OBSERVABILITY AND REDUNDANCY DECOMPOSITION

APPLICATION TO DIAGNOSIS

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This paper describes different ways of generating analytical redundancy equations in the case of systems represented by either static or dynamic equations. A relation is called a redundancy equation when known variables only, i.e. measured variables or controlled inputs of the system, appear in its expression. Redundancies are a powerful tool for monitoring processes as they can be used to detect and isolate sensor and actuator faults. The classical methods for generating redundancy equations are presented first. A method using the decomposition of the process equations by the observability concept is also described. The general character of the methods of generation of redundancy equations is demonstrated by showing that the same formalism can be applied to static and dynamic systems. This general character also appears when redundancy equations are used for detecting faults. Indeed, the statistical tests, presented in this paper, which are used for detecting and localizing the faults, apply indifferently to static and dynamic representations.

1.1 INTRODUCTION

The safety of processes can be greatly enhanced through the detection and isolation of the changes indicative of modifications in the process performances. If the models describing the process are accurate, the problem of fault detection may be solved by observer-type filters. These filters generate the so-called residuals computed from the inputs and the outputs of the process. This residual generation is the first stage in the problem of fault detection and identification (FDI). For them to be useful in FDI, the residuals must be insensitive to modelling errors and highly sensitive to the failures under consideration. In that regard, the residuals are designed so that the effects of possible failures are enhanced which in turn increases their detectability. The residuals must also respond

quickly. The second stage of FDI is concerned with residuals analysis and decision making; the residuals are examined for detecting the presence of failures. The use of simple decision rules such as threshold tests or more sophisticated approaches using pattern recognition, sequential probability ratio test or sensitivity analysis is very helpful at this stage. Various FDI method have been reported in the literature, notably in the excellent survey papers of Willsky (1976), Iserman (1984), Frank (1990), Gertler (1988 and 1991), Patton (1991). Among the classical books on the subject are those of Himmelblau (1978), Pau (1981), Basseville (1986a), Patton (1989b), Dubuisson (1990). The particular point of view of data reconciliation is addressed in the book of Ragot (1990a).

This paper is devoted to the generation of residuals using observability analysis and to the residual analysis allowing the detection of gross errors or more generally, failures. It must be recalled that redundancy represents the relationships between the known inputs (actuators) and the known outputs (sensors) of a process. Unfortunately, in a general situation, processes have other inputs and outputs which are not necessarily measured or known. One of the first methods for generating redundancy, in the case of physical redundancy between sensors, was developed by Potter (1977): this method yielded the parity space formulation. A second classical method uses an approach based on a Kalman filter. The innovation sequence can then be treated as the residual. In the case of deterministic systems, the problem can easily be solved by the use of state observers. In the case of observable processes, residuals can be obtained by calculating, for each variable, the difference between the observed sensor outputs and the modelled values. In the absence of errors, these residuals should be of zero mean showing a close agreement between the observed and expected behavior of the process. These residuals can then be used for testing the functioning of the process. The use of parity functions has then been extended to systems exhibiting temporal redundancy (see for instance papers by Chow (1984) and Massoumnia (1988)). Special redundancy equations have been proposed for enhancing a particular subset of variables. These have been used by Gertler (1985), Massoumnia (1988) and Frank (1990). The particular case in which each redundancy equation is affected by all but one fault or by only one fault (sensor or actuator fault) has been considered by some authors (Clark 1978). The robustness FDI problem, due to model uncertainties and unknown inputs, has been considered by a number of investigators such as Watanabe (1982), Wünnenberg (1987) and Frank (1990); this problem has also been investigated in the frequency domain by Ding (1989). Staroswiecki (1989b) proposed some extensions for interconnected transfer functions described by linear bloc diagrams. Recently, Ueng (1990) developed a new concept of decision-making using model cost analysis through the energy concept. Finally, several authors have pointed out and demonstrated the equivalence between residual generation by observer and parity space approaches; this equivalence is addressed in the following references: Frank (1989), Gertler (1991), Patton (1991), Magni (1991), Staroswiecki (1991).

In some cases, the number of redundant equations is not sufficient to achieve a diagnosis of the process. It is then important to decide, amongst unmeasured variables, which one must be assigned a sensor device. The problem of sensors location covers different aspects such as: number, location, type, scheduling. Because of the higher demand for accuracy and tighter finances, considerable effort has been devoted to the optimized design of sensor systems. Unfortunately, due to multiple constraints, no general method for solving this problem has yet been found. In many diagnosis situations, changes in the location of the sensors can greatly improve the measurements quality as modifications in the sensor location change the observability conditions (Kretsovalis 1988) and the observation equations. This consequently modifies the structure and performances of the observer system. Except in the case of distributed parameter systems, only a few papers concerned with the design of sensor systems have been published. Optimizing the variance matrix of the estimation with respect to the coefficients of the measurement matrix is the first solution proposed by Arbel (1982); Basseville (1986b) addresses the problem of sensor location by monitoring the eigenstructure of a multivariable dynamical system. In most cases, this problem has been addressed mainly for obtaining correct parameter and state estimations rather than for monitoring and diagnosing a system. However, in the field of electric power networks, specific developments on this subject have been published. They use topological observability algorithms (Krumholtz 1980, Clements 1990).

The purpose of generation of analytical redundancy equations is to extract redundant variables in a partially observed system; concomitantly, redundancy equations can be obtained. These equations yield variables, referred to as residuals, which in turn can reveal the presence of measurement errors. The first part of the paper reviews the case of static systems. The second part is devoted to dynamic systems. In both cases, we propose a new presentation which "unifies" the two points of view. The third part is concerned with the problem of residual signal analysis for failure detection.

1.2 STATIC REDUNDANCY EQUATIONS

Historically speaking, likely due to measurement availability, static redundancy equations have first been utilized in the mineral processing and chemical industries as well as for electrical distribution networks. The first studies (Ripps 1962, Vaclavek 1969, Schweppe 1970, Broussolle 1978) were concerned with data reconciliation using the now classical technique of equilibration of production's balances. In the following stages this data reconciliation principle has been generalized to processes which are described by algebraic equations either linear (Crowe 1983, Zasadzinski 1990) or non linear (Sood 1979, Crowe 1986, Héraud 1991). At the same time, data reconciliation went into use for more general applications than establishing statistically coherent balances. It was then applied to more fundamental problems such as: detection and estimation of gross errors (Narasimhan 1989, Ragot 1990a, Kratz 1991), diagnosis and observability of systems (Kretsovalis 1988, Crowe 1989,

Ragot 1990b), optimization of sensors location (Maquin, 1986) and study of the reliability of a measurement system (Turbatte, 1991). In this first part we will present the principles for generating redundancy equation i.e. equations containing only redundant variables. Let us recall here that a measured variable is called redundant if it can be calculated uniquely from the remaining variables. As previously mentioned, this redundancy generally leads to a discrepancy between the data and the equations which have to be reconciled; so it provides a check on the reliability of a given set of measurements.

1.2.1 Presentation

The linear relationship between the measurements Y and the actual values X of a process variables vector is given in a simple matrix form as:

$$Y = C X + \varepsilon \tag{1}$$

where Y is the (c.1) data vector whose entries are obtained from either sensors or analytic relationships, C the (c.m) measurement matrix, X is the (m.1) actual values of the vector process variables vector and ε the (c.1) noise vector associated to the data. It is assumed that the noise is zero mean and characterized by a known variance matrix V (which is diagonal if the measurement errors are independent).

For obtaining the vector X, a minimum m, out of the c measurements, is needed. Therefore, redundancy in measurements always appears when the inequality $c > m$ holds (Ray, 1991). Data inconsistency can be easily pointed out by eliminating the unknown variables X from equation (1) when ε is null. This yields (c-m) linearly independent equations known as parity equations. Potter and Suman (1977) have established a general formulation of this problem when the covariance matrix of the measurement errors is equal to unity. When the taking covariance matrix is not equal to unity, the (c-m) generalized parity vector can be defined as follows:

$$p = \Omega V^{-1/2} Y \tag{2}$$

The ((c-m).c) projection matrix Ω is selected so that it fulfills the following conditions:

$$\Omega V^{-1/2} C = 0 \tag{3a}$$

$$\Omega \Omega^T = I_{c-m} \tag{3b}$$

$$\Omega^T \Omega = I_c - V^{-1/2} C (C^T V^{-1} C)^{-1} C^T V^{1/2} \tag{3c}$$

The first condition expresses the fact that the measurements space is orthogonal to the parity space. Because of this, the parity vector becomes independent of the measured values. The second

condition is a normality condition which ensures the isotropy of the parity space (Bath, 1982). Finally, the third condition links the parity vector to the estimator of actual values, X, in the sense of least squares (Daly 1979, Kratz 1991). The measurements vector Y can then be projected from the measurement space into an orthogonal sub-space, called the parity space. Inconsistency of the measurements can be enhanced in this space which is spanned by the rows of the matrix Ω (see last section). It should also be noted that the analytical redundancy equation is given by equation (2) when ε is equal to zero. Under these conditions, the redundancy equations can be written under the following form:

$$\Omega \, V^{-1/2} \, Y = 0 \tag{4}$$

When the hypotheses about measurement noises are not valid, different situations can appear. In the case of errors whose mean values are not equal to zero, because of a systematic error of a sensor, the problem can easily be solved. Indeed, this systematic error can be statistically estimated which allows a correction of the measurements. For this estimation to be performed, the faulty measurement has first to be localized and identified. Many authors have studied this difficult localization problem in the case of static redundancy equations. The main localization methods will be presented in the last section of this chapter. In the same way, when the variance of measurement errors is unknown, it is possible to estimate it. The technique for this estimation was first proposed by Almasy (1984). It was then treated again by Darouach (1989) and extended by Keller (1991).

