Linear Feedback Control Input under Actuator Saturation: a Takagi-Sugeno Approach

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***Abstract*— In this paper, the Takagi-Sugeno representation is used to represent the nonlinear behaviour of a saturated actuator. The control design is based on a state feedback controller function of the saturation levels. Stabilization conditions in the sense of Lyapunov method are derived and expressed as a linear matrix inequality problem. An academic example is presented with a comparaison between the proposed approach and a conventional anti-windup controller.**

***Index Terms*— Takagi-Sugeno models, actuator saturation, linear matrix inequality.**

I. INTRODUCTION

Actuator saturation or control input saturation is probably the most usual nonlinearity encountered in control engineering because of the physical impossibility of applying unlimited control signals and/or safety constraints. Classical examples of such limits are the deflection limits in aircraft actuators, the voltage limits in electrical actuators and the limits on flow volume or rate in hydraulic actuators [11].

Motivated by these practical issues, many approaches have been developed to deal with actuator saturations in the existing literature (see, for example, [3], [5], [13], [4] and the references therein).

In general, there are three main design strategies to deal with actuator saturation for linear plants. First, by taking into account the effect of saturation throughout the design procedure, a controller that may be linear or nonlinear is constructed such that the stability of the closed loop system is guaranteed and a certain performance is achieved. In ([1], an optimal control approach with bounded control leads to a bang-bang type controller that is rarely used in application due to implementation difficulty of the resulting switching surfaces. A more practical solution is developed in [12] where a linear and non-linear output feedback dynamic compensators are synthesized. It is applicable to unstable but controllable and observable systems.

The second strategy is model predictive control (MPC), also known as receding or moving horizon control, which has been a popular control design for discrete time systems recently both in theory and application [8]. In MPC, an optimisation -often a quadratic programming (QP)- is performed at every time instant k by considering both current and future actuator constraints over a certain horizon of length N.

The third strategy is a two-step approach in which a nominal linear controller is first constructed by ignoring actuator saturation. Once this controller has been designed, usually by using standard linear design tools (it may be noticed that some anti-windup schemes are designed for a state feedback nominal controller (e.g. [13]) and some others are for an output feedback dynamic controller (e.g. [7], [2])) a so called anti-windup compensator is designed to handle the saturation constraints [9]. A typical anti-windup scheme consists in augmenting a nominal pre-designed linear controller with a compensator based on the discrepancy between unsaturated and saturated control signals fed to the plant [8].

In this paper, we propose a state feedback controller such that the control gain depends on the saturation levels. The advantage of this approach is to synthetize only one controller (instead of two for the anti-windup case) such that the input saturation is straightly taken into account in the controller design process. For that, the Takagi-Sugeno (T-S) formalism is used to represent the nonlinear behaviour of the saturated actuator and a state feedback control law is synthetized. The stability conditions of the controlled system are derived from the Lyapunov theory and expressed as matrix inequality (LMI) problem. A numerical example with a comparison between conventional anti-windup controller is given to illustrate the effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 introduces the Takagi-Sugeno structure for modeling, some preliminary results, mathematical notations and a brief description of the saturation and anti-windup configuration. It is followed by the representation of the nonlinear saturation by a T-S structure in section 3. In section 4 is designed a state feedback control law depending on the saturation bounds. A numerical example and some simulation results are given to show the effectiveness of the proposed methods in section 5. Conclusions and future works are detailed in section 6.

II. PRELIMINARIES

A. Takagi-Sugeno structure for modeling

The T-S modeling allows to represent the behavior of nonlinear systems by the interpolation of a set of linear submodels. Each sub-model contributes to the global behavior of the nonlinear system through a weighting function $\mu_i(\xi(t))$ [10]. The T-S structure is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{n} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state variable, $u(t) \in \mathbb{R}^{n_u}$ is the control input and $y(t) \in \mathbb{R}^m$ is the system output. $\xi(t) \in \mathbb{R}^q$

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is the decision variable vector assumed to be measurable (as the system output) or known (as the system input). The weighting functions $\mu_i(\xi(t))$ of the r submodels satisfy the convex sum property

$$\begin{cases} \sum_{i=1}^{n} \mu_i(\xi(t)) = 1 \\ 0 \le \mu_i(\xi(t)) \le 1, \quad i = 1, \dots, n \end{cases} \tag{2}$$

In the remaining of the paper, the two following lemmas are used.

