State estimation for wastewater treatment plant with slow and fast dynamics using multiple models

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***Abstract*— This paper addresses the state estimation of two-time scale nonlinear systems by designing an unknown input multi-observer (UIMO). In order to design such an observer, the nonlinear system is transformed into an equivalent multiple model form and the fast dynamics are considered as unknown inputs. An application to a model of Wastewater Treatment Plants (WWTP) is considered and gives encouraging results.**

I. Introduction

Nonlinear models are often needed to represent real system behaviors. As a consequence, there is a need to extend linear methods to nonlinear systems (such as the observer synthesis), which is an a priori difficult problem. In order to overcome this difficulty, the concept of multiple model (MM) has received much attention in the last two decades. The MM structure gives the possibility to reduce the complexity of nonlinear systems, by constructing linear submodels aggregated using weighting functions [12]. A MM form can be obtained by applying a method proposed in [8] to represent nonlinear systems into an equivalent MM. Only the general steps of these technique are given here.

Real systems can have multiple time scale dynamics. In order to deal with such systems, the singularly perturbed theory is often used to highlight the systematic decomposition of the system into various scales of time. Nevertheless, it is not obvious to model a process under the standard singularly perturbed form.

The *first* difficult point is the separation of the slow and fast dynamics. In [2], [3] this separation is realized by comparing the kinetic parameters of the biological process. But, in a general nonlinear case this comparison is difficult to acheive. So, more general methods to identify different time scales were proposed in the literature ([11]). These methods are based on the evaluation of the jacobian eigenvalues of the linearized system and will be used here.

After the separation of the multiple-time scale dynamics, the standard singularly perturbed form is obtained. In the limit case, this form has a dynamic part and a static part expressed by an algebraic system. Thus, a *second* difficult point is the resolution of the algebraic system which is not always a trivial problem. The method mainly used to deal with this problem is based on a change of coordinates ([2], [13]) requiring a linear transformation in order to eliminate the fast dynamic components. In order to be able to apply this method, the nonlinear system to be studied has to respect some structural constraints. However, not all nonlinear systems can be put under the proposed particular form, thus other methods of identification of the slow and fast modes must be implemented.

By considering the standard singularly perturbed system, an equivalent MM can be written. The classical MM form is slightly modified in order to separate the slow and the fast dynamics. The main contribution of this paper is to estimate the state variables of a multiple time scale nonlinear system. Due to the limited number of sensors, this is done by considering the fast varying state variables as unknown inputs, thus an unknown input multi-observer (UIMO) can be designed by using the MM singularly perturbed form. Most of the existing works are dedicated to MM with measurable decision variables (inputs / outputs). Unfortunately, in many practical situations these variables are not accessible. Only a few works [5], [6] are devoted to the case of unmeasurable decision variables. This last case will be treated here. The convergence conditions of the state and unknown input estimation error are expressed through LMIs (Linear Matrix Inequalities) by using the Lyapunov method and the $\mathscr{L}_2$ approach.

In the present paper, the MM structure and the singularly perturbed theory are used in order to deal with the complexity of an ASM1 (Activate Sludge Model 1)[10] describing a biological degradation process, which is characterized by two-time scale dynamics. Most of the previous works using the MM representation and dedicated to activated sludge systems were based on linearization techniques despite the drawbacks mentioned as follows: the loss of information and the delicate choice of different operating points or trajectories. In addition to that, the choice of the decision variables expressing the nonlinearities of the system still remains a delicate point.

In section II are given the essential tools for modeling nonlinear systems, in section III is presented the observer design. Section IV proposes a real application to WWTP.

II. Main tools for modeling

A. Multiple model representation

Generally, a dynamic nonlinear system can be described by the following ordinary differential equations:

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

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The measure equation is generally linear and time invariant since, in most practical situations, the sensors do not change according to the operating point.

