

Robust PI observer design for hybrid heterogeneous multiple models with delayed measurements

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Abstract: This paper deals with the problem of state estimation for hybrid systems modelled via multiple models and subject to delayed measurements. In contrast to the most popular results found in the multiple model literature, we consider heterogeneous multiple models, also known as *decoupled multiple model*, which allow to use submodels whose state space dimension can be different. On the basis of this multiple model a proportional-integral observer (PIO) is designed in order to cope with the robust state estimation with respect to disturbances. Sufficient delay-independent conditions for ensuring robust performances (attenuation level) and dynamic performances (exponential convergence) of the estimation error are provided in terms of LMIs using the Lyapunov-Krasovskii method. The validity of the proposed methodology is illustrated by an academic example.

Keywords: hybrid heterogeneous multiple models; state estimation; proportional-integral observer; delayed measurements

1. INTRODUCTION

Context. Time-delay, also known as dead-time, appears frequently in several real-world processes, such as chemical and thermal processes, electrical and communications networks, etc. Consequently, in many industrial applications time-delays must unavoidably be considered as an inherent part of the system and they must be accounted for in the mathematical model of the system under investigation (Smith and Corripio, 1985). Therefore, considerable attention has been paid in the past decades to systems with delays and appropriate theoretical tools, in the frequency and in the time domain, for the analysis, control and state estimation are well established (Bliman, 2001; Sename, 2001; Richard, 2003). Among them, the well known *Lyapunov-Krasovskii* functional method is largely adopted for the stability analysis of linear or nonlinear delayed systems. This method provides generally sufficient conditions in a LMI form which can be solved in polynomial-time using appropriated standard convex optimization algorithms (Boyd et al., 1994).

On the other hand, complex industrial systems generally present a nonlinear dynamic behaviour in particular when the whole operating range (*global behaviour*) must be considered instead of a reduced operating range in the neighbourhood of an operating point (*local behaviour*). This situation is commonly encountered in the state estimation problem where all admissible trajectories of the system must be taken into consideration. Hence, adequate nonlinear model representations must be used to cope

effectively with the global modelling and the state estimation of these systems. A number of quite diverse similar modelling frameworks such as hybrid systems, fuzzy systems, switched systems, multiple models, etc. are proposed to dealing with the modelling of systems that evolves according to different dynamics in the operating space.

Tools and proposed method. The nonlinear behaviour of a system can accurately be represented using the multiple model approach (Murray-Smith and Johansen, 1997; Boukhris et al., 1999). Multiple models have been recognized as a popular and a powerful modelling tool of complex nonlinear systems. In this approach the operating space of the system under investigation is decomposed into a number of operating zones, each of them being characterised by a *submodel* of reduced dimension. The interpolation of submodels with the help of *weighting functions*, associated to each operating zone, allows the global representation of the nonlinear system. It should be noted that the hybrid character of multiple models is given by the interaction of a set of submodels according to a dynamic law which can be smooth (interpolation law) as well as crisp (switching law).

At this point, it is important to point out that the interpolation of submodels can be operated using many architectures (Filev, 1991; Gregorcic and Lightbody, 2008), among them two main architectures can be distinguished: via an interpolation of parameters (e.g. *Piecewise Affine Systems* or *Takagi-Sugeno multiple model*) or via an interpolation of outputs (e.g. *decoupled multiple model*). Many

contributions concern the analysis, the control and the state estimation of the first multiple model realisation (with or without delays) (Murray-Smith and Johansen, 1997; Babuska, 1998; Tanaka and Wang, 2001; Lin et al., 2007; Yan et al., 2008), much less are devoted to observer design using decoupled multiple models. In this work, the latter realisation will be employed for dealing with the problem of state estimation for nonlinear hybrid systems with delayed measurements as an extension of the ideas proposed previously (Orjuela et al., 2007, 2008).

Related work on decoupled multiple model. Uppal et al. (2006) have shown a successful implementation of this structure in order to cope with the state estimation for fault detection and isolation. However, only a note of the convergence estimation error is proposed in this work and neither any analytic proof of the convergence of the estimation error is given, nor the time delay is taken into account. On the other hand, the state estimation in presence of delayed measurements is investigated in Orjuela et al. (2007). Sufficient conditions, under a LMI form, are given for ensuring the state estimation error convergence towards zero. However, the robust state estimation with respect to disturbances (e.g. noise) acting on the outputs has not been addressed in this work.