1.2.2 Generation of redundancy equations by direct elimination

A redundant equation is an equation which involves known variables only. In other words, it is a relation only between the different entries of the measurement vector. Thus, to obtain a set of redundant equations, the unknown variables X in equation (1) have to be eliminated. When there are no measurement errors, system (1) consists of c equations with m unknowns X. The existence of solutions can be determined using the general analysis of linear systems. If this system is consistent, it is possible to eliminate the whole set of variables X; the remaining equations are then explicating the redundancy relations between variables Y. As C is a (c.m) matrix of rank m, it is always possible to extract a full rank (m.m) matrix C_1. C is then partitioned into two sub-matrices (C_1 and C_2). The components of the vector X have sometimes to be permuted due to the extraction of the regular part of C. Through this treatment, equation (1) is expanded into:

$$Y = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} X \tag{5}$$

By premultiplying equation (5) by the regular matrix ($C_2\ C_1^{-1}$ -I), X can be eliminated. One then obtains the redundancy equation:

$$(C_2\ C_1^{-1} \quad -I\ )\ Y = 0 \tag{6}$$

1.2.3 Generation of redundancy equations by projection

In this type of treatment it is necessary to look for a matrix T which allows the elimination of X by direct multiplication of the measurement equation (1). As C is a (c.m) matrix of rank m, the transformation matrix T can be defined as $T = \begin{pmatrix} N \\ C^l \end{pmatrix}$ where N represents a left annihilator and C^l a left inverse of C. Premultiplying the measurement equation (1) by T yields:

$$T\ Y = \begin{pmatrix} 0 \\ I \end{pmatrix} X + T\ \varepsilon \tag{7}$$

According to the definition of T, the redundancy and deduction equations can then be written:

$$N\ Y = 0 \tag{8a}$$

$$C^l\ Y = X \tag{8b}$$

If c<m, it is impossible to find a non trivial solution N. This means that, either the model of the system is not well defined, or not enough process variables are known to generate the redundancy equations. When C is not a full column rank matrix (rank(C) = r < m), the technique previously defined can still be applied. Another possibility for solving this system is to decompose C using two orthogonal matrices Q (dimension c.c) and S (dimension m.m) and one upper triangular matrix R (dimension r.r) (Golub, 1983):

$$C = Q \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} S \tag{9}$$

In the absence of measurement errors, equation (1) writes:

$$Q^T\ Y = \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} S\ X \tag{10}$$

This allows an easy formulation for the redundancy and deduction equations. It also yields the list of variables which are deducible or not and redundant or not.

1.2.4 The constraint static case

Let us now consider the system described by a linear constraint and a measurement equation:

$$A X = 0 \tag{11a}$$

$$Y = C X \tag{11b}$$

X is a v-dimensional state vector, Y is a m-dimensional vector of measured outputs, A and C are known matrices of appropriate dimensions. Using simple transformations this equation can be reduced to those used for the unconstrained case.

In the first transformation, the constraint equation is eliminated. The matrix A is partitioned into two matrices A_1 and A_2, where A_1 states for its regular part. The X vector and the measurement equation are then rewritten:

$$X = H X_2 \tag{12a}$$

$$Y = C H X_2 \tag{12b}$$

with $$H = \begin{pmatrix} - A_1^{-1} A_2 \\ I \end{pmatrix} \tag{12c}$$

Equation (12b) is similar to equation (1). Consequently the transformations previously described apply to equation (12b).

In the second transformation, a projection matrix is used. First, let us notice that the two equations (11) may also be grouped as:

$$\begin{pmatrix} C \\ A \end{pmatrix} X = \begin{pmatrix} I \\ 0 \end{pmatrix} Y \tag{13}$$

In order to remove the unknown variables X, it is necessary to find two vectors α and β such that:

$$(\alpha^T \quad -\beta^T) \begin{pmatrix} C \\ A \end{pmatrix} = 0 \tag{14}$$

Let us define the T matrix by:

$$T = (M \quad C^r) \tag{15}$$

where M is right orthogonal to C and C^r right inverse of C. Post-multiplying equation (14) by T yields the system:

$$\beta^T A M = 0 \quad (16a)$$

$$\alpha^T - \beta^T A C^r = 0 \quad (16b)$$

Using these two relations the two vectors α and β can easily be determined. In particular, β can be easily obtained from equation (16a) by utilizing the generalized inverse N^- of the matrix $N = (AM)^T$; indeed:

$$\beta = (I - N^- N) \beta_0 \quad (17)$$

where β_0 is defined as an arbitrary vector. A family of independent solutions can then be obtained if β_0 is chosen as the set of vectors of the unity matrix having the same dimension as β_0. Calculating α from equation (16b) is then trivial. The redundancy equations are obtained by multiplying (13) by the vector (α -β). They can be written:

$$\alpha^T Y = 0 \quad (18)$$

In order to partially avoid the preceding decompositions, it is possible to generate the redundancy equations by eliminating X: starting from equation (11a), if A^- corresponds to the generalized inverse of A and Z is an arbitrary vector, X is obtained through the relation:

$$X = (I - A^- A) Z \quad (19)$$

By transferring (19) into (11b), the measure equation of variables Z is obtained:

$$Y = C (I - A^- A) Z \quad (20)$$

This leads back to the initial problem of measurements in the absence of constraints. A similar formulation is obtained when X is expressed from (11b) and then transferred into (11a). The technique of parity space can then be applied to the generated measurement equations.

1.2.5 A systematic decomposition

Linear systems, in which redundancy is present, can be written under various forms depending on the structure of the constraint and measure equations. However, through a few simple transformations they can be reduced to a unique representation defined by equations (11) or under an equivalent form:

$$\begin{pmatrix} 0 & A \\ I & -C \end{pmatrix} \begin{pmatrix} Y \\ X \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{21}$$

These equations describe the following particular situations as well: indirect and non constrained measurement system (A = 0), partially direct and constrained measurement system (C = I), partially indirect and constrained measurement system (C = (D 0)). The dimension of system (21) can also be reduced. Indeed, using the mode of reduction defined for obtaining equation (12), it comes:

$$\begin{pmatrix} -I & H \end{pmatrix} \begin{pmatrix} -Y \\ X_2 \end{pmatrix} = 0 \tag{22}$$

Then after a slight rearrangement, we can consider the unique case:

$$M\,Z = 0 \tag{23}$$

where M is the incidence matrix of the process which is assumed to be full row rank and Z is the vector of the process variables which may be measured or not. If the incidence matrix is rank deficient, then the number of constraints can be reduced to prevent this deficiency. The classical results of system decomposition based on observability (Darouach 1986, Kretsovalis 1988) can be applied to such a system. It leads to the canonical form of the matrix M which explicitly exhibits the deducible and redundant parts of the system. Several methods can be used to perform the process decomposition (Darouach, 1986). They include: singular values decomposition, reduction of the incidence matrix to an echelon form etc... This last method is the simplest and the most efficient as it takes advantage of the particular structure of the incidence matrices which generally contains many zero elements (Maquin, 1987). Let us consider systems described by equation (23). The Z variables can be classified into measured variables Z_m and unmeasured variables $Z_{\overline{m}}$. System (23) becomes:

$$M_m Z_m + M_{\overline{m}} Z_{\overline{m}} = 0 \tag{24}$$

where Z_m is a m.1 vector and $Z_{\overline{m}}$ a (v-m).1 vector. This first classification is achieved through a sorting of the components of Z and a permutation of the columns of M. The global condition of observability of the previous system is given by:

$$\text{rank} \begin{pmatrix} I & 0 \\ M_m & M_{\overline{m}} \end{pmatrix} = v \tag{25}$$

which is equivalent to: rank($M_{\overline{m}}$) = v-m. If this condition is not fulfilled, the system is partially observable. In this case, one has to decompose the vectors Z_m and $Z_{\overline{m}}$, as well as the matrices M_m and $M_{\overline{m}}$ in order to determine the deducible part and the redundant part. This classical problem can be solved by extracting the regular part $M_{\overline{m}\,12}$ out of $M_{\overline{m}}$. The matrix M can then be partitioned as follows:

$$M = \begin{pmatrix} M_{m1} & M_{\overline{m}11} & M_{\overline{m}12} \\ \\ M_{m2} & M_{\overline{m}21} & M_{\overline{m}22} \end{pmatrix} \tag{26}$$

The vector $Z_{\overline{m}}$ has to be partitioned with compatible dimensions. After some elementary row and column operations involving only permutations and linear combinations, the incidence matrix can be written as :

$$M = \begin{pmatrix} M_{\overline{m}12}^{-1} M_{m1} & M_{\overline{m}12}^{-1} M_{\overline{m}11} & I \\ \\ M_{m2} - M_{\overline{m}22} M_{\overline{m}12}^{-1} M_{m1} & 0 & 0 \end{pmatrix} \tag{27}$$

The same expression can be obtained by multiplying the matrix M by the regular matrix P defined as:

$$P = \begin{pmatrix} M_{\overline{m}12}^{-1} & 0 \\ \\ M_{\overline{m}22} M_{\overline{m}12}^{-1} & I \end{pmatrix} \tag{28}$$

The analysis of the canonical form of the incidence matrix yields the observability conditions and the redundancy of the variables: the block 2-1 (second row, first column) concerns the equations of redundancy as it only includes measured variables. The corresponding incidence matrix is:

$$M_r = M_{m2} - M_{\overline{m}\,22} M_{\overline{m}12}^{-1} M_{m1} \tag{29}$$

If the block 1-2 contains rows of zeros, the rows of the same rank in block 1-3 (identity matrix) give the indices of the unmeasured variables which can then be determined; the other variables are unobservable since they do not appear in deduction equations. The measured variables, which can not be estimated are contained in the columns of block 2-1 composed of all zero elements. The decision variables of the global observability are found in the rows of block 1-2 which contain only a non zero value (Gomolka, 1992). If the number of redundancy equations obtained appear insufficient, an analysis of the variables contained in this block can be performed. This allows to propose a choice of complementary sensors. Finally it must be pointed out that the structure of the

matrix M_r is not unique as it depends, in particular, on the vectors composing the regular part $M_{\overline{m}_{12}}$. This last point can be useful for generating structured redundancy equations i.e. equations in which a particular group of variables appear preferentially. As described by Gertler (1990, 1991), some transformations may be used to generate these structured equations which are more sensitive to specific faults.