Lemma 1: Consider two matrices X and Y with appropriate dimensions and G a symmetric positive definite matrix. The following property is verified

$$X^T Y + Y^T X \le X^T G X + Y^T G^{-1} Y \tag{3}$$

Lemma 2: (Congruence) Consider two matrices X and Y, if X is positive (resp. negative) definite and if Y is a full column rank matrix, then the matrix YXY^T is positive (resp. negative) definite.

B. Mathematical notation

The following notations are used throughout the paper.

- A bloc diagonal matrix A denoted $diag(A_1, \dots, A_n)$ is defined as

$$diag(A_1, \dots, A_n) := \begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_n \end{pmatrix} \tag{4}$$

  where A_k are square matrices.

- The integer part of a number x is denoted:

$$int(x) := \lfloor x \rfloor \tag{5}$$

- The smallest and largest eigenvalues of the matrix M are respectively denoted $\lambda_{min}(M)$ and $\lambda_{max}(M)$.

- The saturation function for a signal $v(t)$ is defined as

$$sat(v(t)) := \begin{cases} v(t) & \text{if} \quad v_{min} \le v(t) \le v_{max} \\ v_{max} & \text{if} \quad v(t) \ge v_{max} \\ v_{min} & \text{if} \quad v(t) \le v_{min} \end{cases} \tag{6}$$

  where v_{max} and v_{min} denote the saturation levels.

C. Anti-windup configuration

Classical anti-windup designs were mostly based on an adjustment on the controller input [8]. Consider the block diagrams in Figure 1 with nominal control input $\overline{v}$ and anti-windup compensator Ψ. The conventional anti-windup controller uses the difference $v - v_{sat}$ as an input for a large gain matrix $\Psi = \alpha I$ where $\alpha >> 1$ is a scalar [6].

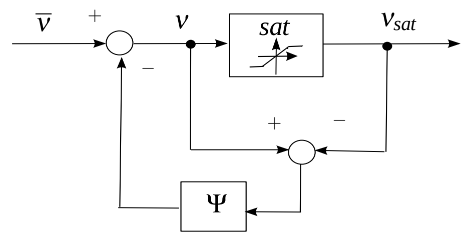

Fig. 1. Conventional anti-windup controller

III. Problem statement

A. Takagi-Sugeno saturation control

The main idea of this work is to model the nonlinear actuator saturation using the Takagi-Sugeno representation (section II-A). For that, it is proposed to re-write the saturation equation (6) for each component of the control input vector under a particular form.

Let us consider a control input vector $u(t) \in \mathbb{R}^{n_u}$, such that

$$u(t) = \begin{pmatrix} u_1(t) \\ \vdots \\ u_{n_u}(t) \end{pmatrix} \tag{7}$$

The contol input under actuator saturation constraint is given by

$$u_{sat}(t) = \begin{pmatrix} u_{sat}^1(t) \\ \vdots \\ u_{sat}^{n_u}(t) \end{pmatrix} \tag{8}$$

Each component of this vector is written as

$$u_{sat}^j(t) = \sum_{i=1}^{3} \mu_i^j(u_j(t)) \, (\lambda_i^j u_j(t) + \gamma_i^j), \quad j = 1, \dots, n_u \tag{9}$$

with

$$\begin{cases} \lambda_1^j & = & 0 \\ \lambda_2^j & = & 1 \\ \lambda_3^j & = & 0 \end{cases} \tag{10}$$

$$\begin{cases} \gamma_1^j & = & u_{min}^j \\ \gamma_2^j & = & 0 \\ \gamma_3^j & = & u_{max}^j \end{cases} \tag{11}$$

and the activation functions

$$\begin{cases} \mu_1(u_j(t)) & = & \frac{1 - sign(u_j(t) - u_{min}^j)}{2} \\ \mu_2(u_j(t)) & = & \frac{sign(u_j(t) - u_{min}^j) - sign(u_j(t) - u_{max}^j)}{2} \\ \mu_3(u_j(t)) & = & \frac{1 + sign(u_j(t) - u_{max}^j)}{2} \end{cases} \tag{12}$$

