FINITE MEMORY OBSERVER FOR SWITCHING SYSTEMS: APPLICATION TO DIAGNOSIS

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Abstract: In this paper, we develop a fault detection method for switching dynamic systems with unknown inputs. These systems are represented by several linear models, each of them being associated to a particular operation mode. The proposed method is based on the use of Finite Memory Observers and mode probabilities with the aim to finding the operating mode of the system and estimating the unknown input. The resulting method also uses the knowledge of a priori information on the mode transition probabilities represented by a Markov chain. The proposed algorithm belongs to the class of the supervised algorithms where the fault to be detected are a priori indexed and modelled. First, the method is used for fault detection in the case of a linear system characterized by a normal model of operation and several fault models. Then, it applies for fault detection in the case of a linear system with unknown input where state and unknown input estimation are done simultaneously. A comparison with the Generalized Pseudo-Bayesian method is carry out showing the advantages of the suggested method.

Keywords: Diagnosis, switching system, multiple model, finite memory observer, unknown input, state estimation.

1. INTRODUCTION

As an evidence, control of systems is becoming more and more sophisticated; that is due to the combined fact that systems are naturally complex but also because it is often desired to manage all things affecting the system. This motivates researches on reliability, availability and security. In this field, FDI (Fault Detection and Isolation) has been developed over the two last decades (Patton *et al.*, 1989), (M. Blanke and Staroswiecki, 2003). FDI is mainly based on the state estimation of a process which also produces an estimation of the process output. Comparison of estimated and measured outputs is used to design residuals that have the property to be sensitive to faults. Thus, state estimation is a key point of FDI. Generalized Pseudo-Bayesian approach of first order approach (GPB1) (Bar-Shalom, 1990) is a powerful tool to track evolution of the process functioning due to faults, that is also based on residual computing. In fact, process can be characterised by one or several models for normal operating conditions and by another set of models describing the different situations of misfunctionning affecting sensors and actuators (that are the consequence of damage of process components). Thus, this set of models can be used to describe the whole functioning of the process. The multiple model representation is often used for that purpose and is naturally exploited in GPB1 approach. In the field of estimation, the GPB has found a great success in tracking targets (Bar-Shalom *et al.*, 1989),(Bar-Shalom, 1990). In the last few years, the multiple model approach was popular and was largely used for estimation (Bar-Shalom *et al.*, 1989),(Bar-Shalom, 1990),(Bar-Shalom and Li, 1993),(Hanlon and P.S.Maybeck, 1998), control (Murray-Smith and Johansen, 1997) and modelling (Gasso *et al.*, 2001). For that purpose, a parallel bank of filters is used; each filter is based on a local model representing a particular behaviour of the real process. Evaluating residuals between filter outputs and observed process outputs allows one to design fault detectors. The GPB method is based primarily on the Kalman filter and mode probabilities. In this work, the Kalman filter is replaced by a Finite Memory Observer (FMO) (Kratz *et al.*, 1994),(Medvedev, 1996),(Nuninger *et al.*, 1998) which shows interesting characteristics owing to the fact that the estimate at the moment k is independent of the one at the moment $k-1$. Moreover, a FMO is less influenced by system noises compared to the Kalman filter.

In the second section, we present the development of the finite memory observer. In the third section, the FMO is used for the estimation of the unknown input. The fourth section presents an original method based on an FMO, within the framework of switching systems, in order to detect the changing regime. In the fifth part, the FMO with unknown input is used for simultaneously detecting the changes of modes and estimating the unknown input. Finally, a conclusion is drawn on the use of FMO for switching systems and on the suggested techniques.

2. A FINITE MEMORY OBSERVER

A Finite Memory Observer uses measurements in a finite time interval only. Consider the discrete time and invariant following system:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Gw_k \\ y_k = Cx_k + v_k \end{cases} \tag{1}$$

where x_k is the state vector at time k, A is the state matrix, u_k is the input vector at time k, B is the input gain matrix, C is the output gain matrix, v_k and w_k are respectively the state and measurement noises and y_k is the output of the system at time k.