In order to cope with this problem, we introduce the so-called *proportional-integral observer* (PIO) which provides very interesting robustness properties with respect to perturbations (Beale and Shafai, 1989; Weinmann, 1991; Busawon and Kabore, 2001). This observer is characterised by the use of two corrective injection terms, proportional and integral, instead of the conventional proportional correction frequently employed in the Luenberger observer and/or in its classical extensions. The PIO has also been successfully employed in the synchronization of chaotic systems (Hua and Guan, 2005). The extension of the PIO design, based on dissipativity framework, to a particular nonlinear system whose non linearity is assumed to satisfy a sector bounded constraint, has been recently proposed in Jung et al. (2007). However, the PIO design for standard delayed linear systems and for delayed multiple models (Takagi-Sugeno and/or decoupled multiple models) seems poorly investigated in the literature.

Contribution and paper organisation. In this communication, the robust PIO design for decoupled multiple models with delayed measurements is investigated. The outline of this paper is as follows. A brief description of the decoupled multiple model is presented in section 2. The state estimation problem is presented in section 3 and sufficient conditions, on the basis of the Lyapunov-Krasovskii method, are established in order to ensure the stability of the PIO. Finally, in section 4, an academic example illustrates the state estimation of a decoupled multiple model.

Notations: The following standard notations will be used throughout the paper: $P > 0$ ($P < 0$) denotes a positive (negative) definite matrix P ; X^T denotes the transpose of matrix X ; I is the identity matrix of appropriate dimension and $\text{diag}\{\}$ is a block diagonal matrix of appropriate dimension. The \mathcal{L}_2 -norm of a signal, quantifying its energy is denoted and defined by $\|e\|_2^2 = \int_0^\infty e^T(t)e(t)dt$.

We shall simply write $\mu_i(\xi(t)) = \mu_i(t)$ and $x(t - \tau(t)) = x(\nabla)$ where $\tau(t) > 0$ is a variable time-delay.

2. DECOUPLED MULTIPLE MODEL

The structure of the decoupled multiple model can be viewed as a parallel interconnection of several submodels via a weighted sum of their outputs (Gatzke and Doyle III, 1999). In other words, as a particular class of hybrid heterogeneous models. By considering a state space representation, this structure takes the following form (Orjuela et al., 2008):

$$\dot{x}_i(t) = A_i x_i(t) + B_i u(t) + D_i w(t) , \quad (1a)$$

$$y_i(t) = C_i x_i(t) , \quad (1b)$$

$$z(t) = \sum_{i=1}^L \mu_i(\xi(t)) y_i(t) , \quad (1c)$$

$$y(t) = z(t - \tau(t)) + W w(t) . \quad (1d)$$

where $x_i \in \mathbb{R}^{n_i}$ and $y_i \in \mathbb{R}^p$ are respectively the state vector and the output of the i^{th} submodel; $u \in \mathbb{R}^m$ is the multiple model input, $z \in \mathbb{R}^p$ is the multiple model output and $y \in \mathbb{R}^p$ the measured output subject to a variable time-delay $\tau(t)$ and a perturbation $w \in \mathbb{R}^r$. The matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m}$, $D_i \in \mathbb{R}^{n_i \times r}$, $C_i \in \mathbb{R}^{p \times n_i}$ and $W \in \mathbb{R}^{p \times r}$ are known and constant.

Assumption 1. The perturbation is bounded energy signal, i.e. $\|w\|_2^2 < \infty$.

The variable time-delay $\tau(t)$ acting on the output is known and modelled by a continuous function that satisfies the following conditions (Wu et al., 2004):

$$0 \leq \tau(t) \leq \tau_{max} \quad \text{and} \quad \dot{\tau}(t) \leq \beta < 1 \quad (2)$$

where τ_{max} and β are constant. This time-delay can be due for example to the inherent nature of the system, the sensor reactivity or the transmission data time-delay.

