SWITCHING TIME ESTIMATION OF PIECEWISE LINEAR SYSTEMS. APPLICATION TO DIAGNOSIS

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Abstract: Analysing the behaviour of physical systems often leads to realise that a natural representation consists in building mixed continuous / discrete models. Multi-models or hybrid models are adapted representations for complex physical systems by introducing transitions (smooth or not) between local behaviours. This paper presents some technical points dealing with the determination of the time transition from one local model to another one. More generally, our purpose is to estimate, at each time, the state of the associated process.

Keywords: Multi-model, hybrid systems, parity space, switching, state estimation.

1. INTRODUCTION

The problem of finding a model representing the behaviour of an unknown system by observing a set of input-output data has received a lot of attention since several decades (Johansen, 1995). It is clear that the principle of "simplicity" often applies and leads to use local models (with the most simple structure as possible) which are linked either by a switching procedure or by an interpolation mechanism. This model involves multiple structures that can characterise the time series behaviours in different regimes ; by permitting switching between these structures the model is able to capture more complex dynamical patterns. Moreover, the natural behaviour of a physical system may naturally differ from one operating region to another one. This fact is a strong motivation to build as many local models as there are operating conditions. Moreover many industrial processes naturally exhibit a hybrid behaviour because they are constituted by dynamical components at a low level and by logical components at an upper level. The consequence of this is that multiple models have to be employed and model switching has to be managed to execute control tasks such as diagnosis, monitoring, identification (Bemporad, 2001), regulator calibration, control (Banerjee, 1995), etc. Therefore, and this constitutes a true difficulty, the hybrid nature of the models requires new forms of analysis (stability, observability, identifiability (Hiskens, 2000) ...) because discrete changes are not handled well by continuous algorithms. Taking into account our field of interest which is process diagnosis, our purpose is to estimate the state of the system under consideration. State estimation is understand as three complementary aspects: at each time instant determination of the active local model, estimation of the time switching between two local models, estimation of the current state of the system (this task is easy to achieve since the two first have been solved).

Several authors investigated for years the design of observers in the discrete and continuous domain. Probably, Ackerson (1970) was the first who introduced the state estimation for switching systems. Recently (Alessandri, 2001), Alessandri considered the case where continuous evolution is linear assuming knowledge of the discrete state at each time. Balluchi (2000) tries to remove this assumption on state knowledge and designs an hybrid observer that estimates the state from the knowledge of the hybrid plant inputs and outputs. Our presentation has the same objective, the way for designing the observer being quite different.

The paper starts in section 2 with introductive examples allowing to present some typical problems about multi-models. Then section 3 is dedicated to our main results about the design of an observer based on parity state space equations. Finally, numerical results are discussed.

2. MODEL OF SWITCHING SYSTEMS

A simple two regimes case may be described as

$$x(k+1) = A_1 x(k) + Bu(k) \qquad \text{1st regime} \qquad (1a)$$
$$x(k+1) = A_2 x(k) + Bu(k) \qquad \text{2nd regime} \qquad (1b)$$

$$y(k) = Cx(k) \tag{1c}$$

The choice between the two regimes (which can be easily extended to more than two) is operated by a transition function $\mu(z(k),\theta)$ (depending on a decision variable z and on parameters θ) which values are chosen between zero and one. Thus, the global model may be written:

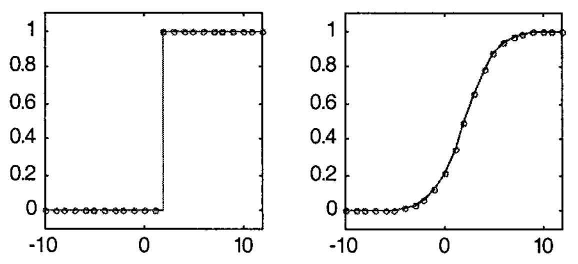
$$x(k+1) = \big((1-\mu(z(k),\theta))A_1 + \mu(z(k),\theta)A_2\big)x(k) + Bu(k) \tag{2a}$$
$$y(k) = Cx(k) \tag{2b}$$

