

Linear Feedback Control Input under Actuator Saturation : a Takagi-Sugeno Approach

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Objective

Design a stabilizing static state feedback control ensuring the stability of the system
under actuator saturation

Contribution

Polytopic representation of the nonlinear saturated behaviour

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 - Vectorial case
 - Illustrative example
 - Saturated state feedback control input
 - Numerical example
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- The Takagi-Sugeno structure is given by

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^n \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^n \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

$x(t) \in \mathbb{R}^{n_x}$ is the system state variable, $u(t) \in \mathbb{R}^{n_u}$ the control input and $y(t) \in \mathbb{R}^m$ the system output.

$\xi(t) \in \mathbb{R}^q$ is the decision variable vector assumed to be measurable (as the system output) or known (as the system input).

- Based on a nonlinear interpolation between certain linear submodels with adequate weighting functions $\mu_i(\xi(t))$ satisfying the convex sum property

$$\begin{cases} \sum_{i=1}^n \mu_i(\xi(t)) = 1 & \text{and} \\ 0 \leq \mu_i(\xi(t)) \leq 1, & i = 1, \dots, n, \quad \forall t \end{cases}$$

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How to systematically obtain a multi-model ?

- **Nonlinear sector approach** : a systematic procedure which guarantees an exact T-S model construction from nonlinear systems with bounded nonlinearities.
- Starting with an existing general form of nonlinear systems, a quasi-LPV state representation is realized. The T-S form is obtained by using the convex polytopic transformation.

Each vertex defines a linear submodel and nonlinear parts are rejected into the weighting functions.

$$\text{NL} \left\{ \begin{array}{l} \dot{x}(t) = f_x(x(t), u(t)) \\ y(t) = f_y(x(t), u(t)) \end{array} \right. \Rightarrow$$

$$\text{Quasi-LPV} \left\{ \begin{array}{l} \dot{x}(t) = A(x(t), u(t))x(t) + B(x(t), u(t))u(t) \\ y(t) = C(x(t), u(t))x(t) + D(x(t), u(t))u(t) \end{array} \right. \Rightarrow$$

$$\text{T-S model} \left\{ \begin{array}{l} \dot{x}(t) = \sum_{i=1}^n \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^n \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{array} \right.$$

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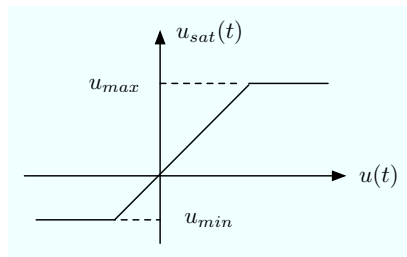
$$\text{T-S model} \left\{ \begin{array}{l} \dot{x}(t) = \sum_{i=1}^n \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^n \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{array} \right.$$

Polytopic saturated control

Objective

Model the nonlinear actuator saturation using the T-S representation

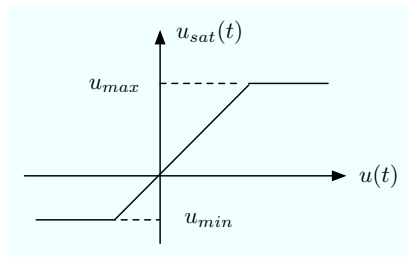
$$u_{sat}(t) = \begin{cases} u(t) & \text{if } u_{min} \leq u(t) \leq u_{max} \\ u_{max} & \text{if } u(t) \geq u_{max} \\ u_{min} & \text{if } u(t) \leq u_{min} \end{cases}$$



$$\begin{cases} u_{sat}(t) = \mu_1(t)u_{min} + \mu_2(t)u(t) + \mu_3(t)u_{max} \\ \mu_1(t) = \frac{1 - \text{sign}(u(t) - u_{min})}{2} \\ \mu_2(t) = \frac{\text{sign}(u(t) - u_{min}) - \text{sign}(u(t) - u_{max})}{2} \\ \mu_3(t) = \frac{1 + \text{sign}(u(t) - u_{max})}{2} \end{cases}$$

$$\Rightarrow \begin{cases} u_{sat}(t) = \sum_{i=1}^3 \mu_i(t)(\lambda_i u(t) + \gamma_i) \\ \lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 0 \\ \gamma_1 = u_{min} \quad \gamma_2 = 0 \quad \gamma_3 = u_{max} \end{cases}$$

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$$\begin{cases} u_{sat}(t) &= \mu_1(t)u_{min} + \mu_2(t)u(t) + \mu_3(t)u_{max} \\ \mu_1(t) &= \frac{1 - \text{sign}(u(t) - u_{min})}{2} \\ \mu_2(t) &= \frac{\text{sign}(u(t) - u_{min}) - \text{sign}(u(t) - u_{max})}{2} \\ \mu_3(t) &= \frac{1 + \text{sign}(u(t) - u_{max})}{2} \end{cases}$$