Then, the control input vector $u(t) \in \mathbb{R}^{n_u}$ subject to actuator saturation is modeled by

$$u_{sat}(t) = \begin{pmatrix} \sum_{i=1}^{3} \mu_i^1(u_1(t))(\lambda_i^1 u_1 + \gamma_i^1) \\ \vdots \\ \sum_{i=1}^{3} \mu_i^\ell(u_\ell(t))(\lambda_i^\ell u_\ell + \gamma_i^\ell) \\ \vdots \\ \sum_{i=1}^{3} \mu_i^{n_u}(u_{n_u}(t))(\lambda_i^{n_u} u_{n_u} + \gamma_i^{n_u}) \end{pmatrix} \quad (13)$$

In order to simplify the notations, the weighting functions $\mu(u(t))$ are now noted $\mu(t)$.
Based on the convex sum property of the weighting functions (2), equation (13) can be written in order to have the same activation functions for all the input vector components.

$$u_{sat}(t) = \begin{pmatrix} \sum_{i=1}^{3} \mu_i^1(t)(\lambda_i^1 u_1 + \gamma_i^1) \times \left(\prod_{k=2}^{n_u} \sum_{j=1}^{3} \mu_j^k\right) \\ \vdots \\ \sum_{i=1}^{3} \mu_i^\ell(u_\ell(t))(\lambda_i^\ell u_\ell + \gamma_i^\ell) \times \left(\prod_{k=1 k\neq \ell}^{n_u} \sum_{j=1}^{3} \mu_j^k\right) \\ \vdots \\ \sum_{i=1}^{3} \mu_i^{n_u}(t)(\lambda_i^{n_u} u_{n_u} + \gamma_i^{n_u}) \times \left(\prod_{k=1}^{n_u-1} \sum_{j=1}^{3} \mu_j^k\right) \end{pmatrix} \quad (14)$$

For n_u inputs, 3^{n_u} submodels are obtained. Thus, it is important to note that we have an analytical expression of the actuators saturation $\mu_{sat}(t)$ directly expressed in term of the control variable $u(t)$.
Equation (14) is equivalent to

$$u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t)(\Lambda_i u(t) + \Gamma_i) \quad (15)$$

The global weighting functions $\mu_i(t)$, the matrices $\Lambda_i \in \mathbb{R}^{n_u \times n_u}$ and vectors $\Gamma_i \in \mathbb{R}^{n_u \times 1}$ are defined as follow

$$\begin{cases} \mu_i(t) & = & \prod_{j=1}^{n_u} \mu^j_{\sigma_i^j}(u_j(t)), \\ \Lambda_i & = & diag(\lambda^1_{\sigma_i^1}, \ldots, \lambda^{n_u}_{\sigma_i^{n_u}}) \\ \Gamma_i & = & vect(\gamma^j_{\sigma_i^j}). \end{cases} \quad (16)$$

where the indexes $\sigma_i^j (i = 1, \ldots, 3^{n_u}$ and $j = 1, \ldots, n_u)$, equal to $1, 2$ or 3, indicate which partition of the j th input (μ_1^j, μ_2^j or μ_3^j) is involved in the i th submodel.
The relations between the i th submodel and the σ_i^j indices are given by the following equation

$$i = 3^{n_u-1}\sigma_i^1 + 3^{n_u-2}\sigma_i^2 + \ldots + 3^0\sigma_i^{n_u} - (3^1 + 3^2 + \ldots + 3^{n_u-1})$$