The multiple model allows to represent nonlinear dynamic systems into a convex combination of linear submodels:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(x,u)\,[A_i x(t) + B_i u(t)] \\ y(t) &= C x(t)\end{aligned} \tag{2}$$

where $x \in \mathbb{R}^N$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^l$ the output vector, A_i, B_i, C are constant matrices of appropriate dimensions. The functions $\mu_i(x,u)$ represent the weights of the submodels $\{A_i, B_i, C\}$ in the global model and they have the following properties:

$$\sum_{i=1}^{r} \mu_i(x,u) = 1; \qquad \mu_i(x,u) \geq 0, \forall (x,u) \in \mathbb{R}^N \times \mathbb{R}^m$$

In order to obtain the MM form, a method giving an equivalent rewriting of the nonlinear system (1) is used (see [8] for further details).

Firstly the system (1) is transformed in a quasi-Linear Parameter Varying (quasi-LPV) form:

$$\begin{aligned}\dot{x}(t) &= A(x(t),u(t))\,x(t) + B(x(t),u(t))\,u(t) \\ y(t) &= C x(t)\end{aligned} \tag{3}$$

Secondly, some nonlinear entries of the matrices A and / or B are considered as "decision variables", or "premise variables" and denoted $z_j(x,u) (j = 1,...,q)$. Several choices of these premise variables are possible (for details in the selection procedure see [9]) due to the existence of different equivalent quasi-LPV forms.

Thirdly, a convex polytopic transformation is performed for all premise variables $(j = 1,...,q)$, as follows:

$$z_j(x,u) = F_{j,1}(z_j(x,u)) \cdot z_{j,1} + F_{j,2}(z_j(x,u)) \cdot z_{j,2} \tag{4}$$

where

$$\begin{aligned}z_{j,1} &= \max_{x,u} \{z_j(x,u)\} \\ z_{j,2} &= \min_{x,u} \{z_j(x,u)\}\end{aligned} \tag{5}$$

$$F_{j,1}(z_j(x,u)) = \frac{z_j(x,u) - z_{j,2}}{z_{j,1} - z_{j,2}} \tag{6a}$$

$$F_{j,2}(z_j(x,u)) = \frac{z_{j,1} - z_j(x,u)}{z_{j,1} - z_{j,2}} \tag{6b}$$

Remark 1. For q decision variables, $r = 2^q$ submodels will be obtained. By multiplying the functions F_{j,σ_i^j} the weighting functions are obtained:

$$\mu_i(x,u) = \prod_{j=1}^{r} F_{j,\sigma_i^j}(z_j(x,u)) \tag{7}$$

The indexes σ_i^j $(i = 1,...,2^q$ and $j = 1,...,q)$ are equal to 1 or 2 and indicates which partition of the j^{th} decision variable ($F_{j,1}$ or $F_{j,2}$) is involved in the i^{th} submodel.

The constant matrices A_i and B_i $(i = 1,...,2^q)$ are obtained by replacing the decision variables z_j in the matrices A and B, with the scalars defined in (5):

$$A_i = A(z_{1,\sigma_i^1},...,z_{q,\sigma_i^q}) \tag{8}$$

$$B_i = B(z_{1,\sigma_i^1},...,z_{q,\sigma_i^q}) \tag{9}$$

B. Singularly perturbed systems

The standard form of the singularly perturbed systems with two-time scales can be expressed by the following system:

$$\varepsilon \dot{x}_F(t) = f_F(x_S(t), x_F(t), u(t), \varepsilon) \tag{10a}$$

$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), \varepsilon) \tag{10b}$$

where $x_S \in \mathbb{R}^n$ and $x_F \in \mathbb{R}^p$ are respectively the slow and fast state variables, $f_F(x,u,\varepsilon) \in \mathbb{R}^p$, $f_S(x,u,\varepsilon) \in \mathbb{R}^n$ and ε is a small and positive parameter, known as *singular perturbed parameter*.

In the limit case $\varepsilon \to 0$, the degree of the system (10) degenerate from $n + p$ to n, and the system is approximated by:

$$0 = f_F(x_S(t), x_F(t), u(t), 0) \tag{11a}$$

$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), 0) \tag{11b}$$

By solving all the algebraic equations (11a) the solution $x_F(t) = \varphi(x_S(t), u(t))$ is obtained and used in (11b) to derive the reduced system.