This decomposition leads to a classification of the variables into the following categories: i) measured and observable variables (as these variables are bound by redundancy equations, the consistency of the measurements can be tested by analyzing the magnitude of the equation residuals), ii) measured and non estimable variables (these variables do not appear in a redundancy equation), iii) unmeasured and deducible variables (these variables are deduced from the variables belonging to the previous categories), iv) unmeasured and non estimable variables (these variables do not appear in the deduction equations and cannot be corrected; further measurements are necessary to render these variables observable). This decomposition according to observability can be generalized to the case of bi-linear (Crowe 1989, Maquin 1987) and n-linear systems (Ragot, 1990b). In the second section of our paper, we will show how this decomposition can be applied to the case of dynamic systems.

1.2.6 Example

The procedure can be illustrated using the following simplistic example. In this case the system is described by the measurement equation (1) without measurement errors. It is a priori redundant as there are 4 observations of only 2 variables.

$$Y = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 1 \\ 2 & 0 \end{pmatrix} X$$

The direct elimination procedure can be applied by taking, for instance, the following values of C_1 and C_2:

$$C_1 = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad C_2 = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$$

Then, the redundancy equations are determined by equation (6):

$$\begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 4 & 0 & -2 \end{pmatrix} Y = 0$$

When the measurements are affected by faults, the redundancy equations are not fulfilled and this may be enhanced by defining the residuals:

$$r_1 = y_1 + y_2 - 2\,y_3 \quad \text{and} \quad r_2 = 4\,y_2 - 2\,y_4$$

If the main goal is to detect measurement faults, these residuals are compared to threshold values. Examination of the residual structure shows that faults on y_1 and y_3 are detectable but non isolable. More generally, detectability and isolability mainly depend on the structure of the equations and on the occurrences of the variables in each of them.

The projection method can be applied using the matrix T as defined by equation (7). Using the standard MATLAB routine "null" for computing the orthogonal matrix of C, we obtain:

$$N = \begin{pmatrix} -0.4381 & -0.0473 & 0.8762 & -0.1954 \\ -0.0198 & -0.8975 & 0.0395 & 0.4389 \end{pmatrix}$$

Then the redundancy equations are:

$$\begin{pmatrix} -0.4381 & -0.0473 & 0.8762 & -0.1954 \\ -0.0198 & -0.8975 & 0.0395 & 0.4389 \end{pmatrix} Y = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Using elementary transformations it is possible to show that the forms obtained by either direct elimination or projection are equivalent.

1.3 DYNAMIC REDUNDANCY EQUATIONS

The different concepts that have so far been introduced for studying the observability of linear systems can also be perfectly applicable to the case of dynamic systems. In particular the previously defined classification of variables according to their deducibility and redundancy can be generalized. The only difficulty lies in the increase in dimension of the vector of variables which is a function of the dimension of the observation time-window. Because of this similarity between the static and dynamic case, fault detection (sensors and actuators) can be achieved using the principles defined in the case of static redundancy equations. In a practical sense, it is possible to generate redundancy

equations from state equations under either in time or frequency domain. These two approaches are totally equivalent (Lou, 1982) if some conditions of duration of the observation time-window are respected. Whatever the approach, the basic principle is the same: the unknown variables are eliminated so that only known or measured variables appear in the equations.

1.3.1. Presentation

Consider the following deterministic model (equation 30) where x is the n-dimensional state vector, A is a n.n matrix, B a n.m matrix, C a p.n matrix. The vectors u and y correspond to the known inputs and outputs of the process. In all the following treatments, without loss of generality, the measurement y depends only on the state x and do not include the input u.

$$x(k+1) = A\,x(k) + B\,u(k) \tag{30a}$$

$$y(k) = C\,x(k) \tag{30b}$$

Direct redundancy may exist among sensors whose outputs are algebraically dependent; this corresponds to the situation where the variable measured by one sensor can be determined instantaneously by other sensor measures. This direct redundancy is very useful for sensor failure detection but is not applicable for the detection of actuator failures. In this situation, the temporal redundancy which links sensor outputs and sensor inputs must be established. When integrated on [k, k + r] window, the system (30) is expressed as:

$$Y(k, r) - G(r)\,U(k, r) = H(r)\,x(k) \tag{31}$$

where Y is the p(r+1) vector of the outputs y(k) to y(k+r), U is the mr vector of the inputs u(k) to u(k+r), G a p(r+1).(mr) matrix and H a p(r+1).n matrix; H(r) is called the r-order observability matrix of the process.

With noise on the output measurement, this equation becomes:

$$Y(k, r) - G(r)\,U(k, r) = H(r)\,x(k) + \varepsilon(k) \tag{32}$$

1.3.2 Bases of redundancies generation

In equation (31), the input u as well as the output y of the process are known. For generating redundancy, the unknown state vector, x(k), has to be eliminated. As equation (31) has the same structure as equation (1), the principles described previously for generating the redundancy

equations may be directly applied to equation (31). The redundancy equations which link Y and U, independently of x, are obtained by multiplying the system (31) by the matrix Ω (called the parity matrix) which must be orthogonal to H(r) (for simplicity the covariance matrix of the measurement errors is taken as unity):

$$\Omega\, H(r) = 0 \tag{33}$$

Then, the general form of the parity equations are:

$$P(k) = \Omega \left(Y(k, r) - G(r)\, U(k, r)\right) = \Omega\, \varepsilon(k) \tag{34}$$

P(k) is referred to as the generalized parity vector which is non-zero mean when a failure occurs. Under a no failure situation P(k) characterizes all the possible relationships between the inputs and the outputs. If a measurement is biased, the parity vector is oriented along a specific direction. However, it must be pointed out that parity equations (34) are not necessarily independent, particularly if the observation window [k, k + r] is too "large". This problem can be ironed out by expressing first the redundancy equations for each sensor (self redundancy), then the redundancy equations between different sensors (inter-redundancy) (Brunet, 1990).

1.3.3 Direct redundancy (self-redundancy)

The notion of direct redundancy (self-redundancy) is very useful for describing analytical redundancy as it expresses the relationships between the time output of a single sensor. The j^{th} term of the observation vector is selected; it is characterized by the submatrix C_j. Equation (31) is then reduced to:

$$Y_j(k, r) - G_j(r)\, U(k, r) = H_j(r)\, x(k) \tag{35}$$

H_j and G_j are deduced using the definitions already given for H and G by replacing C by C_j.

In that case, the single sensor parity relation is simply defined as:

$$P_j(k) = \Omega \left(Y_j(k, r) - G_j(r)\, U(k, r)\right) \tag{36}$$

The value of the length of the observation window, r, has not yet been specified. The parity equations with minimal r value are particularly interesting. They can be found very simply by using the well-known Cayley-Hamilton theorem which implies that there is an r_j such that:

$$\text{if } r < r_j \qquad \text{rank}\left(H_j(r)\right) = 1 + r \tag{37a}$$

and

$$\text{if } r \geq r_j \qquad \text{rank}\left(H_j(r)\right) = r_j \tag{37b}$$

As the row $(r_j + 1)$ of the matrix $H_j(r_j)$ is a linear combination of the r_j other rows, there is a vector Ω such that:

$$\Omega \begin{pmatrix} C_j \\ C_jA \\ . \\ C_jA^{r_j} \end{pmatrix} = 0 \qquad (38)$$

We then obtain the redundancy equation for the j^{th} sensor or self-redundancy:

$$P_j(k) = \Omega \left(Y_j(k, r_j) - G_j(r)\, U(k, r_j)\right) \qquad (39)$$

It also represents a temporal redundancy linking the actuator inputs to the temporal behavior of the j^{th} sensor. This equation involves one sensor only. Thus, if the actuators function properly, this relation can be used as a self test for sensor j. However, if both actuators and sensors are defective, the occurrence of a failure can be detected, but the fault cannot be located.

As an example, let us consider the system described by the state matrices:

$$A = \begin{pmatrix} 0.7 & 0.2 \\ 0 & 0.5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and observed on the window [k, k + 2].

For the first sensor, $C_1 = (1 \quad 0)$ and for a window [k, k+2] the rank r_1 of the observability matrix $H_1(2)$ is equal to 2. The third row of the matrix $H_1(2)$ can be expressed as a linear combination of the others. For determining this linear combination the matrix Ω is calculated according to equation (33). Then the application of (39) yields the parity equation and therefore the self-redundancy of the first and the second sensors:

$$(0.35 - 1.2\, q + q^2)\, y_1(k) = 0.2\, u(k)$$
$$(-1 + 2q)\, y_2(k) - 2\, u(k) = 0$$

where q is the shift forward operator for a discrete signal ($q\, f(t) = f(t+1)$). The two last equations represent the direct temporal redundancy of the output sensors.

1.3.4 Redundancy between sensors (inter-redundancy)

Temporal redundancy exists between several sensors. For each observation matrix (built from one single output and all the inputs), let us retain only the r_j first independent rows (r_j has been defined by using the Cayley-Hamilton theorem).

$$Y_i(k, r_i\text{-}1) = H_i(r_i\text{-}1)\, x(k) + G_i(r_i\text{-}1)\, U(k, r_i\text{-}1) \qquad i = 1, ..., p \tag{40}$$

In order to obtain a formulation which is valid for all the outputs, we can introduce the common vector U(k, r) (where $r = \max(r_1, r_2, ..., r_p)$ of all the inputs $U(k, r_i\text{-}1)$; it may also be necessary to complete some matrices G_i with zero columns in order to define a common G matrix. Using trivial definitions system (40) can be written under a more compact form as:

$$Y(k, r_1, ..., r_p) = H(r)\, x(k) + G(r)\, U(k, r) \tag{41}$$

As in the previous case, we define the matrix Ω, orthogonal to H(r). The parity equations are then written as:

$$P(k) = \Omega\, (Y(k, r_1, ..., r_p) - G(r)\, U(k, r)) \tag{42}$$

In practice, the inter-redundancy equations are obtained by linear combinations of the self-redundancy equations. Although inter-redundancy equations are not independent of the self-redundancy equations, their particular structure make them useful for isolating failures.