The σ_i^j are such that $((\sigma_i^1 - 1), \ldots, (\sigma_i^{n_u} - 1))$ corresponds to (i-1) in base 3.
An illustrative example is given for two inputs ($n_u = 2$), such that

$$u_{sat}(t) = \begin{pmatrix} u_{sat}^1(t) \\ u_{sat}^2(t) \end{pmatrix} \quad (17)$$

Since we have three partitions for each input $\mu_j(t)$, the Takagi-Sugeno model for $u_{sat}(t)$ is then composed with 3^2 submodels

$$u_{sat}(t) = \sum_{i=1}^{9} \mu_i(t)(\Lambda_i u(t) + \Gamma_i) \quad (18)$$

with the parameters μ_i, Λ_i and Γ_i given by the following table

submodel i	(σ_i^1, σ_i^2)	$\mu_i(t)$	Λ_i	Γ_i
1	$(1,1)$	$\mu_1^1\mu_1^2$	$\text{diag}(\lambda_1^1, \lambda_1^2)$	$\begin{pmatrix} \gamma_1^1 \\ \gamma_1^2 \end{pmatrix}$
2	$(1,2)$	$\mu_1^1\mu_2^2$	$\text{diag}(\lambda_1^1, \lambda_2^2)$	$\begin{pmatrix} \gamma_1^1 \\ \gamma_2^2 \end{pmatrix}$
3	$(1,3)$	$\mu_1^1\mu_3^2$	$\text{diag}(\lambda_1^1, \lambda_3^2)$	$\begin{pmatrix} \gamma_1^1 \\ \gamma_3^2 \end{pmatrix}$
4	$(2,1)$	$\mu_2^1\mu_1^2$	$\text{diag}(\lambda_2^1, \lambda_1^2)$	$\begin{pmatrix} \gamma_2^1 \\ \gamma_1^2 \end{pmatrix}$
5	$(2,2)$	$\mu_2^1\mu_2^2$	$\text{diag}(\lambda_2^1, \lambda_2^2)$	$\begin{pmatrix} \gamma_2^1 \\ \gamma_2^2 \end{pmatrix}$
6	$(2,3)$	$\mu_2^1\mu_3^2$	$\text{diag}(\lambda_2^1, \lambda_3^2)$	$\begin{pmatrix} \gamma_2^1 \\ \gamma_3^2 \end{pmatrix}$
7	$(3,1)$	$\mu_3^1\mu_1^2$	$\text{diag}(\lambda_3^1, \lambda_1^2)$	$\begin{pmatrix} \gamma_3^1 \\ \gamma_1^2 \end{pmatrix}$
8	$(3,2)$	$\mu_3^1\mu_2^2$	$\text{diag}(\lambda_3^1, \lambda_2^2)$	$\begin{pmatrix} \gamma_3^1 \\ \gamma_2^2 \end{pmatrix}$
9	$(3,3)$	$\mu_3^1\mu_3^2$	$\text{diag}(\lambda_3^1, \lambda_3^2)$	$\begin{pmatrix} \gamma_3^1 \\ \gamma_3^2 \end{pmatrix}$

B. Problem statement

Let us now consider a linear system represented by the following state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (19)$$

The control input $u(t)$ is subject to actuator saturation, then the system described by (19) becomes

$$\dot{x}(t) = Ax(t) + Bu_{sat}(t) \quad (20)$$

From (15), equation (20) can be written as

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t)(Ax(t) + B(\Lambda_i u(t) + \Gamma_i)) \quad (21)$$

IV. SATURATED STATE FEEDBACK CONTROL INPUT

The objective is to design a stabilizing static state feedback control ensuring the stability of the system, even in the presence of control input saturation. The solution is obtained by representing the saturation as a T-S system and by solving

an optimization problem under LMI constraints.
Let us consider a linear feedback control input given by (22)

$$u(t) = -Kx(t) \tag{22}$$

Our objective is to design the state feedback controller (22) to guarantee the stability of the system (21) such that the control gain K takes into account the saturation limits.
Replacing the control law (22) in the T-S system equation, the obtained system is the following

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t)((A - B\Lambda_i K)x(t) + B\Gamma_i) \tag{23}$$

The computation of the static feedback K is detailed in the next theorem.