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$u(t) \in \mathbb{R}^{n_u}$, $u_j(t)$ (resp. $u_{sat}^j(t)$) the j^{th} component of $u(t)$ (resp. $u_{sat}(t)$).

$$u_{sat}(t) = \begin{pmatrix} \sum_{i=1}^3 \mu_i^1(u_1(t))(\lambda_i^1 u_1(t) + \gamma_i^1) \\ \vdots \\ \sum_{i=1}^3 \mu_i^{n_u}(u_{n_u}(t))(\lambda_i^{n_u} u_{n_u}(t) + \gamma_i^{n_u}) \end{pmatrix}$$

To have the same activation functions $\mu_i^j(t)$ for all the input vector components $u_{sat}(t)$

$$u_{sat}(t) = \begin{pmatrix} \left(\sum_{i=1}^3 \mu_i^1(u_1(t))(\lambda_i^1 u_1(t) + \gamma_i^1) \right) \times \left(\prod_{k=2}^{n_u} \sum_{j=1}^3 \mu_j^k(u_k(t)) \right) \\ \vdots \\ \sum_{i=1}^3 \mu_i^\ell(u_\ell(t))(\lambda_i^\ell u_\ell(t) + \gamma_i^\ell) \times \left(\prod_{k=1, k \neq \ell}^{n_u} \sum_{j=1}^3 \mu_j^k(u_k(t)) \right) \\ \vdots \\ \left(\sum_{i=1}^3 \mu_i^{n_u}(u_{n_u}(t))(\lambda_i^{n_u} u_{n_u}(t) + \gamma_i^{n_u}) \right) \times \left(\prod_{k=1}^{n_u-1} \sum_{j=1}^3 \mu_j^k(u_k(t)) \right) \end{pmatrix}$$

T-S representation for the saturation

$$u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t) (\Lambda_i u(t) + \Gamma_i)$$

Weighting functions

$$\begin{cases} \mu_i^{sat}(t) &= \prod_{j=1}^{n_u} \mu_{\sigma_i^j}^j(u_j(t)) \\ \Lambda_i &= \text{diag}(\lambda_{\sigma_i^1}^1, \dots, \lambda_{\sigma_i^{n_u}}^{n_u}) \\ \Gamma_i &= \begin{pmatrix} \gamma_{\sigma_i^1}^1 & \dots & \gamma_{\sigma_i^{n_u}}^{n_u} \end{pmatrix}^T \end{cases}$$

where the indexes $\sigma_i^j (i = 1, \dots, 3^{n_u} \text{ and } j = 1, \dots, n_u)$, equal to 1, 2 or 3, indicate which partition of the j^{th} input (μ_1^j, μ_2^j or μ_3^j) is involved in the i^{th} submodel.

The relations between the i^{th} submodel and the σ_i^j indices are given by the following equation

$$i = 3^{n_u-1} \sigma_i^1 + 3^{n_u-2} \sigma_i^2 + \dots + 3^0 \sigma_i^{n_u} - (3^1 + 3^2 + \dots + 3^{n_u-1})$$

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for two inputs ($n_u = 2$) :

$$u_{sat}(t) = \sum_{i=1}^9 \mu_i(t)(\Lambda_i u(t) + \Gamma_i)$$

$$\left\{ \begin{array}{lll} \lambda_1^1 = \lambda_1^2 = 0 & \lambda_2^1 = \lambda_2^2 = 1 & \lambda_3^1 = \lambda_3^2 = 0 \\ \gamma_1^1 = u_{min}^1 & \gamma_2^1 = 0 & \gamma_3^1 = u_{max}^1 \\ \gamma_1^2 = u_{min}^2 & \gamma_2^2 = 0 & \gamma_3^2 = u_{max}^2 \end{array} \right.$$