The current operating point of the system is taken into account by means of the so-called *decision variable* $\xi(t)$ which is a real-time accessible variable of the system (e.g. inputs, outputs and/or other measured variable). The relative contribution of each submodel to the global model according to the current operating point of the system is quantified by the *weighting functions* $\mu_i(\xi(t))$ (i.e. the interpolation mechanism). They are associated to each operating zone and satisfy the following convex sum properties:

$$\sum_{i=1}^L \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \leq \mu_i(\xi(t)) \leq 1, \forall i = 1 \dots L, \forall t. \quad (3)$$

In this multiple model the contribution of each submodel is taken into account via a weighted sum of the submodel outputs. Consequently, *heterogeneous submodels* (i.e. submodels of different dimensions) can be considered in the modelling stage as a supplementary degree of flexibility and generality. Therefore, this structure is well suited for modelling nonlinear hybrid systems whose structure varies with the operating regime, for example when the complexity of the dynamic behaviour is not uniform across the operating space (Gregorcic and Lightbody, 2008).

Notice that the multiple model approach is an efficient way to deal with hybrid systems: no assumption is made on the continuity of the weighting functions blending the submodel outputs. Consequently, switches between submodels can be considered with the help of discontinuous and even non differentiable weighting functions taking boolean values in the set $[0, 1]$.

3. STATE ESTIMATION PROBLEM

In a previous work (Orjuela et al., 2007), the state estimation of a decoupled multiple model subject to delayed measurements has been investigated on the basis of a proportional observer given by:

$$\dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i u(t) + K_i(y(t) - \hat{y}(t)) , \quad (4a)$$

$$\hat{z}(t) = \sum_{i=1}^L \mu_i(\xi(t)) C_i \hat{x}_i(t) , \quad (4b)$$

$$\hat{y}(t) = \hat{z}(t - \tau(t)) , \quad (4c)$$

where $K_i \in \mathbb{R}^{n_i \times p}$ is the gain associated to the i^{th} observer. Sufficient conditions for ensuring the exponential convergence towards zero of the estimation errors $e_i(t) = x_i(t) - \hat{x}_i(t)$ have previously been established (Orjuela et al., 2007). Note however that disturbances acting on the system are not taken into consideration. Consequently, the sensitivity problem of the state estimation with respect to disturbances has not been addressed in this work. On the other hand, the proportional observer (4) offers only one degree of freedom K_i for reducing the influence of the disturbance and providing at the same time good dynamic performances (two antagonist design goals).

In this paper, the proportional observer (4) is replaced by a more general observer, known as proportional-integral observer (PIO), which provides very interesting robustness properties with respect to disturbances. In particular, a good trade-off between dynamic and robust performances of the observer can be obtained with the PIO because two degrees of freedom are available for the observer design.

3.1 Preliminaries

For the simplicity of manipulations, the decoupled multiple model (1) is rewritten in the following compact form (Orjuela et al., 2007, 2008):

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + \tilde{D}w(t) , \quad (5a)$$

$$z(t) = \tilde{C}(t)x(t) , \quad (5b)$$

$$y(t) = z(\nabla) + Ww(t) , \quad (5c)$$

where:

$$x(t) = [x_1^T(t) \cdots x_i^T(t) \cdots x_L^T(t)]^T \in \mathbb{R}^n, n = \sum_{i=1}^L n_i , \quad (6a)$$

$$\tilde{A} = \text{diag}\{A_1 \cdots A_i \cdots A_L\} \in \mathbb{R}^{(n \times n)} , \quad (6b)$$

$$\tilde{B} = [B_1^T \cdots B_i^T \cdots B_L^T]^T \in \mathbb{R}^{(n \times m)} , \quad (6c)$$

$$\tilde{D} = [D_1^T \cdots D_i^T \cdots D_L^T]^T \in \mathbb{R}^{(n \times r)} , \quad (6d)$$

$$\tilde{C}(t) = [\mu_1(t)C_1 \cdots \mu_i(t)C_i \cdots \mu_L(t)C_L] \in \mathbb{R}^{(p \times n)} . \quad (6e)$$

The matrix $\tilde{C}(t)$ is time-varying because the weighting functions are taken into consideration in this matrix.

3.2 Proportional-integral observer structure

In a PIO, the classic used proportional action given by the gains K_i in (4) is replaced by the use of two correction actions: proportional and integral. For this purpose, a supplementary integral variable $y_I(t) = \int_0^t y(\eta) d\eta$ must be taken into consideration in the PIO architecture. Note however that this integration can be replaced by a more general action, for example, by considering the filtered output signal as follows:

$$\dot{y}_I(t) = Ny_I(t) + y(t) \quad (7)$$

where the matrix N is a *fading effect* coefficient matrix that regulates the transient response of $y_I(t)$ (Jung et al., 2007).

