} \left(\mu_i(x) - \mu_i(\hat{x})\right)\left(A_i x + B_i u\right)$$

Thanks to convex sum property, we have

$$\sum_{i=1}^{r} (\mu_i(x) - \mu_i(\hat{x}))X_i = \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x})(X_i - X_j)$$

where

$$X_i \in \{A_i, B_i, C_i, D_i\}$$

Let us define the following notations

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Then the system can be transformed into the following representation

$$\dot{x} = \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x})((A_j + \Delta A_{ij})x + (B_j + \Delta B_{ij})u)$$

The output equation can similarly be written in the following form

$$y = \sum_{i,k=1}^{r} \mu_i(x)\mu_k(\hat{x})((C_k + \Delta C_{ik})x + (D_k + \Delta D_{ik})u)$$

The system is written like an uncertain system but the considered "uncertain terms" ΔX_{ij} are completely known and are constant matrices.

Observer structure

$$\begin{cases} \dot{\hat{x}} = \sum_{j=1}^{r} \mu_j(\hat{x}) \left(A_j \hat{x} + B_j u + G_j (y - \hat{y})\right) \\ \hat{y} = \sum_{k=1}^{r} \mu_k(\hat{x}) (C_k \hat{x} + D_k u) \end{cases}$$

Observer structure

$$\begin{cases} \dot{\hat{x}} = \sum_{j=1}^{r} \mu_j(\hat{x}) \left(A_j\hat{x} + B_ju + G_j(y - \hat{y})\right) \\ \hat{y} = \sum_{k=1}^{r} \mu_k(\hat{x}) (C_k\hat{x} + D_ku) \end{cases}$$

Taking into account the convex sum property, the equations of the observer can be multiplied by $\sum_{i=1}^{r} \mu_i(x)$ to obtain

$$\dot{\hat{x}} = \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x}) \left(A_j\hat{x} + B_ju + G_j(y - \hat{y})\right)$$

$$\hat{y} = \sum_{i,k=1}^{r} \mu_i(x)\mu_k(\hat{x}) (C_k\hat{x} + D_ku)$$

The state estimation error

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$$\dot{e} = \sum_{i,j,k=1}^{r} \mu_i(x)\mu_j(\hat{x})\mu_k(x)(\Phi_{jk} e + \Gamma_{ijk} x + S_{ijk} u)$$

where

$$\begin{aligned} \Phi_{jk} &= A_j - G_j C_k \\ \Gamma_{ijk} &= \Delta A_{ij} - G_j \Delta C_{ik} \\ S_{ijk} &= \Delta B_{ij} - G_j \Delta D_{ik} \end{aligned}$$

$$i,j,k \in \{1,\ldots,r\}$$

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Let us define the augmented state $\tilde{x} = [e^T \ \ x^T]^T$ which dynamics is described by the augmented system:

$$\begin{aligned} \dot{\tilde{x}} &= \sum_{i,j,k=1}^{r} \mu_i(x)\mu_j(\hat{x})\mu_k(x)\left(\mathscr{M}_{ijk}\tilde{x} + \mathscr{B}_{ijk} u\right) \\ z &= H\tilde{x} \end{aligned}$$

where

$$\mathscr{M}_{ijk} = \begin{bmatrix} \Phi_{jk} & \Gamma_{ijk} \\ 0 & A_i \end{bmatrix}, \ \mathscr{B}_{ijk} = \begin{bmatrix} \Sigma_{ijk} \\ B_i \end{bmatrix}, \ H = [I_n \ \ 0]$$

Augmented state dynamics

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Observer design

The goal is to determine the observer gain matrices G_j to guarantee the stability of the system which generates the state estimation error while attenuating the effect of the input $u(t)$ on $z(t)$.

Theorem 1: State estimation

The system generating the state estimation error is stable and the $\mathscr{L}_2$-gain of the transfer from $u(t)$ to $z(t)$ is bounded, if there exists symmetric matrices P_1 and P_2, matrices K_i and a positive scalar $\bar{\gamma}$, such that the following conditions hold:

$$\begin{bmatrix} A_j^T P_1 + P_1 A_j - K_j C_k - C_k^T K_j^T + I & P_1 \Delta A_{ij} - K_j \Delta C_{ik} & P_1 \Delta B_{ij} - K_j \Delta D_{ik} \\ (P_1 \Delta A_{ij} - K_j \Delta C_{ik})^T & A_i^T P_2 + P_2 A_i & P_2 B_i \\ (P_1 \Delta B_{ij} - K_j \Delta D_{ik})^T & B_i^T P_2 & -\bar{\gamma} I \end{bmatrix} < 0,$$