In the noise-free case, the system is described by:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases} \tag{2}$$

Observing the system evolution on the time horizon $[k-m, k]$, we can write:

$$Y_k = P_m x_{k-m} + B_m U_k + G_m W_k + V_k \tag{3}$$

with :

$$Z_k = \begin{bmatrix} z_{k-m}^T & z_{k-m+1}^T & \dots & z_k \end{bmatrix}^T, Z \in \{Y, U, W, V\} \tag{4}$$

$$P_m = \begin{bmatrix} C^T & (CA)^T & \dots & (CA^m)^T \end{bmatrix}^T \tag{5}$$

$$B_m = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ CB & 0 & \ddots & \ddots & 0 \\ CAB & CB & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ CA^{m-1}B & CA^{m-2}B & \dots & CB & 0 \end{bmatrix} \tag{6}$$

$$G_m = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ CG & 0 & \ddots & \ddots & 0 \\ CAG & CG & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ CA^{m-1}G & CA^{m-2}G & \dots & CG & 0 \end{bmatrix} \tag{7}$$

The estimate of the state $\hat{x}_{k-m}$, at the moment $k-m$, can be obtained easily using the least square method, by minimizing the criterion $J_k = \|P_m x_{k-m} + B_m U_k - Y_k\|^2$ subject to x_{k-m}. We obtain:

$$\hat{x}_{k-m} = (P_m^T P_m)^{-1} P_m^T (Y_k - B_m U_k) \tag{8}$$

The state estimate at the final moment k of the observation window is obtained by integrating the system (2):

$$\hat{x}_k = A^m \hat{x}_{k-m} + T_m U_k \tag{9}$$

with

$$T_m = \begin{bmatrix} (A^{m-1}B)^T & (A^{m-2}B)^T & \dots & B^T & 0 \end{bmatrix}^T \tag{10}$$

Thus, at each time k, the expression of the state estimate is defined by:

$$\hat{x}_k = R_m Y_k + S_m U_k \tag{11}$$

with

$$R_m = A^m (P_m^T P_m)^{-1} P_m^T \tag{12a}$$
$$S_m = T_m - A^m (P_m^T P_m)^{-1} P_m^T B_m \tag{12b}$$

3. A FINITE MEMORY OBSERVER WITH UNKNOWN INPUT

The finite memory observer can be used for the estimation of unknown input by considering the unknown input as a state of the system. The unknown input system is written as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ed_k + Gw_k \\ y_k = Cx_k + v_k \end{cases} \tag{13}$$

where d_k is the unknown input at time k and E is the unknown input gain matrix.

The following assumption is made:

$$d_{k+1} = d_k + \delta_k$$

where δ_k is a random noise. With that definition, time-varying unknown input can be taken into account.

An augmented system can be written as follows:

$$\begin{cases} x'_{k+1} = A_a x'_k + B_a u_k + G_a w'_k \\ y_k = C_a x'_k + v_k \end{cases} \qquad (14)$$

with

$$x'_k = \left[\begin{array}{cc} x_k^T & d_k^T \end{array}\right]^T, \quad w'_k = \left[\begin{array}{cc} w_k^T & \delta_k^T \end{array}\right]^T$$

$$A_a = \begin{bmatrix} A & E \\ 0 & I \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_a = \begin{bmatrix} C & 0 \end{bmatrix} \text{ and } G_a = \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix}$$

The augmented state estimate $\hat{x}'_{k-m}$ can be carried out as in the previous case:

$$\hat{x}'_{k-m} = (P_{m,a}^T P_{m,a})^{-1} P_{m,a}^T (Y_k - B_{m,a} U_k) \qquad (15)$$

where the matrix $P_{m,a}$ and $B_{m,a}$ are built as the matrix P_m (5) and B_m (6), by replacing matrices A, B and C respectively by A_a, B_a and C_a.