Let us notice that the equations (5) and (7) can be gathered as follows:

$$\begin{aligned} \dot{x}_a(t) = & \left(\bar{C}_1 \tilde{A} \bar{C}_1^T + \bar{C}_2 N \bar{C}_2^T \right) x_a(t) + \bar{C}_2 \tilde{C}(\nabla) \bar{C}_1^T x_a(\nabla) \\ & + \bar{C}_1 \tilde{B} u(t) + \left(\bar{C}_2 W + \bar{C}_1 \tilde{D} \right) w(t) , \end{aligned} \quad (8a)$$

$$y(t) = \tilde{C}(\nabla) \bar{C}_1^T x_a(\nabla) + W w(t) , \quad (8b)$$

$$y_I(t) = \bar{C}_2^T x_a(t) \quad (8c)$$

where the augmented vector $x_a \in \mathbb{R}^{(n+p)}$ and the constant bloc matrices \bar{C}_1 and \bar{C}_2 of appropriate dimensions are respectively given by:

$$x_a(t) = \begin{bmatrix} x(t) \\ y_I(t) \end{bmatrix}, \bar{C}_1 = \begin{bmatrix} I_{(n \times n)} \\ 0_{(p \times n)} \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} 0_{(n \times p)} \\ I_{(p \times p)} \end{bmatrix} . \quad (9)$$

The structure of the proposed PIO can be obtained on the basis of the augmented form (8):

$$\begin{aligned} \dot{\hat{x}}_a(t) = & \left(\bar{C}_1 \tilde{A} \bar{C}_1^T + \bar{C}_2 N \bar{C}_2^T \right) \hat{x}_a(t) + \bar{C}_2 \tilde{C}(\nabla) \bar{C}_1^T \hat{x}_a(\nabla) \\ & + \tilde{K}_P(y(t) - \hat{y}(t)) + \tilde{K}_I(y_I(t) - \hat{y}_I(t)) + \bar{C}_1 \tilde{B} u(t) , \end{aligned} \quad (10a)$$

$$\hat{y}(t) = \tilde{C}(\nabla) \bar{C}_1^T \hat{x}_a(\nabla) , \quad (10b)$$

$$\hat{y}_I(t) = \bar{C}_2^T \hat{x}_a(t) . \quad (10c)$$

where the gains \tilde{K}_P and \tilde{K}_I must be designed and the matrices \tilde{A} , \tilde{B} and $\tilde{C}(t)$ have been previously defined in (6). Let us notice that an additional injection term, given by the filtered (or integral) output estimation error $y_I(t) - \hat{y}_I(t)$, is included in the dynamic equation of the observer. Hence some degrees of freedom are in this way introduced for the observer design. The use of the two gains \tilde{K}_P (proportional action) and \tilde{K}_I (integral action) is at the origin of the name Proportional-Integral Observer.

3.3 Problem formulation

The dynamics of the estimation error

$$e_a(t) = x_a(t) - \hat{x}_a(t) \quad (11)$$

can be easily established by considering the gathered forms of the decoupled multiple model (8) and the PIO (10):

$$\dot{e}_a(t) = \Lambda_1 e_a(t) + \Lambda_2(\nabla) e_a(\nabla) + \Lambda_3 w(t) , \quad (12)$$

where

$$\Lambda_1 = \bar{C}_1 \tilde{A} \bar{C}_1^T + \bar{C}_2 N \bar{C}_2^T - K_I \bar{C}_2^T, \quad (13a)$$

$$\Lambda_2(\nabla) = (\bar{C}_2 - K_P) \tilde{C}(\nabla) \bar{C}_1^T, \quad (13b)$$

$$\Lambda_3 = (\bar{C}_2 - K_P) W + \bar{C}_1 \tilde{D}. \quad (13c)$$

Let us remark, from equations (13a) and (13c), that the proportional gain K_P can be used to modulate the influence of the disturbances on the estimation error $e_a(t)$ and the observer dynamics can be improved with the help of the integral gain K_I .