This representation may be modified by taking into account additive noises or errors on the dynamic part (2a) of the system and also on the measurement equation (2b). The model often represents a transition regression model or a regression model with several regimes where the change from one regime to another one depends on the value of a linear combination of the elements of $z(k)$; for example z may be a combination of the past input u and output y. In some situations $z(k)$ may be an unobservable variable. A very similar structure may be proposed dealing with the case where the transition is modelled as a random variable (Hamilton, 1989):

$$x(k+1) = \big((1-\mu(k))A_1 + \mu(k)A_2\big)x(k) + Bu(k) \tag{3a}$$
$$y(k) = Cx(k) \tag{3b}$$

where, for example, $\mu(.)$ is a Bernoulli random variable being one with probability p and zero with probability $1-p$.

Returning to (1), there is a various choice to specify the transition function $\mu(.)$ and the switching variables contained in $z(.)$. As a first possibility, $\mu(.)$ can be a step function which means that the transition between two regimes occurs abruptly (fig. 1a). Second, the transition may be selected more smoothly (fig. 1b).



Figures 1a et 1b

Concerning the switching variables $\mu(.)$, it can simply contains a time index and a constant which consists to compare k to an unknown date τ (the switching is said exogenous and occurs at a single point of time):

$$\mu(k-\tau) = \begin{cases} 0 & \text{if } k \geq \tau \\ 1 & \text{if } k < \tau \end{cases}$$

The time index itself may be a continuous variable where the value of an endogenous or exogenous variable is compared to a known threshold:

$$\mu(z(k),\Delta) = \begin{cases} 0 & \text{if } z^T(k)\theta \geq \Delta \\ 1 & \text{if } z^T(k)\theta < \Delta \end{cases}$$

Instead of a step transition function, a smooth transition may also be considered:

$$\mu(z(k),\Delta) = \frac{1}{2}\left(1 + tanh\frac{z^T(k)\theta - \Delta}{\gamma}\right)$$

where the parameters Δ and γ specify the shape of the smoothing. There are a lot of interesting problems associated with multimodels analysis: determining the number of local models, identifying the parameters of the local models (Bemporad, 2001), identifying the transition functions (Verdult, 2001) and its active parameters, estimating the switching time, identifying a basis for the multimodel representation, finding pattern in time series, controling system operating in multiple regimes (Banerjee, 1995), diagnosing the process functionning (Simani, 2000). Among all these questions, our main interest is concerned here with the state estimation of the system.

3. EXAMPLE OF SWITCHING SYSTEM

Let us consider an academic example that would illustrate some typical problems concerning the switching systems. This example deals with a first order AR system:

$$x(k+1) = \big(a_1(1-\mu(k)) + a_2\mu(k)\big)x(k)$$
$$y(k) = x(k)$$
$$\mu(k) = \begin{cases} 0 & \text{if } z(k) - 0.5 \geq 0 \\ 1 & \text{if } z(k) - 0.5 < 0 \end{cases}$$

where $z(k)$ is an exogenous variable generated from a random distribution).

Let us consider a given sequence $y(k)$ which is provided by a perfect measurement device. Figure (2a) shows the measurements and reveals the switching from one model to the other. A very simple and systematically way to point out the switching consists in using one of the following estimates:

$$\mu(k) = \frac{1}{a_2 - a_1}\left(\frac{y(k+1)}{y(k)} - a_1\right) \tag{4a}$$

$$a(k) = \frac{y(k+1)}{y(k)} = a_1(1-\mu(k)) + a_2\mu(k) \tag{4b}$$

It is clear that (4b) is less demanding than (4a), since only the measurement sequence is needed, whereas (4a) is based on the knowledge of the parameters of the system. Figure 2b shows the evolution of the ratio (4b) and more precisely explains that the system has a time constant taking one of the two values $\{a_1 = 0.95,\ a_2 = -0.90\}$ (the phase portrait of figure 2c also confirms this result) ; moreover, it is possible to estimate accurately the switching time by considering the different jumps from a_1 to a_2 or conversely. Indeed, comparing (4a) and (4b) leads to:

$$\mu(k) = \frac{a(k) - a_1}{a_2 - a_1} \tag{5}$$

Without other knowledge, it is however impossible to explain the origin of the switching and particularly the variable which causes the different switches. However, in some cases, if it is assumed that the switch is due to a known variable, one may try to express this dependency and to fit μ with the input for example.