Remark 2. The fast variables cannot always be explicitly expressed from (11a). The most popular method used to deal with this problem is based on a change of coordinates [2], [13], requiring a linear transformation in order to eliminate the fast dynamics. So, this method can only be applied to systems (e.g. biochemical processes) for which this linear transformation can be founded.

By taking into account the previous drawbacks, no change of coordinates will be considered; the unreduced standard singularly perturbed form (10) is taken into account in this study.

In order to obtain the standard singularly perturbed form, the identification and separation of slow and fast dynamics is the keypoint. This is realized by using the mathematical homotopy method for the linearized system [11]. This method allows to link each state variable with an eigenvalue. By comparing the eigenvalues, the biggest (resp. smallest) one will be associated with the slowest (resp. fastest) dynamic.

Remark 3. It is important to note that the linearized system is only used to identify the slow and fast dynamics, but not to design the multi-observer in order to estimate the state variables. An equivalent MM representation will be used for this purpose.

Let us present in the following the multi-observer design.

III. State estimation

In [7] is presented a state estimation method for singular MM affected by unknown inputs and with measurable decision variables. The proposed observer is not a singular system, but in a usual form in order to simplify the implementation. We suggest to keep the idea of a classical observer form, but to propose an extension to MM affected by unknown inputs and with unmeasurable decision variables. The nonlinear system is under the singularly perturbed form with two time scales and the MM depends on unmeasurable decision variables. The fast dynamic state of the system will be considered as unknown inputs and will be thus estimated. Let us start with a general nonlinear system with two time scale dynamics:

$$\dot{x}_F(t) = \frac{1}{\varepsilon} f_F(x_S(t), x_F(t), u(t), \varepsilon) \quad (12a)$$
$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), \varepsilon) \quad (12b)$$
$$y(t) = Cx(t) \quad (12c)$$

Let us consider the multiple model form of (12) as follows:

$$\begin{aligned} \dot{x}_F(t) &= \sum_{i=1}^{r} \mu_i(x(t),u(t)) \left[A_{FF}^i x_F(t) + A_{FS}^i x_S(t) + B_F^i u(t)\right] \\ \dot{x}_S(t) &= \sum_{i=1}^{r} \mu_i(x(t),u(t)) \left[A_{SF}^i x_F(t) + A_{SS}^i x_S(t) + B_S^i u(t)\right] \\ y(t) &= [C_F \;\; C_S]\, x(t) \end{aligned} \quad (13)$$

where the matrices A_{FF}^i, A_{FS}^i, A_{SF}^i, A_{SS}^i, B_F^i, B_S^i, C_F and C_S are block matrices with appropriate dimensions corresponding to slow and fast dynamics identified in the matrices A_i, B_i and C:

$$A_i = \begin{bmatrix} A_{FF}^i & A_{FS}^i \\ A_{SF}^i & A_{SS}^i \end{bmatrix} \qquad B_i = \begin{bmatrix} B_F^i \\ B_S^i \end{bmatrix} \quad (14)$$

This approach allows to decouple both time scales and the estimation of the slow dynamics x_S is made independently of the value of x_F. If the fast dynamic states are considered as the unknown inputs $d(t) = x_F(t)$ then the state vector becomes:

$$x(t) = \begin{bmatrix} d(t) \\ x_S(t) \end{bmatrix} \quad (15)$$

With the following partitioned matrices:

$$\bar{A}_i = \begin{bmatrix} A_{FF}^i & A_{FS}^i \\ 0 & A_{SS}^i \end{bmatrix} \quad (16a)$$
$$E_i = \begin{bmatrix} 0 \\ A_{SF}^i \end{bmatrix} \quad (16b)$$
$$\bar{C}_S = \begin{bmatrix} 0 & C_S \end{bmatrix} \quad (16c)$$

the system (13) is equivalently written as follows:

$$\begin{aligned} &\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t),u(t)) \cdot \left[\bar{A}_i x(t) + B_i u(t) + E_i d(t)\right] \\ &y(t) = \bar{C}_S x(t) + C_F d(t) \end{aligned} \quad (17)$$