Remark 1. Note, from equation (13c), that the disturbance $w(t)$ can be totally decoupled (i.e. its influence is null) if and only if $K_P W = \bar{C}_2 W + \bar{C}_1 \tilde{D}$. Notice that

$$\begin{bmatrix} \tilde{D} \\ W \end{bmatrix} = K_P W = \begin{bmatrix} K_{P1} \\ I \end{bmatrix} W \quad (14)$$

if $\text{rank}(W) = p$, the gain K_{P1} can be computed as $K_{P1} = \tilde{D} W^T (W W^T)^{-1}$. In the other case total decoupling cannot be achieved. Note however that the total disturbance decoupling case reduce inevitably the freedom degrees to adjust the dynamic performances of the observer.

The robust PIO design problem can thus be formulated as finding the matrices \tilde{K}_P and \tilde{K}_I such that the influence of $w(t)$ on the estimation error (11) is attenuated on the one hand and the state estimation error remains globally bounded for any blend between the submodels on the other hand. To this end, the following *objective signal* $\nu(t)$ which only depends on the estimation error $e_a(t)$ is introduced:

$$\nu(t) = H e_a(t), \quad (15)$$

where H is a matrix of appropriate dimension chosen by the designer. The matrix H in (15) can be selected for totally or partially taking into consideration the transfer from $w(t)$ to $e_a(t)$.

Finally, the expected robust performances of the PIO can be formulated by the following constraints:

$$\lim_{t \rightarrow \infty} e_a(t) = 0 \text{ for } w(t) = 0, \quad (16a)$$

$$\| \nu \|_2^2 \leq \gamma^2 \| w \|_2^2 \text{ for } w(t) \neq 0 \text{ and } \nu(0) = 0, \quad (16b)$$

where γ is the \mathcal{L}_2 gain from $w(t)$ to $\nu(t)$ to be minimized. Notice that condition (16a) ensures convergence towards zero of the estimation error in the perturbation free case. On the other hand, condition (16b) guarantees attenuation level on the estimation error with respect to perturbations. In the following, we will investigate the exponential convergence of the estimation error. Indeed, the exponential convergence is a strong form of convergence that guarantees dynamic performances of the estimation error, in particular a convergence velocity via a decay rate.

3.4 Computation of the gains \tilde{K}_P and \tilde{K}_I

This section focuses on the computation of the PIO gains \tilde{K}_P and \tilde{K}_I . For this purpose, the Lyapunov-Krasovskii functional method is employed in order to provide delay-independent sufficient conditions, in LMIs terms (Boyd et al., 1994), for ensuring the robust performances (16).

Theorem 1. Consider the decoupled multiple model (1), the time-delay conditions (2) and assumption 1. There exists a PIO (10) ensuring the objectives (16) if there exists two symmetric, positive definite matrices P and Q , two matrices G_P and G_I and a positive scalar $\bar{\gamma}$ solution of the constrained optimisation problem:

$$\min \bar{\gamma} \quad \text{subject to} \quad \begin{bmatrix} H^T H + \bar{\Lambda}_1^T P + P \bar{\Lambda}_1 + Q & \bar{\Lambda}_{2,i} & \bar{\Lambda}_3 \\ \bar{\Lambda}_{2,i}^T & -(1-\beta)e^{-2\alpha\tau_{max}} Q & 0 \\ \bar{\Lambda}_3^T & 0 & -\bar{\gamma} I \end{bmatrix} < 0$$

for $i = 1 \dots L$ where

$$\bar{\Lambda}_1 = \bar{C}_1 \tilde{A} \bar{C}_1^T + \bar{C}_2 N \bar{C}_2^T - G_I \bar{C}_2^T + \alpha I,$$

$$\bar{\Lambda}_{2,i} = (P \bar{C}_2 - G_P) \tilde{C}_i \bar{C}_1^T,$$

$$\bar{\Lambda}_3 = (P \bar{C}_2 - G_P) W + P \bar{C}_1 \tilde{D},$$

$$\tilde{C}_i = [0 \dots C_i \dots 0].$$

for a given decay rate α and for prescribed matrices H and N . The observer gains are given by $K_P = P^{-1} G_P$ and $K_I = P^{-1} G_I$; the \mathcal{L}_2 gain from $w(t)$ to $\nu(t)$ is given by $\gamma = \sqrt{\bar{\gamma}}$.