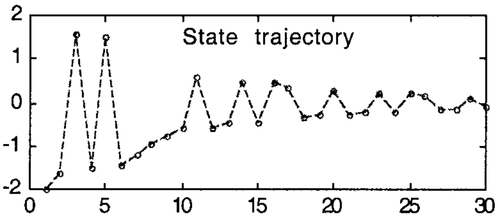


Figure 2a. Measurements versus time

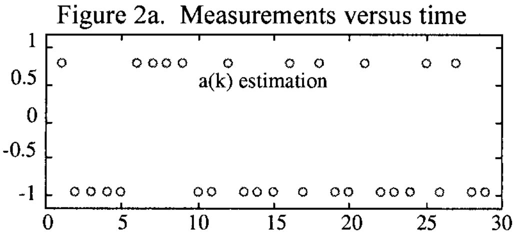


Figure 2b. Ratio $y(k) / y(k-1)$ versus time

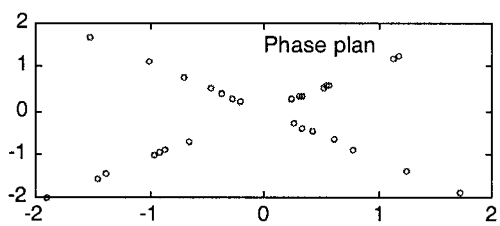


Figure 2c. Phase portrait $y(k), y(k-1)$

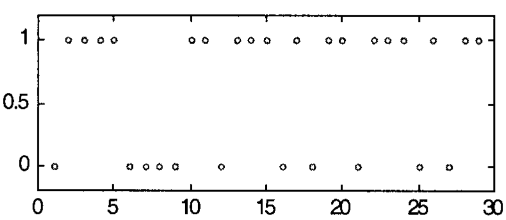


Figure 2d. Estimation of $\mu(k)$

This academic example was mainly used to introduce the different points of interest: the estimation of the parameters of the local models, the estimation of the switching function, the determination of the switching instants. In the rest of the paper, a general approach allowing the characterisation of switching systems is considered.

4. OBSERVER DESIGN

In the following, the plant is characterized by inputs (u,z), output (y) and state (x,q) in which (u,x) denote the variables associated to the continuous part of the process and (z,q) the variables associated to the changing regime. The discrete variables $\{q_1, \ldots, q_r\}$ constitutes the set of locations, each location being associated with a particular regime and a local model:

$$x(k+1) = A_{q(k)} x(k) + Bu(k)$$
$$y(k) = Cx(k)$$
$$A_{q(k)} \in \{A_\omega,\ \omega \in I\} \qquad I = \{1, 2, .., r\} \tag{6}$$

The particular mode q at any given time instant may be the result of a decision-making process. Here, such process is represented by a switching law of the form:

$$q(k+1) = \delta(z(k), q(k))$$
$$x \in \Re^n,\ u \in \Re^p,\ y \in \Re^m$$

Thus at each time k the model assumed to be in effect throughout the plant is one of r possible models (the plant is in one of r modes). It should be noted (but this is not a strong restriction) that only the matrix A changes according to the functioning point of the process and B remains unchanged. $\delta(.)$ is the switching law that describes the logical events system dynamics.