$$\forall (i,j,k) \in \{1,...,r\}^3$$

The gains of the observer are derived from:

$$G_j = P_1^{-1} K_j$$

and the attenuation level is :

$$\gamma = \sqrt{\bar{\gamma}}$$

State and unknown input estimation

Let us consider the following system

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i(x)\left(A_i x + B_i u + E_i f\right) \\ y = Cx + Du + Ff \end{cases}$$

Assumption

The following assumption holds

$$\dot{f}(t) = 0$$

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Let us consider the augmented state $x_a = [x^T \quad f^T]^T$, then the corresponding augmented system is

$$\begin{cases} \dot{x}_a = \sum_{i=1}^{r} \mu_i(x)\left(\tilde{A}_i x_a + \tilde{B}_i u\right) \\ y \;=\; \tilde{C} x_a + Du \end{cases}$$

where:

$$\tilde{A}_i = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C & F \end{bmatrix}$$

Applying the same method used previously, the system can be transformed into the following equivalent form

$$\dot{x}_a = \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x}) \left((\tilde{A}_j + \Delta\tilde{A}_{ij})x_a + (\tilde{B}_j + \Delta\tilde{B}_{ij})u \right)$$

where

$$\Delta X_{ij} = X_i - X_j, \;\; X_i \in \left\{ \tilde{A}_i, \tilde{B}_i \right\}$$

The PI observer is given by

$$\begin{cases} \dot{\hat{x}}_a = \sum_{j=1}^{r} \mu_j(\hat{x}) \left(\tilde{A}_j \hat{x}_a + \tilde{B}_j u + \tilde{G}_j (y - \hat{y}) \right) \\ \hat{y} = \tilde{C} \hat{x}_a + Du \end{cases}$$

where

$$\tilde{G}_j = \begin{bmatrix} G_{Pj} \\ G_{Ij} \end{bmatrix}$$

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The estimation error is given by $e_a = x_a - \hat{x}_a$, and its dynamics is given by

$$\begin{aligned} \dot{e}_a &= \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x})((\underbrace{\tilde{A}_j - \tilde{G}_j \tilde{C})}_{\Phi_j} e_a + \Delta\tilde{A}_{ij} x_a + \Delta\tilde{B}_{ij} u) \\ &= \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x})(\Phi_j e_a + \begin{bmatrix} \Delta A_{ij} \\ 0 \end{bmatrix} x + \begin{bmatrix} \Delta E_{ij} \\ 0 \end{bmatrix} f + \Delta\tilde{B}_{ij} u) \\ &= \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x}) \left(\Phi_j e_a + \begin{bmatrix} \Delta A_{ij} \\ 0 \end{bmatrix} x + \tilde{\Gamma}_{ij} \omega \right) \end{aligned}$$

where

$$\tilde{\Gamma}_{ij} = \begin{bmatrix} \Delta\tilde{E}_{ij} & \Delta\tilde{B}_{ij} \end{bmatrix}, \quad \omega = \begin{bmatrix} f \\ u \end{bmatrix}, \quad \Delta\tilde{E}_{ij} = \begin{bmatrix} \Delta E_{ij} \\ 0 \end{bmatrix}$$

Let us define the augmented state $\tilde{x} = [e_a^T \;\; x^T]^T$, then the augmented system is:

$$\dot{\tilde{x}} = \sum_{i,j=1}^{r} \mu_i(x)\mu_j(\hat{x}) \left(\underbrace{\begin{bmatrix} \Phi_j & \Delta A_{ij} \\ 0 & A_i \end{bmatrix}}_{M_{ij}} \tilde{x} + \underbrace{\begin{bmatrix} \tilde{\Gamma}_{ij} \\ \bar{B}_i \end{bmatrix}}_{R_{ij}} \omega \right)$$

$$z = H\tilde{x}$$

where:

$$\bar{B}_i = \begin{bmatrix} E_i & B_i \end{bmatrix}$$

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The objective is to assure the stability of the system which generates the state estimation error while minimizing the influence of $u(t)$ on the estimation error $e(t)$.