Proof. The exponential convergence of the estimation error (11) is investigated via a Lyapunov-Krasovskii functional as proposed by Mondié and Kharitonov (2005):

$$V(t) = e_a^T(t) P e_a(t) + \int_{-\tau(t)}^0 e_a^T(t+\theta) e^{2\alpha\theta} Q e_a(t+\theta) d\theta \quad (17)$$

where P and Q are symmetric, positive definite matrices. Notice that Mondié and Kharitonov (2005) propose this functional in order to provide sufficient delay-independent conditions for the exponential stability of linear systems with constant time-delay (i.e. $\tau(t) = \tau$). Here, we will use the same functional but a time-varying delay is considered.

It should be noted that the Lyapunov-Krasovskii functional (17) can be rewritten as follows:

$$V(t) = \begin{bmatrix} e_a(t) \\ e_a(\nabla) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_a(\nabla) \end{bmatrix} + \int_{-\tau(t)}^0 e_a^T(t+\theta) e^{2\alpha\theta} Q e_a(t+\theta) d\theta. \quad (18)$$

On the other hand, the robust state estimation objectives (16) are guaranteed if there exists a Lyapunov-Krasovskii functional (17) such that (Boyd et al., 1994):

$$\dot{V}(t) + 2\alpha V(t) < \gamma^2 w^T(t) w(t) - \nu^T(t) \nu(t) \quad (19)$$

where α is the so-called decay rate for convergence velocity and γ is the attenuation level from $w(t)$ to $\nu(t)$ for robust estimation. So, inequality (19) must be ensured in order to provide a solution of the robust state estimation problem.

Let us consider initially the time-derivative of the functional (17) evaluated via the Leibniz-Newton formula:

$$\begin{aligned}\dot{V}(t) &= \dot{e}_a^T(t) P e_a(t) + e_a^T(t) P \dot{e}_a(t) + e_a^T(t) Q e_a(t) \\ &\quad - (1 - \dot{\tau}(t)) e^{-2\alpha\tau(t)} e_a^T(\nabla) Q e_a(\nabla) \\ &\quad - 2\alpha \int_{-\tau(t)}^0 e_a^T(t + \theta) e^{2\alpha\theta} Q e_a(t + \theta) d\theta\end{aligned}\quad (20)$$

that can be upper bounded, using the *a priori* knowledge (2) of the time-delay as follows:

$$\begin{aligned}\dot{V}(t) &\leq \dot{e}_a^T(t) P e_a(t) + e_a^T(t) P \dot{e}_a(t) + e_a^T(t) Q e_a(t) \\ &\quad - (1 - \beta) e^{-2\alpha\tau_{max}} e_a^T(\nabla) Q e_a(\nabla) \\ &\quad - 2\alpha \int_{-\tau(t)}^0 e_a^T(t + \theta) e^{2\alpha\theta} Q e_a(t + \theta) d\theta,\end{aligned}\quad (21)$$

which becomes finally by considering the dynamics of the estimation error (12):

$$\begin{aligned}\dot{V}(t) &\leq -2\alpha \int_{-\tau(t)}^0 e^T(t + \theta) e^{2\alpha\theta} Q e(t + \theta) d\theta \\ &\quad + w^T(t) \Lambda_3^T P e_a(t) + e_a^T(t) P \Lambda_3 w(t) \\ &\quad + \begin{bmatrix} e_a(t) \\ e_a(\nabla) \end{bmatrix}^T \begin{bmatrix} \Lambda_1^T P + P \Lambda_1 + Q & P \Lambda_2(\nabla) \\ \Lambda_2^T(\nabla) P & -(1 - \beta) e^{-2\alpha\tau_{max}} Q \end{bmatrix} \begin{bmatrix} e_a(t) \\ e_a(\nabla) \end{bmatrix}.\end{aligned}\quad (22)$$

Hence using (22), the left-hand side of the inequality (19) is bounded as follows:

$$\begin{aligned}\dot{V} + 2\alpha V &\leq \begin{bmatrix} e_a(t) \\ e_a(\nabla) \\ w(t) \end{bmatrix}^T \left\{ \begin{bmatrix} 2\alpha P & 0 & P \Lambda_3 \\ 0 & 0 & 0 \\ \Lambda_3^T P & 0 & 0 \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} \Lambda_1^T P + P \Lambda_1 + Q & P \Lambda_2(\nabla) & 0 \\ \Lambda_2^T(\nabla) P & -(1 - \beta) e^{-2\alpha\tau_{max}} Q & 0 \end{bmatrix} \right\} \begin{bmatrix} e_a(t) \\ e_a(\nabla) \\ w(t) \end{bmatrix}.\end{aligned}\quad (23)$$