As explained in the introductive section, the observer of a multi-model mainly consists of two parts: a location observer and a continuous observer. On a practical point of view, the two parts may be mixed in a whole procedure ; however, the presentation will be more clear by using this hierarchical analysis.
The location observer is devoted to the identification of the current local plant, i.e. those of the local plan which is active. It receives as inputs the plan input (u) and output (y) and provides the estimate of the discrete location of the local plant at the current time.
The continuous observer produces an estimation of the state continuous variables of the system under consideration. For that, it uses the location information given by the first level and constructs an estimate $\hat{x}$ of the plant continuous state that converges to x. The figure 3 gives the structure of the observer associated to the plant. In our approach, it is important to note that the discrete decision variable z is not directly used in the observer (however, its effects are taken into account through the output y).

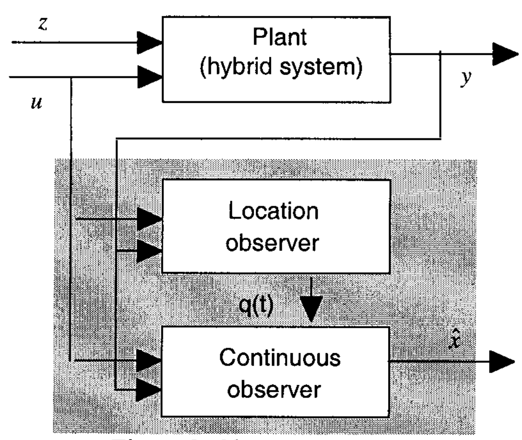


Figure 3. Observer structure

4.1 The location observer

The location observer received as inputs the continuous input and output of the plant. The idea is to design a sensitive residual in a set of known ones.

For that purpose, both discrete and continuous techniques will be employed. The continuous part uses a parity space equation generator while the discrete part is a selection procedure based on the analysis of the parity vector magnitude. Indeed, let us consider the state evolution on a time sliding window $[k-h,\ k]$. Let us define the observability, the controllability and the Toeplitz matrices:

$$\boldsymbol{O} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^h \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} A^h B & \dots & B \end{pmatrix}$$

$$\boldsymbol{T} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & & \\ \vdots & & & \\ CA^{h-1}B & CA^{h-2}B & \cdots & CB \end{pmatrix}$$

Thus, the initial state verifies:

$$\boldsymbol{O}x(k-h) = \varphi(y,u) \tag{7a}$$

with the definitions:

$$\varphi(y,u) = \begin{pmatrix} y(k-h) \\ \vdots \\ y(k) \end{pmatrix} - \boldsymbol{T}\begin{pmatrix} u(k-h) \\ \vdots \\ u(k-1) \end{pmatrix} \tag{7b}$$

That allows to estimate the initial state $x(k-h)$, uniquely, if and only if:

$$rank(\boldsymbol{O}) = rank(\boldsymbol{O} \quad \varphi(y,u)) \tag{8}$$

In fact, the situation is more complicated, matrix A taking different values as explained in definition (6). Considering a possible switching sequence of matrices (denoted by the subscript l which will be more clearly defined later), the previous matrices express as:

$$\boldsymbol{O}_l^{k,h} = \begin{pmatrix} C \\ CA_{i_{k-h,l}} \\ \vdots \\ \underbrace{CA_{i_{k-1,l}}A_{i_{k-2,l}}\cdots A_{i_{k-h,l}}}_{h} \end{pmatrix} \quad \boldsymbol{T}_l^{k,h} =$$

$$\begin{pmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & & \\ \vdots & & & \\ \underbrace{CA_{i_{k-1,l}}\cdots A_{i_{k-h+1,l}}}_{h-1}B & \underbrace{CA_{i_{k-1,k}}\cdots A_{i_{k-h+2,l}}}_{h-2}B & \cdots & CB \end{pmatrix}$$