As previously, expression of the augmented state estimate at time k is deduced from (15):

$$\hat{x}'_k = A_a^m \hat{x}'_{k-m} + T_{m,a} U_k \qquad (16)$$

with

$$T_{m,a} = \left[\begin{array}{ccccc} (A_a^{m-1} B_a)^T & (A_a^{m-2} B_a)^T & \dots & B_a^T & 0 \end{array}\right]^T \qquad (17)$$

This formulation allows one to simultaneously obtain state and unknown input estimates of the system.

Example: Consider the following unknown input system:

$$x_{k+1} = \begin{bmatrix} 0.45 & 0 \\ 0 & 0.4 \end{bmatrix} x_k + \begin{bmatrix} 0.1815 \\ 1.7902 \end{bmatrix} u_k + \begin{bmatrix} 0.0129 \\ -1.2504 \end{bmatrix} d_k + \begin{bmatrix} 1 \\ 10 \end{bmatrix} w_k$$

$$y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + v_k$$

First a constant unknown input (δ_k is centered) that occurs in a specified time interval is considered. Afterwards, we consider an unknown input with a ramp shape (the mean value of δ_k is different from zero).

Examination of figure 1 makes it possible to note a good estimation of the unknown input, in spite of a certain delay due to the horizon of observation of the OMF (chosen here equal to 11 ($m = 10$)). Figure 2 presents similar results when the unknown input evolves according to a ramp.

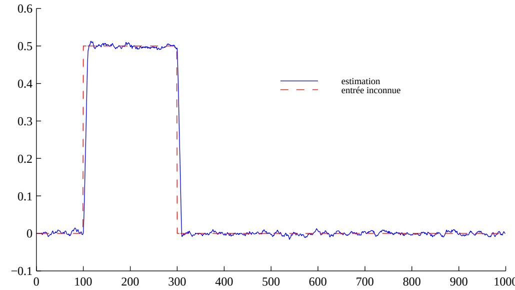

Figure 1. Constant unknown input estimation

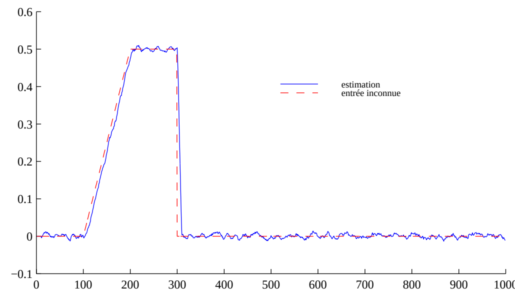

Figure 2. Variable unknown input estimation

4. FMO FOR SWITCHING SYSTEMS

In this section, we consider a system represented by a set of models $M_i, i = 1, \dots, r$; each model representing a particular behaviour of the system. The objective is to detect, at each moment, the active model and simultaneously to estimate the state of the system. The transitions from a model to another one are assumed to be described by a Markovien process governed by the a priori known Markov transition matrix π given by:

$$\Pi = \begin{bmatrix} p_{11} & \cdots & p_{1r} \\ \vdots & \ddots & \vdots \\ p_{r1} & \cdots & p_{rr} \end{bmatrix}$$

where p_{ij} is the mode transition probability from the model M_i to the model M_j; we note μ_j^k the probability that the j^{th} model is active at time k.

4.1 Development of the method

Consider the j^{th} model described by:

$$M_j : \begin{cases} x_{k+1} = A_j x_k + B_j u_k + G_j w_k \\ y_k = C_j x_k + v_k \end{cases} \qquad (18)$$

The state estimation of this model can be carried out using a FMO according to the method described in section 2. We obtain:

$$\hat{x}^j_{k-m} = (P_{j,m}^T P_{j,m})^{-1} P_{j,m}^T (Y_k - B_{j,m} U_k) \qquad (19)$$

and

$$\hat{x}^j_k = A_j^m \hat{x}^j_{k-m} + T_{j,m} U_k \qquad (20)$$

Matrices $P_{j,m}$, $B_{j,m}$ and $T_{j,m}$ are built using the definitions (5), (6) and (10) replacing matrices A,

B and C by matrices A_j, B_j and C_j related to the j^{th} model.