Using the same example as above, we can determine the inter-redundancy equations. For the first and the second sensor, the independent rows are:

$$Y_1(k, 1) = \begin{pmatrix} C_1 \\ C_1A \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ C_1B \end{pmatrix} u(k) \tag{43a}$$

$$Y_2(k, 0) = \begin{pmatrix} C_2 \end{pmatrix} x(k) \tag{43b}$$

By combining equations (43a) and (43b) equation (41) becomes:

$$Y(k, 2, 1) = \begin{pmatrix} C_1 \\ C_1A \\ C_2 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ C_1B \\ 0 \end{pmatrix} u(k)$$

Solving (33) gives $\Omega = (\, 3.5 \quad -5 \quad 1)$, which leads to the inter-redundancy equation:

$$3.5\, y_1(k) - 5\, y_1(k+1) + y_2(k) = 0$$

Finally, the redundancy equations are:

$$5\ y_1(k+2) - 6\ y_1(k+1) + 1.75\ y_1(k) - u(k) = 0$$
$$-\ y_2(k) + 2\ y_2(k+1) - 2\ u(k) = 0$$
$$3.5\ y_1(k) - 5\ y_1(k+1) + y_2(k) = 0$$

It should be noted that the inter-redundancy equation may be obtained as a linear combination of the self-redundancy equations through the elimination of the input u. If the actuators are assumed to be fully reliable, the first equation is only sensitive to the failure of the first sensor whereas the second equation is only affected by the failure of the second sensor. Hence, these two equations can be used to identify sensor failures only. The third equation is only affected by sensor failures even if the actuators are faulty; thus it is possible to isolate the failure of a sensor or of an actuator as their signatures of failure are different.

1.3.5 Direct generation of the redundancy equations

The generation of the redundancy equations can be performed in a more direct way from the state equations of the system. This can be achieved simply by eliminating the unknown variables x so that only known or measured variables u and y are considered. Using the state equations (28) the following relations between x, u et y can be written:

$$(q\ I - A)\ x(k) = B\ u(k) \qquad (44a)$$
$$y(k) = C\ x(k) \qquad (44b)$$

Eliminating X between (44a) and (44b) gives the redundancy relation:

$$r(k) = y(k) - C\ (q\ I - A)^{-1}\ B\ u(k) = 0 \qquad (45)$$

Despite possible difficulties involving the calculation of the inverse of (q I - A), this formulation is quite advantageous as it directly provides redundancy relations for each output which allows to isolate the influence of each sensor. X can also be eliminated by reporting the value of X obtained from equation (44b) into equation (44a). As C is not regular, the general solution of (44b) is written as a function of an arbitrary vector w and the generalized inverse C^- of C:

$$x(k) = C^-\ y(k) + (I - C^-C)\ w(k) \qquad (46)$$

Equation (44a) then depends on y(k), u(k) and w(k). Redundancies are then obtained through the elimination of the arbitrary vector w(k). This can be achieved by premultiplying this equation by a

matrix Ω orthogonal to the matrix $(q\,I - A)(I - C^{-}C)$. This matrix Ω can be obtained in a simple way, taking into account the binomial character of the matrix $(q\,I - A)$.

We can now examine the structure of the redundancy equations when failures of both sensors and actuators are considered. For fault detection purpose, let us assume that the effects of actuator and sensor failures can be modelled by rewriting the dynamics of the process as:

$$x(k+1) = A\,x(k) + B\,u(k) + F_l\,l(k) \qquad (47a)$$

$$y(k) = C\,x(k) + E_m\,m(k) \qquad (47b)$$

where $l(k)$ and $m(k)$ are some unknown time functions identically equal to zero when the actuators and the sensors are functioning properly. The elimination of $x(k)$ between these equations yields the residual equations:

$$r(k) = y(k) - C\,(q\,I - A)^{-1}\,B\,u(k) \qquad (48a)$$

$$r(k) = E_m\,m(k) + C\,(q\,I - A)^{-1}\,F_l\,l(k) \qquad (48b)$$

Equation (48a) represents the computational form of the parity equation (which contains only known quantities) and equation (48b) is the internal form which contains the faults. Furthermore, using equation (48b), it is possible to explicit the conditions of isolability of sensors and actuators failures according to the rank of the matrix $(E_m \quad C\,(q\,I - A)^{-1}\,F_l)$.

Let us return to the redundancy equation (45). The corresponding residuals can be directly computed by determining the resolvent matrix $(q\,I - A)^{-1}$. However, when the order of the system becomes high, this approach is extremely sensitive to numerical round-off errors. Different techniques can be used for solving this problem: decomposition of $(q\,I - A)^{-1}$ using the algorithm of Leverrier-Sourriau (Faddev, 1963), calculation of the transfer function relating every input to every output (Bingulac 1975, Varga 1981), triangularisation of the state equations (Blackwell, 1984) and (Hashim, 1990). The algorithm proposed in Misra (1987) uses orthogonal similarity transformations to find the minimal order subsystem corresponding to each input-output pair. As an alternative, the H_∞ approach (Kailath 1980, Vidyasagar 1985) has been used for generating transfer functions and redundancy equations through the double coprime factorization (Viswanadham 1987, Ding 1990 and Fang 1991). Despite some numerical problems involved in the calculation of the inverse matrix, the direct generation of redundancy equations is still a very powerful method as the equations obtained reveal the influence of each input and output. This is very helpful for detecting sensors and actuators failures. In particular, the influence of the outputs can be separated more clearly by applying equation (48b) separately for each line of the measurement matrix C. This treatment yields a redundancy equation for each output. The same applies to the inputs if they are carefully isolated in the following way:

$$x(k+1) = A\,x(k) + B_i\,u_i(k) + \bar{B}_i\,\bar{u}_i(k) \qquad i=1, ..., r \tag{49}$$

where B_i is the i^{th} column of B and $\bar{B}_i$ is the n(r-1) matrix obtained from B by deleting B_i. Let $u_i(k)$ be the ith entry of u(k) and $\bar{u}_i(k)$ the (r-1) column vector obtained from u(k) by deleting $u_i(k)$. By premultiplying (49) by the matrix E_i (chosen orthogonal to $\bar{B}_i$) we obtain an equation depending only on a single input $u_i(k)$. The principle defined previously for eliminating the state x can still be applied. In a more general sense, this technique affords the generation of redundancy equations in the case of singular systems. Finally the technique of direct elimination can be applied to the case of systems with unknown inputs by generalizing the measurement equation (30b) considering that y depends on x and u.

It is worth mentioning that redundancy equations can also be generated using the observer equations associated with the system (30). In that case the output y is compared with the output modelled by the observer. This technique has been thoroughly explored and used for system diagnosis (Clark 1978, Patton 1989a, Frank 1990). The residuals obtained directly using the state equations or indirectly using the associated observer equations are equivalent. The only difference is the presence of a filter whose poles are those of the transfer function of the system. This equivalence was mentioned by Frank (1990) and reexamined by Staroswiecki (1991) and Magni (1991).

Let us return to the previous example. For the first sensor, $C_1 = (1 \quad 0)$ and for the second one $C_2 = (0 \quad 1)$. The two outputs are written as :

$$y_1(k) = C_1\,(q\,I - A)^{-1}\,B\,u(k) = \frac{1}{(q - 0.7)(q - 0.5)}\;\; 0.2\,u(k)$$

$$y_2(k) = C_2\,(q\,I - A)^{-1}\,B\,u(k) = \frac{1}{(q - 0.5)}\;\; u(k)$$

The parity equations write:

$$5\,y_1(k+2) - 6\,y_1(k+1) + 1.75\,y_1(k) - u(k) = 0$$
$$2\,y_2(k+1) - y_2(k) - 2\,u(k) = 0$$

The first equation is sensitive to faults concerning the first sensor. These equations are self-redundancy equations. By eliminating u(k) between the two equations the inter-redundancy equation can be written as:

$$5\,y_1(k+2) - 6\,y_1(k+1) + 1.75\,y_1(k) - y_2(k+1) + 0.5\,y_2(k) = 0$$

which may be reduced to the minimal form:

$$(5\,q - 3.5)\,y_1(k) - y_2(k) = 0$$

1.3.6 Generation of redundancies by reduction of the state equations

This is an original technique for generating redundancy equations which involves only simple numerical calculations. Let us return to the general state equations (30). If C_1 is the regular part of the matrix C, a simple permutation of the components of x yields the decomposition:

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u(k) \qquad (50a)$$

$$y(k) = C_1\,x_1(k) + C_2\,x_2(k) \qquad (50b)$$

Using the following variable changes:

$$\bar{x}_1(k) = C_1\,x_1(k) + C_2\,x_2(k) \qquad (51a)$$

$$\bar{x}_2(k) = x_2(k) \qquad (51b)$$

the state equations are rewritten:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \end{pmatrix} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{pmatrix} + \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix} u(k) \qquad (52a)$$

$$y(k) = \bar{x}_1(k) \qquad (52b)$$

As definition of the matrices $\bar{A}_{ij}$ and $\bar{B}_i$ is trivial, it will not be further developed. The elimination of $\bar{x}_1(k)$ and $\bar{x}_2(k)$ between equations (52a) and (52b) gives the redundancy equation:

$$\left((z\,I - \bar{A}_{11}) - \bar{A}_{12}\,(z\,I - \bar{A}_{22})^{-1}\,\bar{A}_{21}\right) y(k) - \left(\bar{B}_1 + \bar{A}_{12}\,(z\,I - \bar{A}_{22})^{-1}\,\bar{B}_2\right) u(k) = 0 \qquad (53)$$