Now, by considering (23) then the inequality (19) is guaranteed if the following inequality holds:

$$\begin{aligned}\begin{bmatrix} e_a(t) \\ e_a(\nabla) \\ w(t) \end{bmatrix}^T \left\{ \begin{bmatrix} \Lambda_1^T P + P \Lambda_1 + Q & P \Lambda_2(\nabla) & 0 \\ \Lambda_2^T(\nabla) P & -(1 - \beta) e^{-2\alpha\tau_{max}} Q & 0 \\ 0 & 0 & 0 \end{bmatrix} \right. \\ \left. + \begin{bmatrix} 2\alpha P + H^T H + \Lambda_1^T P + P \Lambda_1 + Q & 0 & P \Lambda_3 \\ 0 & 0 & 0 \\ \Lambda_3^T P & 0 & -\gamma^2 I \end{bmatrix} \right\} \begin{bmatrix} e_a(t) \\ e_a(\nabla) \\ w(t) \end{bmatrix} < 0.\end{aligned}\quad (24)$$

Let us notice that the above inequality is a quadratic form in $[e_a(t) \ e_a(\nabla) \ w(t)]^T$. Consequently, the inequality (19) for ensuring robust objectives (16) is finally guaranteed if the following inequality holds:

$$\begin{bmatrix} 2\alpha P + H^T H + \Lambda_1^T P + P \Lambda_1 + Q & P \Lambda_2(\nabla) & P \Lambda_3 \\ \Lambda_2^T(\nabla) P & -(1 - \beta) e^{-2\alpha\tau_{max}} Q & 0 \\ \Lambda_3^T P & 0 & -\gamma^2 I \end{bmatrix} < 0.\quad (25)$$

Note that a time-varying matrix $\Lambda_2(\nabla)$, given by (13b), appears in this inequality and therefore its resolution cannot be computed using standard LMI algorithms.

However, the time-delayed varying matrix $\tilde{C}(\nabla)$ in $\Lambda_2(\nabla)$ can be rewritten as a weighted sum of constant matrices as follows:

$$\tilde{C}(\nabla) = \sum_{i=1}^L \mu_i(\nabla) \tilde{C}_i, \quad (26)$$

where \tilde{C}_i is a constant block matrix given by:

$$\tilde{C}_i = [0 \ \cdots \ C_i \ \cdots \ 0] \quad (27)$$

such that the term C_i is found on the i^{th} block column of \tilde{C}_i . By considering (26) then the inequality (25) becomes

$$\sum_{i=1}^L \mu_i(\nabla) \begin{bmatrix} 2\alpha P + H^T H + \Lambda_1^T P + P \Lambda_1 + Q & P \tilde{\Lambda}_{2,i} & P \Lambda_3 \\ \tilde{\Lambda}_{2,i}^T P & -(1 - \beta) e^{-2\alpha\tau_{max}} Q & 0 \\ \Lambda_3^T P & 0 & -\gamma^2 I \end{bmatrix} < 0. \quad (28)$$

where the constant matrices $\tilde{\Lambda}_{2,i}$ are given by:

$$\tilde{\Lambda}_{2,i} = (\bar{C}_2 - K_P) \tilde{C}_i \bar{C}_1^T. \quad (29)$$

According to the properties (3) of the weighting functions, the above matrix inequality (28) can be satisfied by considering the simultaneous solution of the set of inequalities expressed for each upper bound of $\mu_i(\nabla)$. On the other hand, the linearisation of the matrix inequality (28) can be achieved by the standard change of variables $G_P = P \tilde{K}_P$, $G_I = P \tilde{K}_I$ and $\gamma^2 = \bar{\gamma}$. Hence, sufficient conditions for ensuring the robust performances (16) are obtained under a LMI form which can be solved via classical LMI tools. The proof of theorem 1 is completed. \square