Without any loss of information, the MM with unmeasurable decision variables (17) can be written as a disturbed MM with measurable decision variables:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x}(t),u(t)) \cdot \left[\bar{A}_i x(t) + B_i u(t) + E_i d(t) + \omega(t)\right] \\ y(t) &= \bar{C}_S x(t) + C_F d(t) \end{aligned} \quad (18)$$

where $\hat{x}$ the estimated state of the system and $\omega(t)$ plays the role of a disturbance:

$$\omega(t) = \sum_{i=1}^{r} (\mu_i(x,u) - \mu_i(\hat{x},u)) \cdot \left[\bar{A}_i x(t) + B_i u(t) + E_i d(t)\right] \quad (19)$$

Let us note the equivalence between the models (17) and (18). An observer with unknown inputs can be built [5], [6] by using the second structure (18), as follows:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t),u(t)) \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \\ \hat{x}(t) = z(t) - H y(t) \end{cases} \quad (20)$$

The state estimation error is given by:

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) \\ &= x(t) - z(t) + H\bar{C}_S x(t) + HC_F d(t) \\ &= Px(t) - z(t) + HC_F d(t) \end{aligned} \quad (21)$$

where:

$$P = I + H\bar{C}_S \quad (22)$$

The dynamic of the state estimation error is:

$$\begin{aligned} \dot{e}(t) &= P\dot{x}(t) - \dot{z}(t) + HC_F \dot{d}(t) \\ &= \sum_{i=1}^{r} \mu_i(\hat{x}(t))[P\bar{A}_i x(t) + PB_i u(t) + PE_i d(t) \\ &+ P\omega(t) - N_i z(t) - G_i u(t) - L_i y(t)] + HC_F \dot{d}(t) \end{aligned} \quad (23)$$

After reorganization of the terms in the right side of (23) and by using the definitions of $y(t)$ and $z(t)$, we obtain:

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\hat{x}(t))[(P\bar{A}_i - N_i - K_i\bar{C}_S)x(t) \\ &+ (PB_i - G_i)u(t) + (PE_i - K_iC_F)d(t) + P\omega(t) \\ &+ N_i e(t)] + HC_F \dot{d}(t) \end{aligned} \quad (24)$$

with $K_i = N_i H + L_i$.
If the following conditions hold:

$$HC_F = 0 \quad (25)$$
$$N_i = P\bar{A}_i - K_i\bar{C}_S \quad (26)$$
$$PB_i = G_i \quad (27)$$
$$PE_i = K_i C_F \quad (28)$$
$$L_i = K_i - N_i H \quad (29)$$

then the dynamic of the state estimation error reduces to:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(N_i e(t) + P\omega(t)\right) \quad (30)$$

showing that the dynamic of the state estimation error is only disturbed by $\omega(t)$. To synthesize the matrices of the observer (20), several methods are proposed [5], [6], based on Lipschitz conditions or on the $\mathscr{L}_2$ approach. Here, the second method is used and presented in **Theorem 1**. But let us recall firstly some tools that lead to this result.
In conformity with [1], the state estimation error $e(t)$ tends

towards zero and the $\mathscr{L}_2$ gain from $\omega(t)$ to $e(t)$ is bounded by γ if the following inequality holds:

$$\begin{bmatrix} N_i^T X + XN_i + I & XP \\ P^T X & -\gamma^2 I \end{bmatrix} < 0 \qquad i = 1,\dots,r \tag{31}$$

By using the expression of N_i from (26) and the notations $\lambda = \gamma^2$ and $M_i = XK_i$ $\forall i = 1,\dots,r$, the inequality (31) becomes for all $i = 1,\dots,r$:

$$\begin{bmatrix} \bar{A}_i^T P^T X + XP\bar{A}_i - \bar{C}_S^T M_i^T - M_i\bar{C}_S + I & XP \\ P^T X & -\lambda I \end{bmatrix} < 0 \tag{32}$$

By multiplying the condition (28) with the matrix X and by using the notations $M_i = XK_i$ and (22) we obtain:

$$(X + S\bar{C}_S)E_i = M_i C_F \tag{33}$$

The notation $S = XH$ is made in order to deal with the nonlinear term.