Considering the size of the matrix to be inverted, this form appears to be more advantageous than (50). However, this size may still be too large. A more interesting presentation of the equation is obtained by eliminating the variable $\bar{x}_1(k)$ in the state equations (52):

$$\bar{x}_2(k+1) = \bar{A}_{22}\ \bar{x}_2(k) + (\bar{A}_{21} \quad \bar{B}_2) \begin{pmatrix} y(k) \\ u(k) \end{pmatrix} \tag{54a}$$

$$z(k) = \bar{A}_{12}\ \bar{x}_2(k) \tag{54b}$$

$$\text{with } z(k) = y(k+1) - \bar{A}_{11}\, y(k) - \bar{B}_1\, u(k) \tag{54c}$$

This form shows the generalized input (y(k), u(k)) which controls the evolution of the state variable $\bar{x}_2(k)$ and the modified measurement z(k). Equations (54) are then structurally similar to equations (30); therefore, the transformation used in equations (52) can be applied to equation (54). In this way, the unobservable variables will be progressively eliminated. The full solution of this treatment will now be presented; the algorithm is applied sequentially and each step is referred to as having an index "n". At step n the state equations are written:

$$x(n, k+1) = A(n)\, x(n, k) + B(n)\, u(n, k) \qquad n = 0, ..., N \tag{55a}$$

$$y(n, k) = C(n)\, x(n, k) \qquad n = 0, ..., N \tag{55b}$$

with the pseudo-measurement defined by:

$$y(n, k) = y(n-1, k+1) - \bar{A}_{11}(n-1)\, y(n-1, k) - \bar{B}_1(n-1)\, u(n-1, k) \tag{55c}$$

At the final step N, different situations can appear:

First, if the matrix A reduces to a scalar matrix, the redundancy equation can be obtained without any matrix inversion:

$$y(N, k)\,(q - A(N)) = C(N)\, B(N)\, u(N, k) \tag{56a}$$

Second, if C(N) becomes null, the pseudo-measurement y(N, k) is null and the redundancy equation is generated from equation (55c) as:

$$y(N-1, k+1) - \bar{A}_{11}(N-1)\, y(N-1, k) - \bar{B}_1(N-1)\, u(N-1, k) = 0 \tag{56b}$$

At each step A(n+1), B(n+1) and C(n+1) are functions of $\bar{A}_{ij}(n)$ and $\bar{B}_i(n)$ which, themselves, are functions of A(n), B(n) and C(n). Due to the change of variable (50), there is only one matrix to be inverted. This stage can still be avoided by applying this algorithm for all the outputs one after the other; in this case, the matrix C reduces to a row-vector and the extraction of its regular part is straightforward.

1.3.7 Generation of redundancies by projection

After a slight rearrangement of the state equations (44), we separate the unknown variables, x, from the known and measured ones, u and y:

$$\begin{pmatrix} C \\ q\,I - A \end{pmatrix} x(k) = \begin{pmatrix} 0 & I \\ B & 0 \end{pmatrix} \begin{pmatrix} u(k) \\ y(k) \end{pmatrix} \tag{57}$$

The projection technique used in the case of static systems can also be applied to this dynamic system. The projection vectors α and β now become polynomials of variable q. Introducing the combined vector $(\alpha(q) \;\; -\beta(q))$ such that:

$$(\alpha(q) \;\; -\beta(q)) \begin{pmatrix} C \\ q\,I - A \end{pmatrix} = 0 \tag{58}$$

and T the matrix such that:

$$C\,T = (\,0 \quad I\,) \tag{59}$$

we know that T may be partitioned as:

$$T = (\; M \quad C^r\;) \tag{60}$$

where M is a right annihilator of C and C^r a right inverse of C. A right multiplication of (58) by T gives the equation:

$$\beta(q)\,(q\,I - A\,)\,M = 0 \tag{61a}$$

$$\alpha(q) - \beta(q)\,(q\,I - A\,)\,C^r = 0 \tag{61b}$$

This last system affords the determination of the polynomials $\alpha(q)$ and $\beta(q)$. In particular, it must be noted that $\beta(q)$ is orthogonal to a binomial matrix. This property facilitates its determination. Lou (1982) proposed an algorithm for a numerical determination of α and β from the nullspace of $(C^T \quad (q\,I - A)^T)$ by forming, in a heuristic way, a minimal base of this space. The redundancy equations can then be obtained by left multiplying equation (57) by the vector $(\alpha(q) \quad -\beta(q))$. It comes:

$$\alpha(q)\,u(k) - \beta(q)\,B\,y(k) = 0 \tag{62}$$

1.3.8 Generation of non independent redundancy equations

The direct approach for generating redundancy equations (transfer functions) yields a set of independent redundancy equations which are function of the inputs and the outputs. The magnitude of their residuals express actuators and/or sensors failures. As they depend on the occurrences of the different input and output variables, the redundancy equations are generally sensitive to specific variables. It means that residuals obtained by linear combinations of the basic redundancy equations may be sensitive to other variables. Typically this technique may be applied in cases where one has to isolate sensor failures from actuator failures. The most reliable coding strategy would then be to have only one residual equation associated to each sensor and others associated with each actuator (fig. 1). Unfortunately, the construction of this ideal set of generalized redundancy equations is not realistic because of the structural connections between inputs and outputs variables. Is is possible to generate all the redundancy equations by considering all the admissible aggregations in the basic equations (two equations may be aggregated if they share at least one common variable).

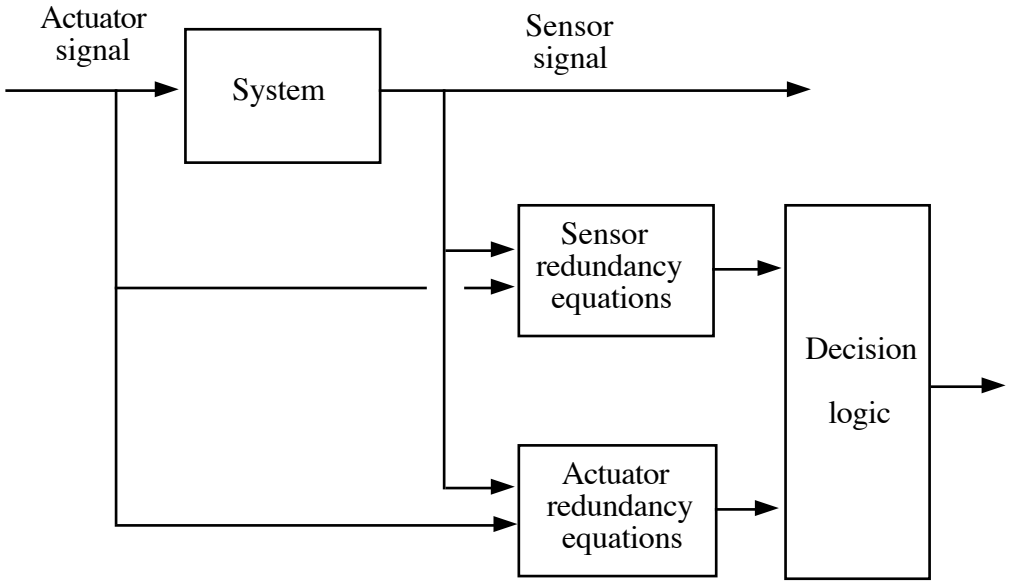


Figure 1 : fault detection and isolation

Let us consider an example taken from Massoumnia (1988) with state matrices:

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Direct calculation of the transfer function yields the two redundancy equations:

$$2\,y_1(k+1) - y_1(k+2) - 2\,u_1(k) + u_1(k+1) + u_2(k+1) = 0$$
$$-\,y_2(k+1) + y_2(k) + u_1(k) + u_2(k) = 0$$

Assuming that the actuators are fully reliable, the first equation is sensitive to the failure of the first sensor, and the second one is sensitive to the failure of the second sensor. Hence these two equations can be used to identify any sensor failure. By aggregation of the two equations, considering u_1 or u_2 as common variables, we obtain:

$$y_1(k+2) - 2\,y_1(k+1) - y_2(k+2) + 2\,y_2(k+1) + u_1(k) = 0$$
$$-\,y_1(k+2) + 2\,y_1(k+1) + y_2(k+2) - 3\,y_2(k+1) + 2\,y_2(k) + 2\,u_2(k) + u_2(k+1) = 0$$

Assuming that the sensors are fully reliable, the first equation is sensitive to the failure of the first actuator and the second equation is sensitive to the failure of the second actuator. Hence these two equations can be used to identify any actuator failure.

It is also possible to use more general redundancy equations with a slight extension. If the basic redundancy equations are expressed in the z domain, the residual may be written:

$$r(k) = D(q)\,y(k) - N(q)\,u(k) = \begin{pmatrix} N(q) & -D(q) \end{pmatrix} \begin{pmatrix} u(q) \\ y(q) \end{pmatrix} \tag{63}$$

if we consider the generalized or structured residual:

$$r_g(q) = J(q)\,\big(D(q)\,y(q) - N(q)\,u(q)\big) \tag{64}$$

in which we have introduced a stable rational transfer function J(q) to add another "degree of freedom". The determination of J(q) is performed so that the new residuals r_g are small under nominal conditions and large when failures are present. Therefore r_g responds quickly to failures and each different actuator and sensor failures should produce different residuals. For instance in Viswanadham (1987), a unimodular matrix J is used so that J D is in upper-triangular form. When the number of outputs is greater than or equal to the number of inputs, a diagonalized residual vector can be obtained based on a coprime factorization approach. Viswanadham shows an example of this procedure applied to a turbine engine control system while Gertler (1988, 1991) presents a method for determining the matrix J by using the concept of occurrence matrix (J is calculated so that the positions of the null terms of the matrix J(q) (D(q) -N(q)) are specified). It should be noted that the so-called inter-redundancy and self-redundancy equations are particular cases of the structured residuals.