Theorem 1: There exists a static state feedback controller (22) for a saturated input system (21) such that the system state converges toward an origin-centred ball of radius bounded by β if there exists matrices $P_1 = P_1^T > 0, R, \Sigma = \Sigma^T > 0$ solutions of the following optimization problem

$$\min_{P_1,R,\Sigma,\beta} \beta \tag{24}$$

s.t.

$$\begin{pmatrix} Q_i & I \\ I & -\beta I \end{pmatrix} < 0 \tag{25}$$

with

$$Q_i = \begin{pmatrix} P_1A^T + AP_1 - R^T\Lambda_i^T B^T - B\Lambda_i R & I \\ I & -\Sigma \end{pmatrix} \tag{26}$$

and

$$\Gamma_i B^T \Sigma B \Gamma_i < \beta \tag{27}$$

The gain of the controller is given by

$$K = RP_1^{-1} \tag{28}$$

Proof:

Let us define the following quadratic Lyapunov function

$$V(x(t)) = x^T(t)Px(t) \tag{29}$$

where $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix.
According to equations (23) and (29), the time derivative of $V(t)$ is given by

$$\dot{V}(x(t)) = \sum_{i=1}^{3^{n_u}} \mu_i(t)(x^T(t)((A - B\Lambda_i K)^T P + P(A - B\Lambda_i K))x(t) + \Gamma_i^T B^T Px(t) + x^T(t)PB\Gamma_i) \tag{30}$$

Using Lemma 1, it follows that

$$\Gamma_i^T B^T Px(t) + x^T(t)PB\Gamma_i \le \Gamma_i^T B^T \Sigma B\Gamma_i + x^T(t)P\Sigma^{-1}Px(t) \tag{31}$$

and thus, the time derivative of the Lyapunov function (30) is bounded as follows

$$\dot{V}(x(t)) \le \sum_{i=1}^{3^{n_u}} \mu_i(t)(x^T(t)((A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P)x(t) + \Gamma_i^T B^T \Sigma B\Gamma_i) \tag{32}$$

Let us define

$$\mathcal{Q}_i = (A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P \tag{33}$$

$$\varepsilon = \min_{i=1:3^{n_u}} \lambda_{min}(-\mathcal{Q}_i) \tag{34}$$

$$\delta = \max_{i=1:3^{n_u}} \Gamma_i^T B^T \Sigma B \Gamma_i \tag{35}$$

Since $\Sigma > 0$ and from equation (32), according to Lyapunov stability theory [14], $\dot{V}(t) < -\varepsilon \parallel x \parallel^2 + \delta$. It follows that $\dot{V}(t) < 0$ for

$$\begin{cases} \mathcal{Q}_i < 0 \\ \text{and} \\ \parallel x \parallel^2 > \frac{\delta}{\varepsilon} \end{cases} \tag{36}$$

Which means that $x(t)$ is uniformly bounded and converges to a small origin-centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}}$.
Since $\mathcal{Q}_i < 0$ is expressed by

$$(A^T - K^T\Lambda_i^T B^T)P + P(A - B\Lambda_i K) + P\Sigma^{-1}P < 0 \tag{37}$$

Applying Lemma 2, the inequality (37) becomes

$$P^{-1}A^T + AP^{-1} - P^{-1}K^T\Lambda_i^T B^T - B\Lambda_i KP^{-1} + \Sigma^{-1} < 0 \tag{38}$$

Using the changes of variables

$$\begin{cases} P_1 = P^{-1} \\ R = KP_1 \end{cases} \tag{39}$$

condition (38) is linearized as

$$P_1A^T + AP_1 - R^T\Lambda_i^T B^T - B\Lambda_i R + \Sigma^{-1} < 0 \tag{40}$$

Applying Schur's complement to equation (40), we finally find the stabilization condition $Q_i < 0$.

$$\begin{pmatrix} P_1A^T + AP_1 - R^T\Lambda_i^T B^T - B\Lambda_i R & I \\ I & -\Sigma \end{pmatrix} < 0 \tag{41}$$