sub-model i	(σ_i^1, σ_i^2)	$\mu_i(t)$	Λ_i	Γ_i
1	(1, 1)	$\mu_1^1 \mu_1^2$	$\text{diag}(\lambda_1^1, \lambda_1^2)$	$[\gamma_1^1 \ \gamma_1^2]^T$
2	(1, 2)	$\mu_1^1 \mu_2^2$	$\text{diag}(\lambda_1^1, \lambda_2^2)$	$[\gamma_1^1 \ \gamma_2^2]^T$
3	(1, 3)	$\mu_1^1 \mu_3^2$	$\text{diag}(\lambda_1^1, \lambda_3^2)$	$[\gamma_1^1 \ \gamma_3^2]^T$
4	(2, 1)	$\mu_2^1 \mu_1^2$	$\text{diag}(\lambda_2^1, \lambda_1^2)$	$[\gamma_2^1 \ \gamma_1^2]^T$
5	(2, 2)	$\mu_2^1 \mu_2^2$	$\text{diag}(\lambda_2^1, \lambda_2^2)$	$[\gamma_2^1 \ \gamma_2^2]^T$
6	(2, 3)	$\mu_2^1 \mu_3^2$	$\text{diag}(\lambda_2^1, \lambda_3^2)$	$[\gamma_2^1 \ \gamma_3^2]^T$
7	(3, 1)	$\mu_3^1 \mu_1^2$	$\text{diag}(\lambda_3^1, \lambda_1^2)$	$[\gamma_3^1 \ \gamma_1^2]^T$
8	(3, 2)	$\mu_3^1 \mu_2^2$	$\text{diag}(\lambda_3^1, \lambda_2^2)$	$[\gamma_3^1 \ \gamma_2^2]^T$
9	(3, 3)	$\mu_3^1 \mu_3^2$	$\text{diag}(\lambda_3^1, \lambda_3^2)$	$[\gamma_3^1 \ \gamma_3^2]^T$

Saturated state feedback control input

Problem statement

$$\dot{x}(t) = Ax(t) + Bu_{\text{sat}}(t)$$

$$u_{\text{sat}}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{\text{sat}}(t) (\Lambda_i u(t) + \Gamma_i), \quad u(t) = -Kx(t)$$

Objectives

Ensure the stability of the system even in the presence of control input saturation

Proposed solution

- Representing the system with a T-S form : $\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{\text{sat}}(t) ((A - B\Lambda_i K)x(t) + B\Gamma_i)$
- Solving an optimization problem under LMI constraints : $\min_K ||x(t)||$

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- Solving an optimization problem under LMI constraints : $\min_K \|x(t)\|$

Lyapunov approach

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t) ((A - B\Lambda_i K)x(t) + B\Gamma_i)$$

$$V(x(t)) = x^T(t)Px(t), \quad P = P^T > 0$$

$$\dot{V}(x(t)) \leq \sum_{i=1}^{3^{n_u}} \mu_i(t) (x^T(t) ((A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P)x(t) + \Gamma_i^T B^T \Sigma B \Gamma_i)$$

Let us define :

$$\mathcal{Q}_i = (A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P$$

$$\varepsilon = \min_{i=1:3^{n_u}} \lambda_{\min}(-\mathcal{Q}_i)$$

$$\delta = \max_{i=1:3^{n_u}} \Gamma_i^T B^T \Sigma B \Gamma_i$$

Thus

$$\dot{V}(t) < -\varepsilon \|x\|^2 + \delta$$

Condition

$$\dot{V}(t) < -\varepsilon \|x(t)\|^2 + \delta$$

$x(t)$ is uniformly bounded and converges to a small origin-centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}}$ if :

$$\mathcal{Q}_i < 0 \quad \text{and} \quad \|x(t)\|^2 > \frac{\delta}{\varepsilon}$$

LMI to solve

$$\mathcal{Q}_i = (A - B\Lambda_i K)^T P + P(A - B\Lambda_i K) + P\Sigma^{-1}P$$

$$\mathcal{Q}_i < 0 \equiv \begin{pmatrix} P_1 A^T + A P_1 - R^T \Lambda_i^T B^T - B \Lambda_i R & I \\ I & -\Sigma \end{pmatrix} < 0, \quad (P_1 = P^{-1}), \quad i = 1, \dots, 3^{n_u}$$

Feedback controller :

$$u(t) = -Kx(t), \quad K = R P_1^{-1}$$

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$x(t)$ is uniformly bounded and converges to a small origin-centred ball of radius $\sqrt{\frac{\delta}{\varepsilon}} \rightarrow$

minimize the radius ?

δ and ε optimisation

$$\delta = \max_{i=1:3^{n_u}} \Gamma_i^T B^T \Sigma B \Gamma_i \quad \text{with} \quad \Gamma_i B^T \Sigma B \Gamma_i < \beta \Rightarrow \delta < \beta$$

$$\begin{cases} 1/\varepsilon < \beta \equiv \varepsilon > 1/\beta \\ \varepsilon = \min_{i=1:3^{n_u}} \lambda_{\min}(-\mathcal{Q}_i) \end{cases} \Rightarrow$$

$$-Q_i > (1/\beta) I, i = 1, \dots, 3^{n_u} \Rightarrow$$

$$\begin{pmatrix} Q_i & I \\ I & -\beta I \end{pmatrix} < 0, i = 1, \dots, 3^{n_u}$$

$$\begin{cases} \delta < \beta \\ \varepsilon > 1/\beta \end{cases} \Rightarrow \text{radius} < \beta \Rightarrow \min(\text{radius}) \equiv \min \beta$$