1.3.9 Interconnected systems

A linear multivariable process can be described by a set of equations:

$$x(k+1) = A\,x(k) + B\,u(k) \tag{65a}$$
$$y(k) = C\,x(k) + D\,u(k) \tag{65b}$$

For a process composed of several sub-processes, the structure (65) is always available. However, it is very abstract and thus, meaningless to the user. The interconnections between the different elements of the system do not explicitly appear. In order to stay closer to the physical structure of the process, it is preferable to use a description of each sub-system and to take into account their connections. In the discrete domain, the i^{th} output S_i can be expressed as a function of the n_i inputs E_{ij} ($j = 1, ..., n_i$):

$$D_i(q)\,S_i(k) = \sum_{j=1}^{n_i} (N_{ij}(q)\,E_{ij}(k)) \tag{66}$$

where $D_i(q)$ and $N_{ij}(q)$ are polynomials in the q variable. For all the outputs it is clear that we can use the matricial polynomial equation:

$$D(q)\,S(k) - N(q)\,E(k) = 0 \tag{67}$$

where D(q) and N(q) are polynomial matrices in q and E(k) and S(k) the discrete forms of the vectors of input and output variables of each linear block considered separately. Because of the interconnections existing between the subprocesses, some components of S represent the same variables as some components of E; the non interconnected variables belonging to E and S define the vector X. With an evident definition for the polynomial matrix M(q), equation (67) becomes:

$$M(q)\,x(k) = 0 \tag{68}$$

Some elements of the X vector are known (measurements by example); we then have a measurement equation:

$$y(k) = C\,x(k) \tag{69}$$

Finally the global system can be put into the following form:

$$\begin{pmatrix} M(q) \\ C \end{pmatrix} x(k) - \begin{pmatrix} 0 \\ I \end{pmatrix} y(k) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{70}$$

This is a form that has been used to describe static and dynamic system. On a structural point of view, the generation of analytical redundancy equations may be obtained, following the same principle: one has to eliminate the unknown variable x in equation (70). As previously explained the

direct elimination or the projection method would be very appropriate; the only difficulty lies in the fact that the operations will have to be carried out on polynomial matrices. For applying the projection method, one has to search a polynomial matrix T, orthogonal to the matrix $P(q) = (M^T(q)\ C^T)^T$. It must be pointed out that the redundancy equations do not always exist (Staroswiecki 1989, 1990). Indeed, if P(q) is a polynomial matrix with n rows (n = dim X + dim Y) whose rank is r, the matrix T exists iff n is greater than r. If this condition is not fulfilled, the system is not redundant. This last condition is a generalization of the condition defined in the case of static systems i.e. the number of variables had to be less than the number of measurement equations.

1.3.10 Observability decomposition

The last technique presented in this section of our paper represents a direct extension of the decomposition of systems according to an observability criterion. The basic idea is to transform a dynamic system into a quasi-static system represented by a vector of measured and non measured variables and a constraint matrix. The decomposition and the classification developed in (27) are then applied to this pair "constraint matrix-variables vector". The state equations (31) can be written under a more condensed form:

$$\begin{pmatrix} -I & G(r) & H(r) \end{pmatrix} \begin{pmatrix} Y(k,r) \\ U(k,r) \\ x(k) \end{pmatrix} = 0 \qquad (71)$$

$$\text{with } G(r) = \begin{pmatrix} 0 & 0 & . & . & 0 \\ CB & 0 & . & . & 0 \\ CAB & CB & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ CA^{r-1}B & CA^{r-2}B & . & . & CB \end{pmatrix} \quad \text{and } H(r) = \begin{pmatrix} C \\ CA \\ . \\ . \\ CA^r \end{pmatrix}$$

On a time-window whose size is r, this equation relates the evolutions of the unknown state vector x with those of the output vector Y and the command vector U. This is similar to the classical expression (23) obtained in the study of the systems described by linear equations. For generalization purposes, M is referred to as the "incidence" matrix and Z as the variables vector defined as:

$$M = \begin{pmatrix} -I & G & H \end{pmatrix} \qquad (72a)$$

$$Z^T = (Y^T \quad U^T \quad x^T) \qquad (72b)$$

The two first blocks in (72a) correspond to the known variables (Y and U) and the last block to the non measured variables. This is a form similar to equation (21). By transforming M into a canonical form (equation (27)), using a decomposition according to observability, the redundancy equations (corresponding to the parity equations) as well as the deduction equations can be obtained under an explicit form.

Let us consider the system described by the following state equations:

$$A = \begin{pmatrix} 0.7 & 0.2 \\ 0 & 0.5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

if it is observed on a time-window whose size is equal to 2, the incidence matrix M is written:

$y_1(k)$	$y_2(k)$	$y_1(k+1)$	$y_2(k+1)$	$y_1(k+2)$	$y_2(k+2)$	u(k)	u(k+1)	$x_1(k)$	$x_2(k)$
-1	.	.	.	.	.	.	.	1	.
.	-1	.	.	.	.	.	.	.	1
.	.	-1	.	.	.	.	.	0.70	0.20
.	.	.	-1	.	.	1	.	.	0.50
.	.	.	.	-1	.	0.20	.	0.49	0.24
.	.	.	.	.	-1	0.50	1	.	0.25

The line located above this matrix recalls the names of the different variables present in equation (71). Using the proposed decomposition procedure (equation (27)), the canonical form of this matrix is:

$y_1(k)$	$y_2(k)$	$y_1(k+1)$	$y_2(k+1)$	$y_1(k+2)$	$y_2(k+2)$	u(k)	u(k+1)	$x_1(k)$	$x_2(k)$
0.49	0.14	.	0.20	-1.00	.	.	.	.	.
0.70	0.20	-1.00	.	.	.	.	.	.	.
.	0.50	.	-1.00	.	.	1.00	.	.	.
.	.	.	.0.50	.	-1.00	.	1.00	.	.
-1.00	.	.	.	.	.	.	.	1.00	.
.	-1.00	.	.	.	.	.	.	.	1.00

where the first line still corresponds to the name of the concerned variables.

The redundancy and deduction equations are then written as:

$$0.49\ y_1(k) + 0.14\ y_2(k) + 0.20\ y_2(k+1) - y_1(k+2) = 0$$
$$0.70\ y_1(k) + 0.20\ y_2(k) - y_1(k+1) = 0$$
$$0.50\ y_1(k) - y_2(k+1) - u(k) = 0$$
$$- y_1(k) + x_1(k) = 0$$
$$- y_2(k) + x_2(k) = 0$$

A simple extension of this decomposition technique deals with the analysis of system with unknown inputs. The structure of such a system can be written:

$$x(k+1) = A\ x(k) + B\ u(k) + E\ d(k) \qquad (73a)$$
$$y(k) = C\ x(k) \qquad (73b)$$

It can be rewritten under its matricial expression for a time-window of size r:

$$(I \quad -G_u \quad -G_d \quad -H) \begin{pmatrix} Y(k, r) \\ U(k, r) \\ D(k, r) \\ x(k) \end{pmatrix} = 0 \qquad (74)$$

This quasi-static form shows the known and unknown parts of the variables vector and the corresponding partitioning of the incidence matrix. The procedure of decomposition according to observability developed in the static case can then be applied to this equation.

1.4 RESIDUALS ANALYSIS

The first two sections were devoted to the generation of redundancy equations for linear static and dynamic systems. As already mentioned, the second stage of FDI concerns the so-called residuals evaluation i.e. the forming of diagnostic decision on the basis of the residuals. To limit the length of the present paper, all the aspects of this stage will not be covered in this section. We will especially focus on the methods issued from static systems analysis and which can be extended to dynamic systems (Maquin, 1991b). As indicated by Gertler (1988, 1990), the decision making stage usually implies statistical testing. There is a close relationship between statistical testing and residual generation. Residuals are variables that are zero under ideal circumstances; they become nonzero as a result of failures, noise and modelling errors. To account for the presence of noise, statistical testing is applied to the residuals. Then, a logical pattern is generated showing which residuals can be considered normal and which ones indicate faults. Such a pattern is called the signature of the

failure. The final step of the procedure is the analysis of the logical patterns obtained from the residuals. The aim is to isolate the failures. Such analysis may be performed by comparison to a set of signature known to belong to simple failures.

1.4.1 Presentation

In this section, residuals generated either from static or dynamic systems are analyzed in a unified framework. A linear system can be described, in the fault-free case, by the following relations:

a model equation : $M X^* = 0$ (75a)
a measurement equation : $Z = H X^* + \varepsilon$ (75b)

where X^* is the v-dimensional vector of process variables, Z the m-dimensional vector of measurements, M the n.v matrix of model equations (without loss of generality, it is supposed of full row rank), H the m.v measurement selection matrix and ε is a vector of random errors characterized by its variance matrix.

For dynamic processes, the model, which relates the state vector x(k) to the input vector $u^*(k)$ and the output vector $y^*(k)$, described in state space discrete form, may be written as :

$$x(k+1) = A\, x(k) + B\, u^*(k) \quad (76a)$$
$$y^*(k) = C\, x(k) \quad (76b)$$

where $u^*(k)$ and $y^*(k)$ denote the actual values of the input and output of the system. Defining, on a time-window of length N, the mixed vector of inputs and states:

$$X^* = (x(0)\ \ u^*(0)\ \ x(1)\ \ u^*(1)\ \ ...\ \ u^*(N)\ \ x(N+1)\ )^T \quad (77)$$

and the corresponding constraints matrix:

$$M = \begin{pmatrix} A & B & -I & . & . & . & . & . \\ . & . & A & B & -I & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & A & B & -I \end{pmatrix} \quad (78)$$

the constraint equation (75a) may be condensed into the form:

$$M X^* = 0 \quad (79a)$$

Similarly, the measurement equation may be written as:

$$Z = H X^* + \varepsilon \tag{79b}$$

As the inputs and only a part of the state (equation (76b)) are measured, the selection matrix H is defined by:

$$H = \begin{pmatrix} C & . & . & . & . & . \\ . & I & . & . & . & . \\ . & . & C & . & . & . \\ . & . & . & I & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & C \end{pmatrix} \tag{80}$$

As equations (79a) and (79b) are strictly identical to (75a) and (75b), static and dynamic systems can be analyzed in a unified framework.