4. SIMULATION EXAMPLE

Let us consider the state estimation of a hybrid heterogeneous multiple model with $L = 2$ submodels. The parameters of the submodels are:

$$\begin{aligned}A_1 &= \begin{bmatrix} -2 & 0.5 & 0.6 \\ -0.3 & -0.9 & 0 \\ -1.3 & 0.6 & -0.8 \end{bmatrix}, & A_2 &= \begin{bmatrix} -0.2 & -0.6 \\ 0.3 & -1 \end{bmatrix}, \\ B_1 &= [1 \ 0.2 \ 0.5]^T, & B_2 &= [-0.5 \ 0.8]^T, \\ D_1 &= [0.1 \ 0.2 \ -0.3]^T, & D_2 &= [-0.1 \ 0.1]^T, \\ C_1 &= \begin{bmatrix} 1 & 0.8 & 0.5 \\ 0.2 & 0.3 & -0.5 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & -0.5 \end{bmatrix}, \\ W &= [0.2 \ -0.3]^T.\end{aligned}$$

The weighting functions are obtained from normalised Gaussian functions given by:

$$\omega_i(\xi(t)) = \exp\left(-(\xi(t) - c_i)^2 / \sigma^2\right), \quad (30)$$

$$\mu_i(\xi(t)) = \omega_i(\xi(t)) / \sum_{j=1}^L \omega_j(\xi(t)), \quad (31)$$

with $\sigma = 0.4$, $c_1 = -0.3$ and $c_2 = 0.3$. The decision variable $\xi(t)$ is the input signal $u(t) \in [-1, 1]$. Here, a sinusoidal input $u(t) = \sin(t)$ is considered in order to explore the operating space of each submodel.

The time-delay function appearing in the output of the systems is given by $\tau(t) = 0.5 + 0.45 \sin(0.5t)$ with $\tau_{max} = 0.95$. The upper bound of its derivative is $\beta = 0.225$.

For $H = I$, $N = -0.3$ and for a decay rate $\alpha = 0.2$, a solution that satisfies the theorem 1, is given by:

$$\begin{aligned}\tilde{K}_P &= \begin{bmatrix} 0.075 & 0.144 & -0.220 & -0.074 & 0.075 & 0.999 & 0.002 \\ -0.237 & -0.444 & 0.678 & 0.225 & -0.238 & 0.002 & 0.992 \end{bmatrix}^T, \\ \tilde{K}_I &= \begin{bmatrix} -0.431 & 0.139 & -0.068 & -0.054 & 0.046 & 2.269 & -0.045 \\ -0.141 & -0.434 & -0.114 & -0.022 & 0.011 & 0.042 & 2.252 \end{bmatrix}^T,\end{aligned}$$

with a minimal attenuation level $\gamma^2 = 0.5$.

In the simulation the disturbance acting on the output is a normally distributed random signal. The state estimation error is plotted in figure 1 (left) where e_i is the i^{th} component of the estimation error vector (11). The measured and the estimated outputs are shown in figure 1 (right). In both cases, the error around the origin time is due to the differences between initial conditions of the multiple model and the observer. The simulation results show the effectiveness of the proposed observer.

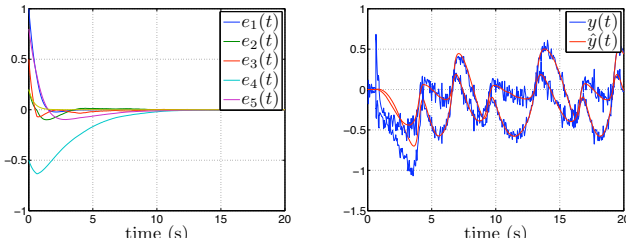


Figure 1. State estimation errors e_i (left). Actual and estimated outputs (right).

5. CONCLUSION

A proportional-integral observer (PIO), on the basis of a decoupled multiple model, is proposed in this contribution in order to cope with the state estimation problem of a nonlinear hybrid system in presence of delayed and perturbed measurements. The decoupled multiple model is an interesting structure for modelling nonlinear systems with variable structure because the dimensions of the employed submodels can be different. On the other hand, the PIO offers more degree of freedom for robust state estimation with respect to the classic proportional observer previously proposed. The robust stability problem of the estimation error is investigated using the Lyapunov-Krasovskii functional method and delay-independent sufficient conditions, under a LMI form, are established.

Further research will be to investigate the conservatism of the proposed LMI conditions on the one hand and the state estimation by considering distributed delays in the submodel dynamic equations on the other hand.

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