Theorem 2: State and unknown input estimation

The system which generates the state estimation error is stable and the $\mathscr{L}_2$-gain of the transfer from $u(t)$ to $z(t)$ is bounded, if there exists symmetric matrices P_1 and P_2, matrices $\tilde{L}_i$ and a positive scalar $\bar{\gamma}$, such that the following conditions hold $\forall(i,j) \in \{1,...,r\}^2$:

$$\begin{bmatrix} \tilde{A}_j^T P_1 + P_1\tilde{A}_j - L_j\tilde{C} - \tilde{C}^T L_j^T + I & P_1\Delta\tilde{A}_{ij} & P_1\Delta\tilde{E}_{ij} & P_1\Delta\tilde{B}_{ij} \\ (P_1\Delta\tilde{A}_{ij})^T & A_i^T P_2 + P_2 A_i & P_2 E_i & P_2 B_i \\ \Delta\tilde{E}_{ij}^T P_1 & E_i^T P_2 & -\bar{\gamma} I & 0 \\ \Delta\tilde{B}_{ij}^T P_1 & B_i^T P_2 & 0 & -\bar{\gamma} I \end{bmatrix} < 0$$

The gains of the observer are derived from:

$$\tilde{G}_j = \begin{bmatrix} G_{Pj} \\ G_{Ij} \end{bmatrix} = P_1^{-1} L_j$$

and the attenuation level is given by:

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Numerical example

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.25 \end{bmatrix}, B_2 = \begin{bmatrix} 1.5 \\ 3 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

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The weighting functions are defined by:

$$\begin{cases} \mu_1(x) = \frac{1-\tanh(x_1)}{2} \\ \mu_2(x) = 1 - \mu_1(x) = \frac{1+\tanh(x_1)}{2} \end{cases}$$

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By solving the LMIs in theorem 1, we obtain the following matrices:

$$L_1 = \begin{bmatrix} -1.556 & 8.556 \\ 6.919 & -8.956 \\ -1.684 & 6.472 \end{bmatrix}, L_2 = \begin{bmatrix} -1.556 & 8.556 \\ 6.919 & -8.956 \\ -1.684 & 6.472 \end{bmatrix}$$

An output noise bounded by 1 is added to the output of the system in order to simulate measurement noise.

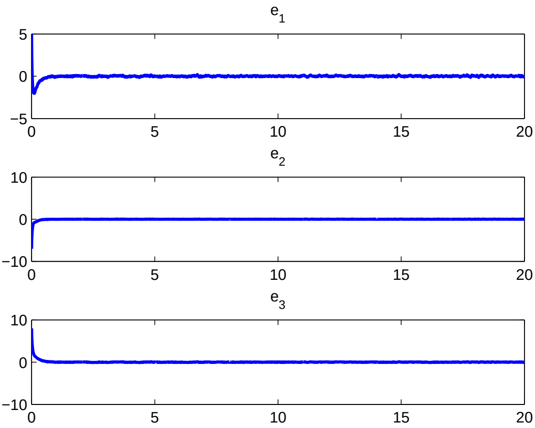

Figure: State estimation errors

Numerical example: unknown input estimation

Actuator and sensor faults are added to the previous example. Their respective influences are defined by the following matrices:

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \; E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \; F = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i(x)\,(A_i x + B_i u + E_i f) \\ y = Cx + Du + Ff \end{cases}$$

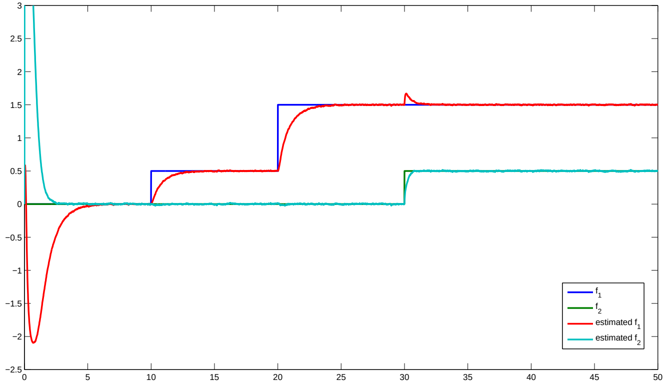

Figure: Unknown inputs and their estimates

Conclusions

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Perspectives

- Study and reduction of the conservatism when searching a common Lyapunov matrix P, which satisfies r LMIs.
- Extension to fault tolerant control of nonlinear systems.

Thank you for your attention!