By using the conditions from (25) to (29) and the previous recalls, the following result is obtained.

Theorem 1. *An unknown input observer can be constructed for* (17) *if there exist a symmetric matrix X, matrices M_i and S and a positive scalar λ such that the following conditions holds for all $i = 1,\dots,r$:*

$$\begin{bmatrix} \bar{A}_i^T \tilde{X}^T + \tilde{X}\bar{A}_i - \bar{C}_S^T M_i^T - M_i\bar{C}_S + I & \tilde{X} \\ \tilde{X}^T & -\lambda I \end{bmatrix} < 0 \tag{34}$$

$$SC_F = 0 \tag{35}$$

$$\tilde{X}E_i = M_i C_F \tag{36}$$

where the notation $\tilde{X} = X + S\bar{C}_S$ was used. The matrices of the observer are given by:

$$\begin{aligned} H &= X^{-1}S \\ K_i &= X^{-1}M_i \\ N_i &= (I + H\bar{C}_S)\bar{A}_i - K_i\bar{C}_S \\ L_i &= K_i - N_i H \\ G_i &= (I + H\bar{C}_S)B_i \end{aligned} \tag{37}$$

IV. Application: Wastewater treatment plant

A. Process description and nonlinear model

The wastewater treatment with activated sludge is widely used in the last two centuries [10], [4]. It consists in putting in contact waste water with a mixture rich in bacteria to degrade and eliminate the polluting constituents contained in the water, in suspension or dissolved. Various configurations are possible: separated basins or single basin, different types of reactions (aerated or not-aerated). For economic considerations, a configuration with a single basin (where both aerobic and anaerobic phases alternate) was developed. The functioning principle of the process is briefly described after. The simplified diagram, given in figure 1, includes a basin of aeration (bioreactor) and a clarifier. In this figure q_{in} represents the input flow, q_{out} the output flow, q_a the air flow, q_R, q_W are respectively the recycled and the rejected flow. The reactor volume is assumed to be constant and thus: $q_{out} = q_{in} + q_R$. In general, q_R and q_W represent fractions of input flow q_{in}:

$$q_R(t) = f_R\, q_{in}(t), \qquad 1 \le f_R \le 2 \tag{38}$$

$$q_W(t) = f_W\, q_{in}(t), \qquad 0 < f_W < 1 \tag{39}$$

The polluted water resulting from an external source circulates in the basin of aeration in which the bacterial biomass degrades the organic matter. Micro-organisms gather together in colonial structures called flocs and produce sludges. The mixed liqueur is then sent to the clarifier where the bacterial separation of the purified water and the flocs is made by gravity. A fraction of settled sludges is recycled towards the ventilator to maintain its capacity of purification. The purified water is thrown back in the natural environment.

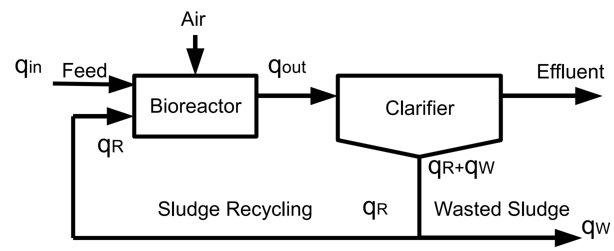

Fig. 1. The diagram of activated sludge wastewater treatment

The ASM1 is a commonly used model to describe this process. Here, a reduced form of the ASM1 model is considered, the carbon pollution of an activated sludge reactor, with three state variables $x = [S_S, S_O, X_{BH}]^T$:

$$\begin{aligned} \dot{S}_S(t) &= -\frac{1}{Y_H}\mu_H\varphi_1(t) + (1 - f_P)b_H\varphi_2(t) + D_1(t) \\ \dot{S}_O(t) &= \frac{Y_H - 1}{Y_H}\mu_H\varphi_1(t) + D_2(t) \\ \dot{X}_{BH}(t) &= \mu_H\varphi_1(t) - b_H\varphi_2(t) + D_3(t) \end{aligned} \tag{40}$$