The methods for fault detection and isolation are often divided into two groups: those which apply a priori, without carrying out the full data reconciliation (estimation of process variables), by directly testing the residuals issued from redundancy equations and those which apply a posteriori on the residuals generated by calculating the differences between the raw measurements and their estimations. In practice, both methods are used together in order to improve the robustness of the procedure of fault detection and isolation.

The estimation or data reconciliation problem of system (75) involves finding a set of adjustments such that the adjusted values verify the model equation (75a). With the classical assumption that the measurement errors ε are normally distributed with zero mean and known variance matrix V, this optimization problem can be stated as maximizing the probability density function (Ragot, 1990a):

$$P(Z) = \frac{1}{(2\pi)^{m/2} |V|^{1/2}} \exp\left(-\frac{1}{2}(Z - H X^*)^T V^{-1} (Z - H X^*)\right) \tag{81}$$

subject to $M X^* = 0$

The solution $\hat{X}$ of this problem is obtained by minimizing the criterion:

$$\phi = \frac{1}{2}(Z - H \hat{X})^T V^{-1} (Z - H \hat{X}) \tag{82a}$$

subject to the constraint $M \hat{X} = 0$ (82b)

Assuming that the system is observable, i.e. the knowledge of the model M and the measurement vector Z is sufficient to determine a unique estimation $\hat{X}$, allows us to write the relation:

$$\text{rank}\begin{pmatrix} H \\ M \end{pmatrix} = \dim(X^*) = v \tag{83}$$

This condition is equivalent to the following one:

$$\text{rank}(H^T V^{-1} H + M^T M) = v \tag{84}$$

Using the Lagrange multipliers technique leads to the classical unbiased estimator:

$$\hat{X} = (G^{-1} - G^{-1} M^T (MG^{-1}M^T)^{-1} M G^{-1}) H^T V^{-1} Z \tag{85}$$

where the regular matrix G is defined by:

$$G = H^T V^{-1} H + M^T M \tag{86}$$

These general expressions may be simplified either if all the variables are measured or if a preliminary extraction of the redundant part of the system is achieved. In this case, system (75) reduces to:

$$M_r X_r^* = 0 \tag{87a}$$
$$Z = X_r^* + \varepsilon \tag{87b}$$

where X_r^* denotes the actual redundant process variables. In order to simplify the further notations, we will drop, in the following, the subscript r. Estimation of the redundant variables X_r are given by the classical result:

$$\hat{X} = (I - V M^T (MVM^T)^{-1}M) Z \tag{88}$$

Using the formula giving the variance of a linear combination of random variables, it can be proved that the variance-covariance matrix of the estimated vector $\hat{X}$ is expressed by:

$$\hat{V} = (I - V M^T (MVM^T)^{-1}M) V \tag{89}$$

The vector E of adjustments (or residuals) and the residual criterion ϕ_R are obtained by direct substitution:

$$E = Z - \hat{X} = V M^T (MVM^T)^{-1}M Z \tag{90}$$

$$\phi_R = E^T V^{-1} E \tag{91}$$

Both the vector of adjustments E (90) and the direct imbalances vector of redundancy equations R defined by :

$$R = M Z \tag{92}$$

can be considered and processed as residuals. However, it should be noticed, that these residuals cannot be analyzed in the same way as each entry of R is associated with an equation and each entry of E with a specific variable.

1.4.2 Residual criterion analysis

A first approach to testing the residuals is to introduce a single scalar statistic like, for example, the residual criterion (91). As was first pointed out by Reilly (1963), the residual criterion:

$$\phi_R = E^T V^{-1} E \tag{93}$$

has a chi-square distribution with a number of degrees of freedom equal to the rank of M. Furthermore, it is also useful to note that the calculation of ϕ_R does not require the estimation stage. It is easy to show that the residual criterion can be expressed as a function of R:

$$\phi_R = R^T(MVM^T)^{-1}R \tag{94}$$

Thus the residuals can be globally tested against tabulated values of chi2. In the fault-free case, the function ϕ_R is below the threshold for the chi-square with the appropriate confidence level and number of degrees of freedom. Unfortunately, if the chi-square test is satisfied, it does not prove that there are no faults in the measurements set since a fault may exist among a large set of measurements. It is then preferable to use further specific tests to diagnose the measurements.

A difficulty with this global test is that, while it indicates well the presence of fault it is not able to identify the source of these errors. The use of a sequential procedure allows the location of the fault. Ripps (1962) proposed a scheme that was also used by Nogita (1972) under a slightly modified form. For the set of all process measurements, one first calculates the global test ϕ_R. If an error is indicated by the test, all measurements are considered as suspect candidates. Then, the measurements are "deleted" sequentially from the process (in groups of size 1, 2, ...). After each deletion the global test is again applied. In this approach, the purpose is to assess the effect of deleting a particular set of measurements on the objective function and on the estimations. Furthermore, it is possible to have the same approach as the one developed in the case of multiple-observer for state reconstruction (Frank, 1989) by comparing together the different estimations obtained after each deletion.

Romagnoli (1981) and later Fayolle (1987) consider suspect measurements by assigning them an infinite variance. The corresponding variation of the criterion ϕ_R is then used to detect the possible

faults. By isolating the measurement z_2, for which the variance will be later modified, let us consider the following partitioning of the matrices:

$$M = (M_1 \quad m_2) \tag{95}$$

$$Z = (Z_1 \quad z_2) \tag{96}$$

At the same time, let us consider a modification Δv_2 of the variance of this measurement. Then, the whole variance matrix is written as:

$$V+\Delta V = \begin{pmatrix} V_1 & 0 \\ 0 & v_2+\Delta v_2 \end{pmatrix} \tag{97}$$

The residual criterion is then modified as:

$$\phi_R+\Delta\phi_R = R^T (M (V+\Delta V) M^T)^{-1} R \tag{98}$$

from which, when Δv_2 is infinite, the following variation can be deduced:

$$\Delta\phi_R = - \frac{R^T K m_2 m_2^T K R}{m_2^T K m_2} \tag{99}$$

with

$$K = (MVM^T)^{-1} \tag{100}$$

Equation (99) gives a simple expression of the reduction in the objective function when deleting a single measurement. Thus, aside from vector-matrix multiplications, the only computation needed is the calculation of K carried out once and only once whatever the suspect measurement. Crowe (1988) has also developed formulas to predict the effects of deleting any set of measurements on the objective function. These formulas can be used without having to compute the reconciliation for each case of deletion.

1.4.3 Imbalances or adjustments vectors analysis

Another approach is the direct parallel testing of the residuals. With the assumption of a Gaussian distribution of the measurement errors, the vector R also follows a normal distribution with zero mean and covariance V_R:

$$V_R = M V M^T \tag{101}$$

In order to compare the elements of the R vector, let us define a standardized imbalance vector R_N:

$$R_N = \text{diag}(V_R)^{-1/2}\, R \qquad (102)$$

Each entry $R_N(i)$ follows a normal distribution with zero mean and unity variance. A simple statistical two tailed test can therefore be used: we may conclude that equation i is a "bad" equation if:

$$| R_N(i) | > t \qquad (103)$$

Classically, one may choose the critical constant t to control the familywise Type I error rate at some pre-assigned level α. Even if we assume the presence of only one gross error, the relationship between the "bad" equation(s) and the suspect measurement is not straightforward. It depends on the structure of the equations and the location of the faults. In some cases, we are not able to suspect one measurement only. For solving this case (Mah, 1976) proposed to apply the preceding test to each equation and also to the aggregates of two or more equations (also known as pseudo-equations). The main assumption underlying this method is that faults do not cancel each other.

This latter approach can also be applied to the adjustments vector E. The variance matrix of this vector is expressed as:

$$V_E = V\, M^T\, (M V M^T)^{-1}\, M\, V \qquad (104)$$

As for the imbalance residuals vector, we define the standardized adjustments vector:

$$E_N = \text{diag}(V_E)^{-1/2}\, E \qquad (105)$$

Each $E_N(i)$ is compared with a critical test value. If at least one entry of E_N is out of the confidence interval then, there is a "bad" measurement. The defective measurement can always be shown to correspond to the greatest standardized adjustment residual (Fayolle, 1987).

For the linear case, instead of (105), Tamhane (1985) has shown that for a non diagonal covariance matrix V, a vector of test statistics with the maximal power for detecting a single fault is obtained by premultiplying E by V^{-1}. Then, the transformed residual, $e = V^{-1} E$, is normally distributed with zero mean and a variance matrix $V_e = V^{-1} V_E V^{-1}$. The power of the test (the probability of correctly detecting and identifying gross errors when they are present in the process data) has been established and discussed by Iordache (1985) under different conditions (various networks, errors location, variance values ...) using the Monte Carlo simulation.

Note that Jongenelen (1988) pointed out the case where the variance V depends on an unknown scale factor σ^2; on this basis, he proposes a new test based on externally Studentized residuals.