In order to clarify the notations of the sequences of matrices let us define the l-th mode history (or sequence of models) through time k by the successive indexes taking by the matrix A:

$$I_l^{k,h} = \{i_{k-h,l},\ i_{k-h+1,l},\ \dots, i_{k-1,l}\} \tag{9a}$$
$$1 \le i_{\delta,l} \le r, \quad \delta = k-h,\dots,k-1$$

where l is the index of a particular sequence. Considering all the possible sequences (with eventually h switching on the time window $k-h{:}k$ and r possible matrices A), the set of model histories is defined by the indexes:

$$I^{k,h} = \{I_l^{k,h}\} \quad l = 1,\dots,r^h \tag{9b}$$

For example, with $r=2$, $k=4$ and $h=3$ one has the following $r^h = 8$ possible sequences:

l	1	2	3	4	5	6	7	8
$i_{1,l}$	1	1	1	1	2	2	2	2
$i_{2,l}$	1	2	2	2	1	1	2	2
$i_{3,l}$	1	2	1	2	1	2	1	2

The particular sequence $I_5^{4,5} = \{2,1,1\}$ resumes that at time $k=1$ the state matrix takes the value A_2, then a switch occurs and the matrix takes the value A_1 for $k=2$ and $k=3$.

In fact, among all the possible regimes (9b) only one has occurred on the given horizon $[k-h{:}k]$. Let us note $I_*^{k,h}$ this particular sequence ; our problem is to recognise this sequence among all the set $I^{k,h}$ knowing the input-output data of the process.

For example, if the system operates on the following way:

$$\begin{cases} & y_{k-3} = Cx_{k-3} \\ x_{k-2} = A_1 x_{k-3} + Bu_{k-3} & y_{k-2} = Cx_{k-2} \\ x_{k-1} = A_1 x_{k-2} + Bu_{k-2} & y_{k-1} = Cx_{k-1} \\ x_k = A_2 x_{k-1} + Bu_{k-1} & y_k = Cx_k \end{cases}$$

and knowing the data $\{y_{k-3}{:}y_k,\ u_{k-3}{:}u_{k-1}\}$, the realised sequence $\{A_1, A_1, A_2\}$ has to be recognized.

The problem now consists in finding the initial state $x(k-h)$ and the sequence of state matrices which are compatible with the input output set of data. Thus this may be considered as a mixed estimation problem involving the estimation of a sequence of integers and a vector of real values.

Assumption. All the local systems are observable:

$$rank\left(\boldsymbol{O}_l^{k,h}\right) = \dim(x) = n, \quad \forall h \ge n \tag{10}$$

Lemma. (Fredholm alternative)
Given $A \in \Re^{m \times n}$ and $b \in \Re^m$, there exists an $x \in \Re^n$ such that $Ax = b$ iff $b \in Ker\left(A^T\right)^{\perp}$. Moreover this condition may be expressed by the condition:

$$\left(I - AA^+\right)b = 0 \tag{11}$$

where A^+ is the (Moore-Penrose) pseudo-inverse of A such that $AA^+A = A$.

Theorem. The time sequence $I_*^{k,h}$ is the one satisfying the condition

$$\left(I - \boldsymbol{O}_*^{k,h}\left(\boldsymbol{O}_*^{k,h}\right)^+\right)\varphi(u,y) = 0 \tag{12}$$

4.2 *The continuous observer*

The true sequence being identified, let us remember that, on the horizon $[k-h:k]$, we have:

$$y_{k-h:h} = \boldsymbol{O}_*^{k,h} x_{k-h} + \boldsymbol{T}_*^{k,h} u_{k-h:k-1}$$

where $y_{k-h:k}$ and $u_{k-h:k-1}$ are respective concatenation of the given output and input. The initial state will be estimated using the data available on the horizon $[k-h:k-1]$ and for that a least square technique is used:

$$\hat{x}_{k-h} = P_*^{k,h}\left(y_{k-h:k} - \boldsymbol{T}_*^{k,h} u_{k-h:k-1}\right) \tag{13}$$

with $P_*^{k,h} = \left(\left(\boldsymbol{O}_*^{k,h}\right)^T \boldsymbol{O}_*^{k,h}\right)^{-1} \left(\boldsymbol{O}_*^{k,h}\right)^T$

Clearly, this estimate corresponds to those established is the context of finite memory observer (Medvedev, 1998), (Bousghiri, 1994). Using the state equation, (6) allows to give the estimation of the state at the end of the time window:

$$\hat{x}_k = \left(A_{i_{k-1},*} \cdots A_{i_{k-h},*}\right)\hat{x}_{k-h} + \boldsymbol{C} u_{k-h:k-1} \tag{14}$$