As the weighting functions satisfy (2) and $\Sigma > 0$, if (41) is satisfied for $i = 1, \dots, 3^{n_u}$ and $\parallel x \parallel^2 > \frac{\delta}{\varepsilon}$, then $\dot{V}(x(t)) < 0$, which implies that $x(t)$ converges to an origin centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}}$.
The objective is now to minimize the radius $\sqrt{\frac{\delta}{\varepsilon}}$. Firstly δ is bounded by $\delta < \beta$ from the definition (35) and the LMIs (27). Secondly $1/\varepsilon$ is bounded by β or equivalently $\varepsilon > 1/\beta$. From the definition (34) it is equivalent to

$$-Q_i > (1/\beta)\ I,\ i = 1, \dots, 3^{n_u} \tag{42}$$

Applying Schur's complement, (42) is equivalent to (25). Finally, the radius is bounded by β. ■

V. Numerical example

The proposed static state feedback controller design for systems with saturated control input is illustrated by an academic example. Let consider the linear system (19) defined by

$$A = \begin{pmatrix} -2 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0.2 & 0.1 & -3 \end{pmatrix},\ B = \begin{pmatrix} 0.1 & 1 \\ 0.5 & 1 \\ 0.8 & 0.6 \end{pmatrix} \tag{43}$$

with two control inputs $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$. The inputs are subject to the following actuator saturations.

$$\begin{cases} u_{1max} &= & 1 \\ u_{1min} &= & -1 \\ u_{2max} &= & 1.5 \\ u_{2min} &= & -1.5 \end{cases} \tag{44}$$

In order to illustrate the effectiveness of the proposed approach, three different control laws are synthesized for the same system. A so-called nominal controller is computed by a conventional pole placement, without taking into account the input saturation, although the saturation acts on the controller. Then in the first part of this section, a comparison is provided between the nominal closed loop system without saturation, the nominal closed loop system with saturated actuators and the closed loop system with the controller proposed in this paper where the saturation bounds are taken into account in the controller design.

In a second case, a conventional anti-windup controller is synthetized (as shown in section II). The obtained performances of this controller and of the proposed T-S control are compared.

A. Comparison with nominal control

Let us consider the nominal controller K_n such that the gain does not take into consideration the actuator saturations. For the considered example,

$$K_n = \begin{pmatrix} 0.3308 & 0.5586 & -1.0239 \\ -0.0453 & 0.7816 & 0.4507 \end{pmatrix} \tag{45}$$

For the T-S controller, the control gain K depending on the saturation limits and computed from theorem 1 is equal to

$$K = \begin{pmatrix} 1.9543 & 0.5029 & 2.3005 \\ 1.0800 & 0.3243 & 1.8291 \end{pmatrix} \tag{46}$$

with a convergence to an origin-centred ball of radius $r = 3.04$.

The following figures (2 and 3) depict the system states and control of the nominal closed loop system without saturation (respectively denoted x_1, x_2,x_3, u_1 and u_2), those of the nominal closed loop system with saturation (respectively denoted x_{1sat}, x_{2sat}, x_{3sat}, u_{1sat} and u_{2sat}) and those of the proposed approach (respectively denoted x_{1TS}, x_{2TS}, x_{3TS}, u_{1TS} and u_{2TS}).

B. Comparison with anti-windup controller

In this subsection, a second simulation is performed in order to compare a conventional anti-windup controller with the proposed one. For the conventional anti-windup, the main idea is to synthetize a nominal feedback control and to add a compensator to handle the saturated input. The anti-windup compensator is taken as a large gain matrix $\Xi = \alpha I$ with $\alpha = 50$. Figures 4 represents the system states for the two controllers, where x_{1TS}, x_{2TS} and x_{3TS} correspond to the proposed approach whereas x_{1AW}, x_{2AW} and x_{3AW} correspond to the anti-windup controller.