The state estimate $\hat{x}_k$ of the switching system is then computed as a weighted sum of the states of the "local" models:

$$\hat{x}_k = \sum_{j=1}^{r} \hat{x}_k^j \mu_k^j \tag{21}$$

Following the work of Bar-Shalom (Bar-Shalom, 1990), the probability that model j is in effect at time k is calculated in the following way:

$$\mu_k^j = P\{M_j(k)|Y_k\} \tag{22}$$

Define $\tilde{Y}_{k-1}$, the observation vector carried out on the horizon $[k-m, k-1]$; we have:

$$Y_k = \begin{bmatrix} \tilde{Y}_{k-1}^T & y_k^T \end{bmatrix} \tag{23}$$

Equation (22) can then be written as:

$$\mu_k^j = P\{M_j(k)|\tilde{Y}_{k-1}, y_k\} \tag{24}$$

Using the Bayes formula, this probability can be transformed into:

$$\mu_k^j = \frac{p\left[y_k|M_j(k), \tilde{Y}_{k-1}\right] P\{M_j(k)|\tilde{Y}_{k-1}\}}{\sum_{l=1}^{r} p\left[y_k|M_l(k), \tilde{Y}_{k-1}\right] P\{M_l(k)|\tilde{Y}_{k-1}\}} \tag{25}$$

In order to alleviate the notations, let us introduce:

$$L_i(k) = p\left[y_k|M_i(k), \tilde{Y}_{k-1}\right] \tag{26}$$

Using the total probability theorem, the activation probability of the model j at time k, according to the active model at the time $k-1$ can be written as:

$$P\{M_j(k)|\tilde{Y}_{k-1}\} = \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), \tilde{Y}_{k-1}\} P\{M_i(k-1)|\tilde{Y}_{k-1}\} \tag{27}$$

To obtain a recurrence on the computation of the μ_k^j, we carried out the following approximation:

$$P\{M_i(k-1)|\tilde{Y}_{k-1}\} \approx P\{M_i(k-1)|Y_{k-1}\} = \mu_{k-1}^i \tag{28}$$

That amounts considering that the information given by the first vector of observation y_{k-m-1} of the vector Y_{k-1} defined on the horizon $[k-m-1, k-1]$ is not very important and can be neglected (that depends obviously on the selected horizon). In this case, considering equations (25) to (28) and noticing that, by definition, $P\{M_j(k)|M_i(k-1), \tilde{Y}_k\} = p_{ij}$, the following recurrence on the probability that the system operates according to the model j at the moment k can be established:

$$\mu_k^j = \frac{L_j(k) \sum_{i=1}^{r} p_{ij} \mu_{k-1}^j}{\sum_{l=1}^{r} L_l(k) \sum_{i=1}^{r} p_{il} \mu_{k-1}^j} \tag{29}$$

4.2 Fault models

An actuator fault can be modelled by "modifying" the appropriate column of the control input matrix B. Thus, a fault on the i^{th} actuator is represented by writing the following equation:

$$x_{k+1} = Ax_k + (B + \Delta B_i)\, u_k + w_k$$

where ΔB_i is a matrix of same dimension that B; all of its columns are null except the i^{th} which characterizes the fault on the i^{th} actuator.

On a same way, a sensor fault is described by:

$$y_k = (C + \Delta C_i)\, x_k + v_k$$

where ΔC_i and C have the same dimension; all of its columns are null except the i^{th} that characterizes the fault on the i^{th} sensor.

4.3 Application

For the application of the suggested method, we consider a model of normal operating (A_1, B_1, C_1), a model of fault actuator (A_2, B_2, C_2) and a model of sensor faults (A_3, B_3, C_3), the various matrices being defined by:

$$A_i = \begin{bmatrix} 0.45 & 0 \\ 0 & 0.4 \end{bmatrix}, \; i = 1 \ldots 3$$

$$B_1 = \begin{bmatrix} 0.1815 & 1.7902 \end{bmatrix}^T, \; C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.1815 & 1.7902 \end{bmatrix}^T, \; C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.1815 & 1.7902 \end{bmatrix}^T, \; C_3 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}.$$

To test the method, the following scenario was established: initially the system normally operates, then at time 100, an actuator fault occurs, at time 500, the system returns to the normal operating mode and, at time 800, sensor faults are introduced.