It should be noted, that at the next sampling instant $k+1$ (13) and (14) allows to express the new estimated. Indeed, considering the expressions $\hat{x}_k$ (or $\hat{x}_{k-h}$) may be a temptation to compute the current estimate $\hat{x}_{k+1}$ based on the previous estimate $\hat{x}_k$. This can be easily be performed for linear systems (Bousghiri, 1994), but, in the present case the switching of the state matrix does not allow to establish such recursive solution.

4.3 *Some comments*

Initial window and moving constant length window: considering definition (9) we have to note that the number of histories increases exponentially with time (r^h possible sequences). Thus, for the first time window $[1:h+1]$ the identification consists in testing the existence of one sequence among r^h sequences. However, the situation is more simpler for the next window $[2:h+2]$. Indeed, more generally the realised sequence on $[k-h+1:k+1]$ depends on those realised on $[k-h:k]$:

$$I_*^{k+1,h} = \left\{I_*^{k,h-1},\ \sigma_{k+1}\right\} \qquad 1 \le \sigma_{k+1} \le r \tag{15}$$

where σ_{k+1} is the index of the state matrix at time $k+1$. Thus, on the new window only the index σ_{k+1} has to be identified.

Noise influence: the test given in (8) or (12) is efficient only for a free noise system. When input and output signals are corrupted by noise, so is $\varphi(u,y)$ and the condition (12) is never strictly satisfied. Under those circumstances, we propose to evaluate the "residual" of eq. (12):

$$r_l^{k,h} = \left(I - \boldsymbol{O}_l^{k,h}\left(\boldsymbol{O}_l^{k,h}\right)^+\right)\varphi(u,y) \quad l = 1,..,r^h \tag{16}$$

Thus, the realised sequence correspond to the smaller residual and is determined by:

$$* = \arg\min_{l=1,..,r^k} r_l^{k,h} \tag{17}$$

Following (8), another solution is to analyse the rank of the matrix:

$$R_l^{k,h} = \left(\boldsymbol{O}_l^{k,h} \quad \varphi(y,u)\right) \quad l = 1,..,r^h \tag{18}$$

With free noise data, this matrix has a degenerate rank according its dimension, i.e. its minimum eigenvalue is zero. Therefore, with noisy data, this eigenvalue is not null, but a good test consists in computing the conditioning factor of his matrix:

$$\tau_l^{k,h} = \lambda_{\max}\left(R_l^{k,h}\right) / \lambda_{\min}\left(R_l^{k,h}\right) \quad l = 1,..,r^h \tag{19}$$

and to select the sequence corresponding to the greater factor.

5. EXAMPLE

Consider the following numerical values for a second order system, with two locations

$$A_1 = \begin{pmatrix} 0.45 & 0.00 \\ 0.00 & 0.40 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0.25 & -0.20 \\ 0.04 & 0.40 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.30 \\ 0.40 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 1 \end{pmatrix}$$

The length time window is $h = 2$ and therefore the set of possible indexes is:

$$I^{k,2} = \left\{I_1^{k,2},\ I_2^{k,2},\ I_3^{k,2},\ I_4^{k,2}\right\}$$

$$I_1^{k,2} = \{1,1\} \quad I_2^{k,2} = \{1,2\} \quad I_3^{k,2} = \{2,1\} \quad I_4^{k,2} = \{2,2\}$$

For state estimation, the sets of matrices of interest are

$$R_{i_1,i_2}^{k,2} = \begin{pmatrix} y_{k-2} & C \\ y_{k-1} - CBu_{k-2} & CA_{i_1} \\ y_k - CA_{i_2}Bu_{k-2} - CBu_{k-1} & CA_{i_2}A_{i_1} \end{pmatrix}$$

$$O_{i_1,i_2}^{k,2} = \begin{pmatrix} C \\ CA_{i_1} \\ CA_{i_2}A_{i_1} \end{pmatrix} \quad 1 \le i_1 \le 2 \quad 1 \le i_2 \le 2$$