where:

$$\begin{aligned} D_1(t) &= \frac{q_{in}(t)}{V}\left[S_{S,in}(t) - S_S(t)\right] \\ D_2(t) &= \frac{q_{in}(t)}{V}\left[S_{O,in}(t) - S_O(t)\right] + Kq_a(t)\left[S_{O,sat} - S_O(t)\right] \\ D_3(t) &= \frac{q_{in}(t)}{V}\left[X_{BH,in}(t) - X_{BH}(t) + f_R\frac{1 - f_W}{f_R + f_W}X_{BH}(t)\right] \end{aligned} \tag{41}$$

The process kinetics are:

$$\varphi_1(t) = \frac{S_S(t)}{K_S + S_S(t)}\frac{S_O(t)}{K_{OH} + S_O(t)}X_{BH}(t) \tag{42}$$

$$\varphi_2(t) = X_{BH}(t) \tag{43}$$

The variables involved are: V the reactor volume, S_S the readily biodegradable substrate, S_O the dissolved oxygen, X_{BH} the active heterotrophic biomass. The "R", "in" and "out" indexes correspond respectively to the reactor recycling, input and output.

We suppose that the dissolved oxygen concentration at the reactor input ($S_{O,in}$) is null. Thus, the vector input is defined by:

$$u(t) = \begin{bmatrix} S_{S,in}(t) \\ q_a(t) \\ X_{BH,in}(t) \end{bmatrix} \tag{44}$$

The clarifier is supposed to be perfect i.e. with no internal dynamic process and no biomass in the effluent. In this case we can write at each time instant:

$$[q_{in}(t)+q_R(t)]X_{BH}(t) = [q_R(t)+q_W(t)]X_{BH,R}(t) \quad (45a)$$
$$S_{S,R}(t) = S_S(t) \quad (45b)$$

The following heterotrophic growth and decay kinetic parameters are considered [10]: $\mu_H = 3.733[1/24h]$, $K_S = 20[g/m^3]$, $K_{OH} = 0.2[g/m^3]$, $b_H = 0.3[1/24h]$. The stoichiometric parameters are $Y_H = 0.6$[g cell formed], $f_P = 0.1$ and the oxygen saturation concentration is $S_{O,sat} = 10[g/m^3]$. The following numerical values are considered here for the fractions f_R and f_W: $f_R = 1.1$ and $f_W = 0.04$.

B. Slow and fast variables

Let us consider the linearization of the nonlinear system (40) around various equilibrium points (x_0, u_0):

$$\dot{x}(t) = A_0 x(t) + B_0 u(t) \quad (46)$$

where $A_0 = \frac{\partial f(x,u)}{\partial x}\Big|_{(x_0,u_0)}$ and $B_0 = \frac{\partial f(x,u)}{\partial u}\Big|_{(x_0,u_0)}$.
If we consider $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_N$ the ordered eigenvalues of A_0, the biggest (resp. smallest) eigenvalue correspond to the slowest (resp. fastest) dynamic. This separation will be made by fixing a threshold of separation of both time scales, τ, such as:

$$\lambda_1 \leq ... \leq \lambda_n << \tau \leq \lambda_{n+1} \leq ... \leq \lambda_N$$

For the considered model ASM1 (40), the slow and fast separation is confirmed by the eigenvalues of the jacobian A, as we can notice on figure 2 where we represented these eigenvalues for forty operating points. We notice that two eigenvalues (λ_2 and λ_3) are included between -40 and -0.7 and that the other (λ_1) between -175 and -250. The mathematical method of homotopy (see [11] for details) requires to consider a system, such that an obvious relation relates the eigenvalues to the state variables (e.g. the diagonalized matrix of the jacobian matrix A). By fixing a threshold of separation $\tau = -50$, we can deduct that the system has one fast dynamic (S_S) and two slow dynamics (X_{BH} and S_O).