1.4.4 Generalized likelihood ratio approach

A new formulation of the problem of gross error detection is due to Narasimhan (1987). This approach, the generalized likelihood ratio test, was first developed by Willsky and Jones (1976) to identify abrupt failures in dynamic systems. It is based on the classical likelihood ratio test. If no faults are present, the mathematical expectation of R (equation (92)) is null and its variance matrix is given by (101). If a fault of magnitude b is present in the measurement of variable i, the mathematical expectation of R can be written:

$$E(R) = b\, M\, e_i = b\, f_i \tag{106}$$

where e_i is an elementary vector with 1 at the position i and zeros elsewhere. If we define μ as the expected value of R, we can formulate the hypotheses for faults detection as:

$$H_0 \; : \; \mu = 0 \tag{107}$$

$$H_1 \; : \; \mu = b\, f_i \tag{108}$$

where H_0 is the null hypothesis that no faults are present and H_1 is the alternative hypothesis H_1 that a measurement bias is present. In order to test the hypothesis H_1 and estimate the unknown parameters b and f_i we use the likelihood ratio test statistics:

$$\lambda = \sup_{b,\, f_i} \left(\frac{Prob(R/H_1)}{Prob(R/H_0)} \right) \tag{109}$$

Using the normal probability density function for R, we can write (109) as:

$$\lambda = \sup_{b,\, f_i} \left(\frac{\exp(-\frac{1}{2}(R - b\, f_i)^T\, V_R^{-1}\, (R - b\, f_i))}{\exp(-\frac{1}{2} R^T V_R^{-1} R)} \right) \tag{110}$$

Since the log function is monotonic, we can simplify the calculation by choosing the test statistics:

$$T = 2\, Log\, \lambda = \sup_{b,\, f_i} \left(R^T\, V_R^{-1}\, R - (R - b\, f_i)^T\, V_R^{-1}\, (R - b\, f_i) \right) \tag{111}$$

The computation proceeds in two steps. First, for any vector f_i, we compute the estimate of b:

$$\hat{b} = (f_i^T\; V_R^{-1}\; f_i)^{-1}\, (f_i^T\; V_R^{-1}\; R) \tag{112}$$

Then, using equation (111), we obtain the corresponding value of T:

$$T_i = \frac{(f_i^T V_R^{-1} R)^2}{(f_i^T V_R^{-1} f_i)} \qquad (113)$$

This calculation is performed for every vector f_i and the test statistics T is therefore obtained as:

$$T = \sup_{f_i} T_i$$

The test statistic T is compared with a prespecified threshold. If T is greater than this threshold, then a fault has been detected and its magnitude is estimated with (112).

1.4.5 Parity space approach

In the early developments of fault diagnosis methods, the parity space approach was applied to hardware redundancy schemes (Potter 1977, Daly 1979, Hamad 1986). In the fault-free case, the measurement equation is :

$$Z = H X^* + \varepsilon \qquad (114)$$

where X^* is the n-dimensional vector of redundant process variables, Z the m-dimensional measurement vector and H an m.n measurement matrix. For such systems, the number m of measurements is greater than the number n of variables (m > n). The noise vector ε has a variance matrix V.

Equations (75), which describe the general structure of redundancy equations of a linear static system, can be transformed into the formulation (114) if we proceed to the elimination of the model equation (Ragot, 1991a). For that purpose, let us extract from M, the regular part M_1:

$$M = (M_1 \quad M_2) \qquad (115)$$

The vector X^* may be decomposed following this partitioning:

$$X^* = \begin{pmatrix} X_1^* \\ X_2^* \end{pmatrix} \qquad (116)$$

As M_1 is a regular matrix, X_1^* may be expressed :

$$X_1^* = - M_1^{-1} M_2 X_2^* \qquad (117)$$

Then, the measurement equation takes a form which looks like (114):

$$Z = H X_2^* + \varepsilon \qquad (118)$$

with:

$$H = \begin{pmatrix} - M_1^{-1} M_2 \\ I \end{pmatrix} \qquad (119)$$

The parity vector is related to the measurement vector Z through a projection matrix Ω of dimension n.v (with n = v-m) :

$$p = \Omega V^{-1/2} Z \qquad (120)$$

Parity equations show that, in the absence of fault, the magnitude of the parity vector is small (presence of measurement noise). If a failure occurs in only one of the sensors, then the parity vector grows in a fixed direction associated with the failed sensor. Furthermore, the components of the parity vector have the same probability distribution as the measurement errors which are independent Gaussian of zero mean value. By definition (120), the variance-covariance matrix V_p of the parity vector p is an identity matrix.

As the variable $c^2 = p^T V_p^{-1} p$ is the sum of squares of (v-m) normally distributed variables, it has a chi-square probability distribution with (v-m) degrees of freedom and may be compared to the threshold $c^2_{1-\alpha}$ where $c^2_{1-\alpha}$ is the value of chi-square at a confidence level α. Once the detection of faults is made, they can be located. For each column Ω_j of the projection matrix Ω, we compute the projection of the parity vector. It is given by:

$$p_j = \frac{\Omega_j^T p}{\| \Omega_j \|} \qquad j = 1, ..., v \qquad (121)$$

The defective sensor then corresponds to the greatest projection p_j of p. Next, the suspect variable is deleted from the system and the detection test is recalculated after this deletion. The procedure is stopped when the magnitude of the parity vector p corresponding to the remaining measurements no longer fulfills the detection test.

It must be noted that, in the linear case, it is possible to establish the complete equivalence between all the preceding methods: residual criterion, imbalances or adjustments vectors analyses, generalized likelihood ratio or parity space approaches (Maquin, 1991a).

In order to illustrate one of the preceding methods, let us consider the third order dynamic system described by system (76) with the following matrices:

$$A = \begin{pmatrix} 0.77 & 0.19 & 0.00 \\ -0.39 & 0.58 & 0.30 \\ -0.60 & -0.45 & 0.86 \end{pmatrix} \qquad B = \begin{pmatrix} 0.053 \\ 0.389 \\ 0.126 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Figure 2 shows the states and the input measurements; differents measurements faults have been incorporated (for state 1 between time 222 and 242, for state 2 between time 111 and 131, for input between time 333 and 343). Figure 3 presents the evolutions of the absolute values of the adjustments, defined by (90), using all the state measurements according to the definition of C. A detection of jumps using, for example, the Page-Hinkley test enables an easy detection of the faults (Basseville, 1986a).

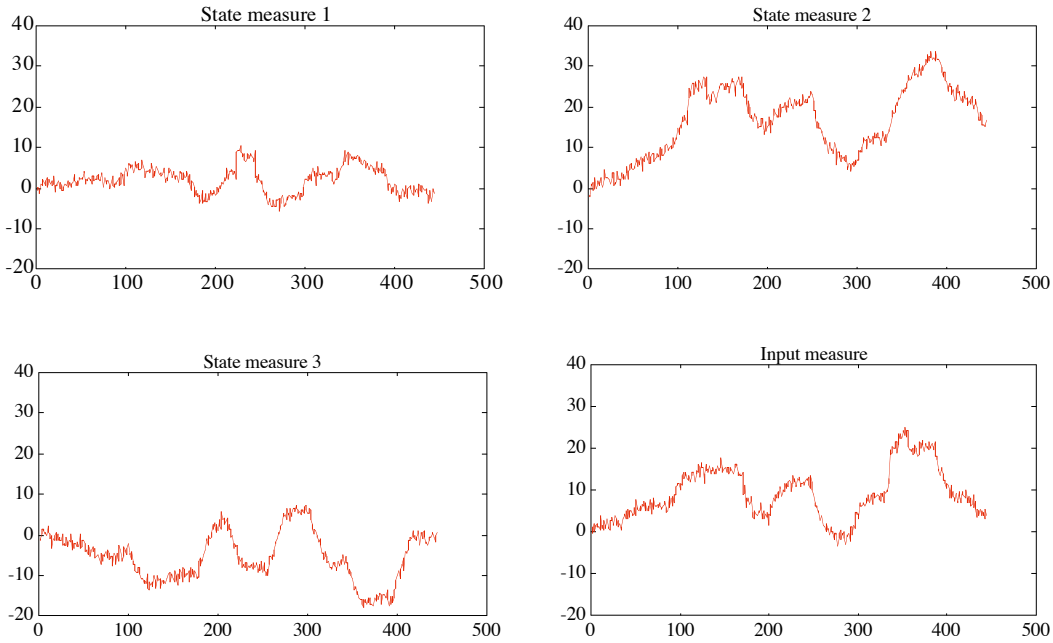
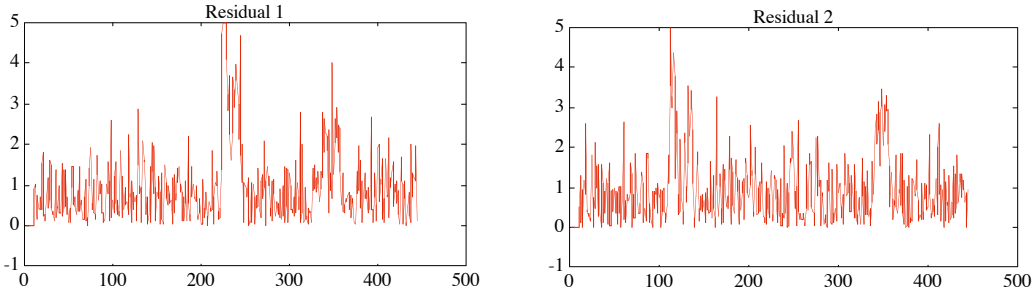


Figure 2 : states and input measurements



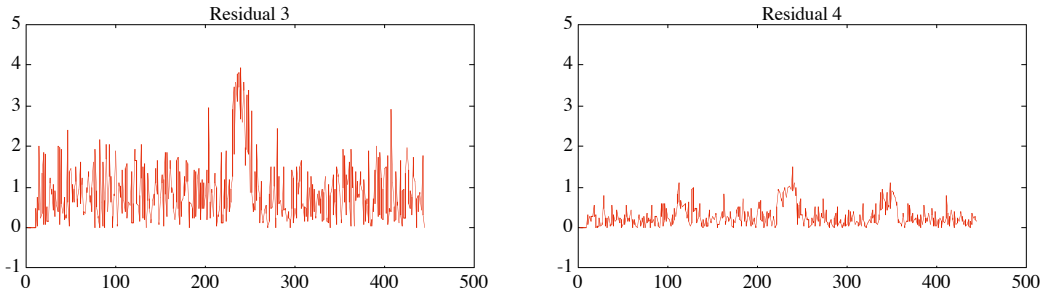


Figure 3 : residuals

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