Results are summarised in figure 4 (input, output, true state), 5 (true and estimated states) and 6 (estimated switches). The vertical dashed lines on figure 5 mark the time instants where switching occur. Figure 6 shows that the true switching have been exactly determined ; the different sub-figures visualise the successive sequence patterns (A_1, A_1), (A_1, A_2),

(A_2, A_1) and (A_2, A_2). As it is understandable after sequence (A_1, A_1) there are two possible sequences: (A_1, A_1) or (A_1, A_2), the first one if the state matrix does not change and the second one if a switch occurs. Analogous comment deals with other sequences.

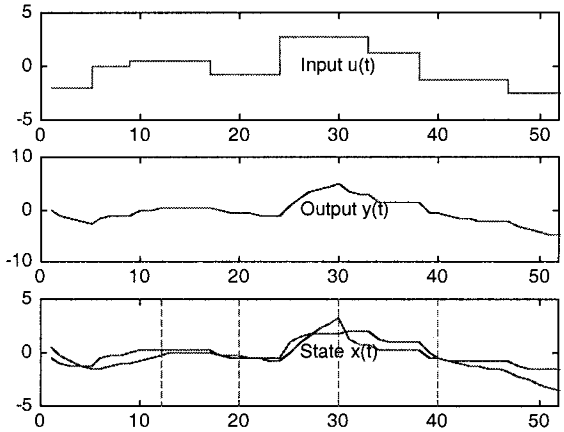


Figure 4. Input, output and state

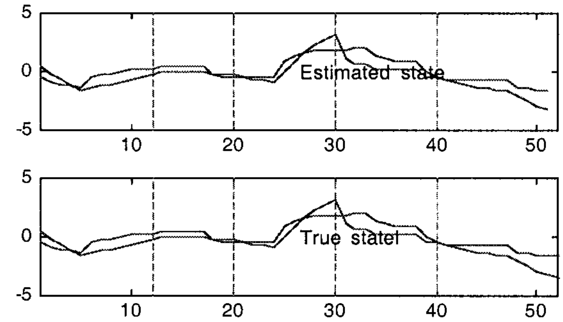


Figure 5. True and estimated states

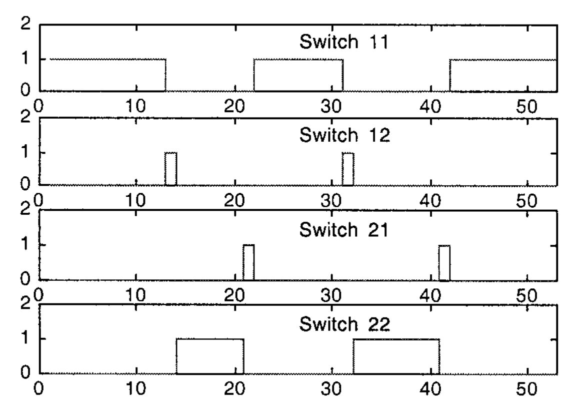


Figure 6. Switching

6. CONCLUSION

We have proposed, based on a residual analysis, the design an observer for a multi-model system. The two fundamental aspects of the design have been considered: the first part of the observer is dedicated to the location, while the second part is devoted to the state estimation.

However, it is necessary to keep in mind, that hybrid estimation and other multi-model estimation schemes have in common that they require models that are 'close' mathematical description of the system to diagnose. They can fail or be imprecise whenever unforeseen situations occur. As a consequence, models must be providedf or each situation or operational mode. Moreover, the parameters of the model have to be known or correctly estimated. Consequently a natural development of the proposed procedure would be: first, to analyse the sensitivity of the observer results in respect to model parameters, second, to improve the robustness of the observers.

Furthermore, following some ideas given in (Narasimhan, 2000) and (McIlraith, 2000), application of such estimations would be developped in the field of process diagnosis.

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