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Theorem

There exists a static state feedback controller for a saturated input system such that the system state converges toward an origin-centred ball of radius bounded by β if there exists matrices $P_1 = P_1^T > 0, R, \Sigma = \Sigma^T > 0$ solutions of the following optimization problem

$$\Gamma_i B^T \Sigma B \Gamma_i < \beta, \quad \min_{P_1, R, \Sigma} \beta \quad \left(\begin{array}{cc} Q_i & I \\ I & -\beta I \end{array} \right) < 0, \quad Q_i = \left(\begin{array}{cc} P_1 A^T + A P_1 - R^T \Lambda_i^T B^T - B \Lambda_i R & I \\ I & -\Sigma \end{array} \right)$$

$$i = 1 : 3^{n_u}$$

The state feedback controller is given by :

$$u(t) = -Kx(t)$$

$$K = R P_1^{-1}$$

Numerical example

Linear system under actuator saturation

$$\dot{x}(t) = Ax(t) + Bu_{sat}(t)$$

$$u_{sat}(t) = \sum_{i=1}^{3^{n_u}} \mu_i^{sat}(t) (\Lambda_i u(t) + \Gamma_i)$$

$$u(t) = -Kx(t)$$

$$\dot{x}(t) = \sum_{i=1}^{3^{n_u}} \mu_i(t) ((A - B\Lambda_i K)x(t) + B\Gamma_i)$$

$$A = \begin{pmatrix} -2 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0.2 & 0.1 & -3 \end{pmatrix}, B = \begin{pmatrix} 0.1 & 1 \\ 0.5 & 1 \\ 0.8 & 0.6 \end{pmatrix}, u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{cases} u_{1max} & = & 1 \\ u_{1min} & = & -1 \\ u_{2max} & = & 1.5 \\ u_{2min} & = & -1.5 \end{cases}$$

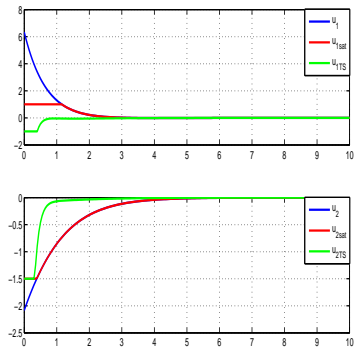
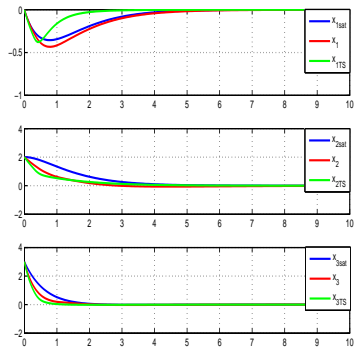


FIGURE: System states (left) - input (right) for nominal, nominal saturated and T-S saturated controllers

Conventional anti-windup controller

The conventional anti-windup controller uses the difference $v(t) - v_{sat}(t)$ as an input for a large gain matrix $\Psi = \alpha I$ where $\alpha \gg 1$ is a scalar

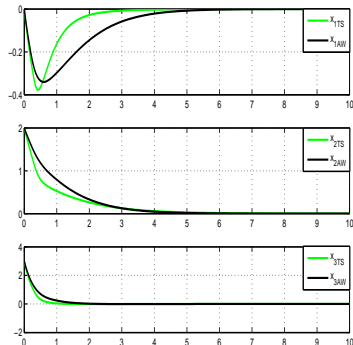
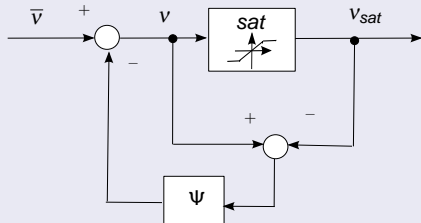


FIGURE: Conventional AW controller (left) - system states for AW and T-S saturated controllers (right)

Conclusions

- **New systematic approach for the saturation representation**
- Procedure based on the T-S representation (sector nonlinearity and convex polytopic transformation)
- Convergences conditions expressed in an optimisation problem with LMI constraints

Perspectives

- Extension to nonlinear systems with a T-S representation
- Extension to systems with unknown parameters
- Extension to the output feedback controller
- Diagnostic application and Fault Tolerant control

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Conclusions

- New systematic approach for the saturation representation
- Procedure based on the T-S representation (sector nonlinearity and convex polytopic transformation)
- Convergences conditions expressed in an optimisation problem with LMI constraints

Perspectives

- Extension to nonlinear systems with a T-S representation
- Extension to systems with unknown parameters
- Extension to the output feedback controller
- Diagnostic application and Fault Tolerant control

Thanks for your attention

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