C. Multiple model

A multiple model is built and used to design an observer allowing slow and fast state estimation.
Considering the process equations, it is natural to define the following decision variables:

$$z_1(u(t)) = \frac{q_{in}(t)}{V} \quad (47a)$$
$$z_2(x(t)) = \frac{1}{K_S + S_S(t)} \cdot \frac{S_O(t)}{K_{OH} + S_O(t)} \cdot X_{BH}(t) \quad (47b)$$
$$z_3(u(t)) = q_a(t) \quad (47c)$$

We consider the quasi-LPV form of the model (40) characterized by matrices $A(t) = A(x(t),u(t))$ and $B(t) = B(x(t),u(t))$

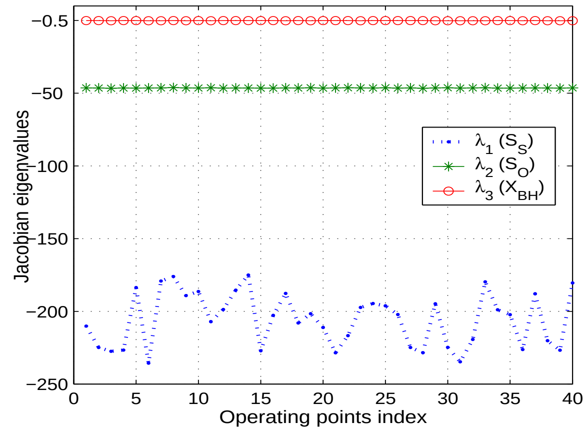

Fig. 2. The jacobian eigenvalues in various points of the operating space

decomposed in the following way:

$$A(t) = \begin{bmatrix} A_{FF}(t) & A_{FS}(t) \\ A_{SF}(t) & A_{SS}(t) \end{bmatrix} \qquad B(t) = \begin{bmatrix} B_F(t) \\ B_S(t) \end{bmatrix} \quad (48)$$

where

$$A_{FF}(t) = -z_1(t) - \frac{1}{Y_H}\mu_H z_2(t) \quad (49)$$

$$A_{FS}(t) = \begin{bmatrix} 0 & (1-f_P)b_H \end{bmatrix} \quad (50)$$

$$B_F(t) = \begin{bmatrix} z_1(t) & 0 & 0 \end{bmatrix} \quad (51)$$

$$A_{SF}(t) = \begin{bmatrix} \frac{Y_H - 1}{Y_H}\mu_H z_2(t) \\ \mu_H z_2(t) \end{bmatrix} \quad (52)$$

$$A_{SS}(t) = \begin{bmatrix} -K z_3(t) - z_1(t) & 0 \\ 0 & \left(\frac{f_R(1-f_W)}{f_W + f_R} - 1\right) z_1(t) - b_H \end{bmatrix} \quad (53)$$

$$B_S(t) = \begin{bmatrix} 0 & K So_{sat} & 0 \\ 0 & 0 & z_1(t) \end{bmatrix} \quad (54)$$

The decomposition of the three premise variables (47) is realized by using the convex polytopic transformation, as in (4) and by using the scalars defined in $z_{\cdot,\cdot}$ (5) and the functions $F_{\cdot,\cdot}$ defined in (6).
By multiplying between themselves the functions $F_{\cdot,\cdot}$, we obtain the $r = 8$ weighting functions $\mu_i(z(t))$:

$$\mu_i(z) = F_{1,\sigma_i^1}(x,u) F_{2,\sigma_i^2}(x,u) F_{3,\sigma_i^3}(x,u)$$

The constant matrices A_i and B_i representing the 8 submodels are defined as in (14) by using the block matrices A and B and the scalars (5):

$$\begin{aligned} A_{FF}^i &= A_{FF}(z_{1,\sigma_i^1}, z_{2,\sigma_i^2}) \\ A_{FS}^i &= \begin{bmatrix} 0 & (1-f_P)b_H \end{bmatrix} \\ A_{SF}^i &= A_{SF}(z_{2,\sigma_i^2}) \\ A_{SS}^i &= A_{SS}(z_{1,\sigma_i^1}, z_{3,\sigma_i^3}) \\ B_F^i &= B_F(z_{1,\sigma_i^1}) \\ B_S^i &= B_S(z_{1,\sigma_i^1}) \qquad & i = 1,...,8 \end{aligned} \quad (55)$$