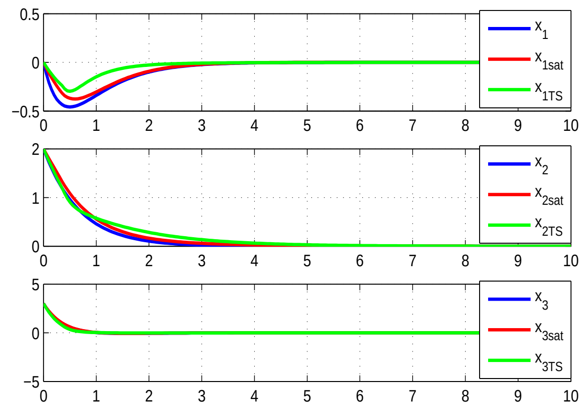

Fig. 2. System states for nominal, nominal saturated and T-S saturated controllers

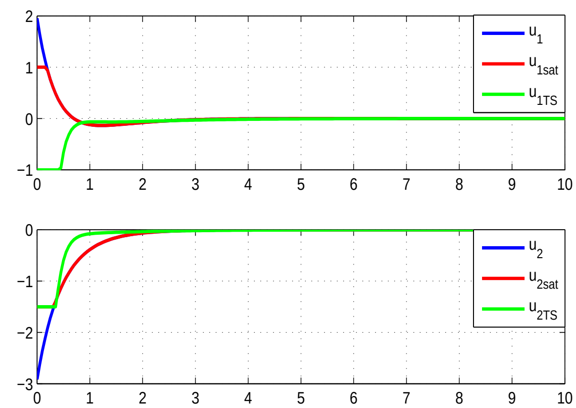

Fig. 3. Control input for nominal, nominal saturated and T-S saturated controllers

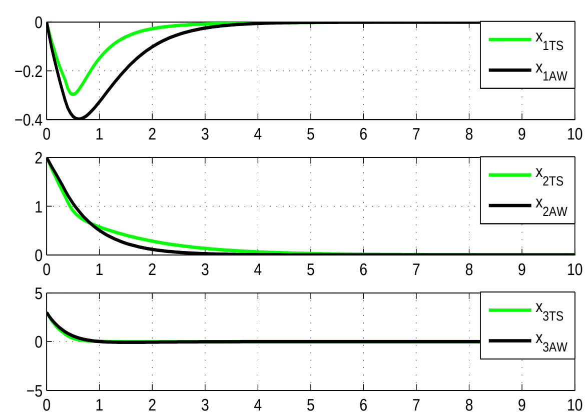

Fig. 4. System states for conventional anti-windup and T-S saturated controllers

C. Comments

In the present work, a new representation of the actuator saturation problem was introduced. Thus, it is important to note as first that with the proposed approach, the actuator saturation is directly expressed in terms of the control variable and the saturation limits. The proposed approach allows us to describe and study a nonlinear behavior using linear tools.

As an illustrative example, a linear feedback controller was applied to an academic system. Three cases were presented: nominal controller, T-S controller and anti-windup controller. For the nominal case, the control gain is computed without considering the saturation limits. For the anti-windup case, the control input is adjusted using the difference $(u_{sat} - u)$, but the controller gain is computed without taking into account the saturation on the input control..

On the other hand, for the proposed approach the control gain is computed depending on the saturation limits by solving the LMIs given in theorem 1.

The obtained results illustrates the effectiveness of the proposed approach for the studied example.

VI. Conclusions and Future Works

Considering the saturation nonlinearity, a linear system can be represented in a T-S form. The main advantage of the proposed approach is to synthetize the control by taking into consideration the saturation limits, since they are added in the new system representation. Then, the stabilizing state feedback gain can be computed by solving an optimization problem under LMI constraint, as usually done in the T-S framework. A numerical example was presented in order to illustrate the proposed approach with a comparaison with a conventional anti-windup controller. It was shown that the obtained results are slightly better for the T-S saturated controller.

As future works, the proposed approach may be generalized to nonlinear systems represented by a T-S model with measurable and unmeasurable decision variable. It is also noted that the saturation may also affect not only the actuators, but also the sensors, this is why our approach may also be applied to sensors saturations.

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