The model (40) is thus written equivalently under the MM form (13) by using the separation into slow and fast states. The output matrix C is taken under the form:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

D. State estimation results

As seen on figure 2 the fast state variable is the biodegradable substrate S_S. This variable is considered as unknown input in the observer proposed in section III. Let us also consider the system output under the form (18), where:

$$C_F = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \bar{C_S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{56}$$

By applying the *Theorem 1* to the ASM1 model (40), which has the equivalent MM form build in the previous section IV-C, the following state estimation results are obtained and presented in figure 3. The estimation of the fast dynamic S_S (considered as unknown input in the global multiple model) is presented first and is followed by the estimation results of the slow dynamics S_O and X_{BH}. A noise measurement is considered on the output as it can be remarked in figure 4.

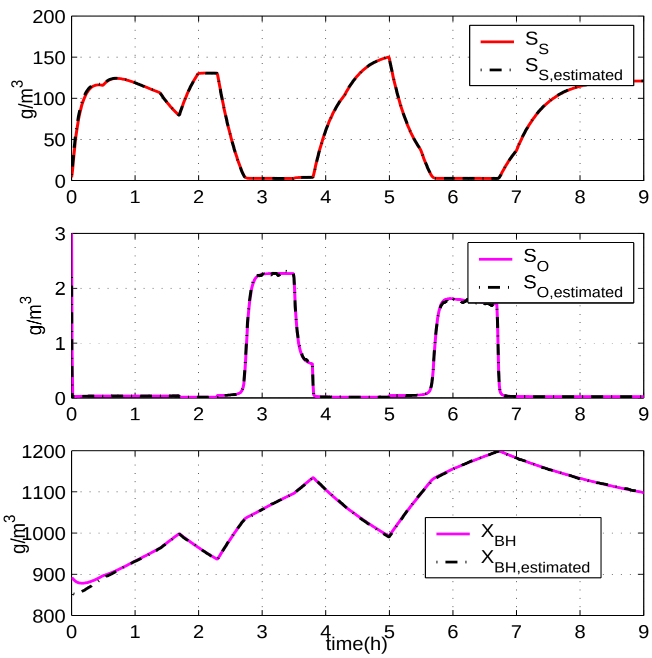

Fig. 3. Estimation of the fast state considered as unknown input S_S and of the slow states X_{BH} and S_O

V. Conclusion

Nonlinear systems with two time scales are considered and they are represented using the standard singularly perturbed form. The slow and fast dynamics are identified using the eigenvalues evaluation of the linearized system. The MM form is obtained by equivalently rewrite the initial nonlinear system, thus no reduction is made. In the same time, the classical MM form is slightly modified in order to separate the slow and the fast dynamics. This modification allows to highlight the fast dynamics as unknown inputs. Based on this equivalent MM representation, an unknown input observer is proposed. The simulation results show good state estimations for both slow and fast dynamics although a noise measurement was considered on the outputs.

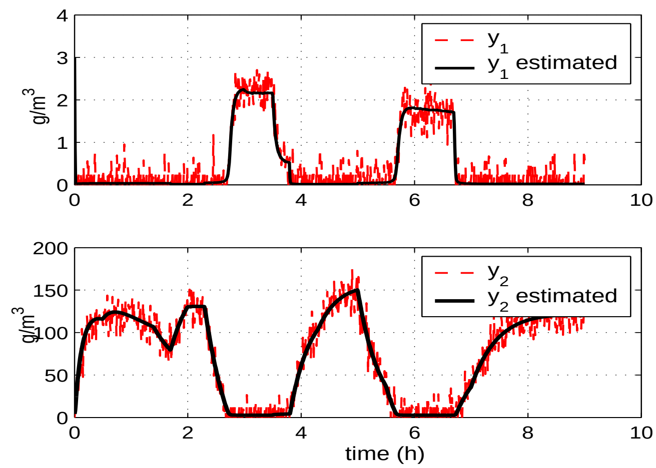

Fig. 4. Estimated outputs

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