The results are presented at the figures 3, 4 and 5 where the changes of mode clearly appears; the mode probabilities of the models, in their respective operation zones, fluctuates around one and thus a detection of the default is carried out. It is noticed that the results of the suggested method are better than those of GPB1 method. That is due to the sensitivity of the latter to the noise of the system. It is thus concluded that the use of a finite memory

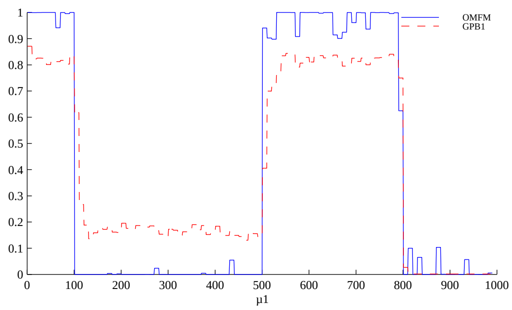

Figure 3. Activation probability of model 1

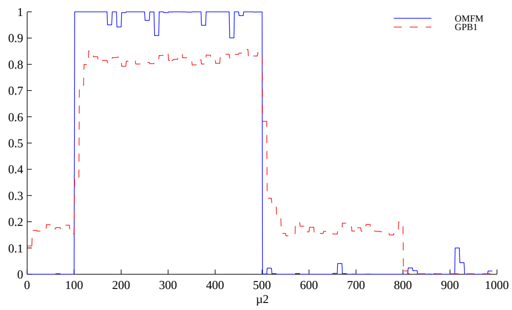

Figure 4. Activation probability of model 2

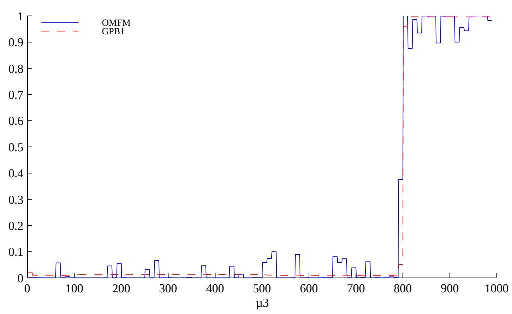

Figure 5. Activation probability of model 3

observer, for fault detection within the framework of switching systems, gives good results which are less sensitive to the noise than traditional GPB method.

5. EXTENSION OF THE METHOD TO UNKNOWN INPUT SYSTEM

The suggested method in preceding section can be applied to unknown input systems. For that, the finite memory observer of the second section is replaced by a finite memory observer with unknown input (see the third section).

The j^{th} model is written as follows:

$$M_j \left\{ \begin{array}{l} x_{k+1} = A_j x_k + B_j u_k + E d_k + G w_k \\ \quad y_k = C_j x_k + v_k \end{array} \right.$$

where d_k is the unknown input at the time k and where E is the unknown input gain matrix.

Using an augmented model for each model M_j, as indicated in section 3, the state and the unknown input can be simultaneously estimated. The estimation of the system state is then obtained following the steps exposed in subsection 4.1.

Example: We consider the same models of normal operation, actuator fault and sensor faults than previously, disturbed by the unknown input d_k whose influence matrix is:

$$E = \left[\, 0.0129 \;\; -1.2504 \, \right]^T$$

To test the method, the following scenario was established: initially the system normally operates, at time 100 occurs an unknown input with a constant magnitude, then at time 200, an actuator fault occurs, at time 300, the unknown input becomes null, at time 500, the system returns to the normal operating mode and, at time 800, sensor faults are introduced.

The results, in the presence of noise, are shown on figures 6 to 8. They clearly exhibit the changes from one mode to another, allowing the detection of faults. The figure 9 shows the estimation of the unknown input in the absence of noise. In the presence of noise, this estimate is represented in the figure 10. We can affirm that the use of a finite memory observer with unknown input for fault detection and unknown input estimation, within the framework of switching systems, gives good results in spite of the presence of noise.

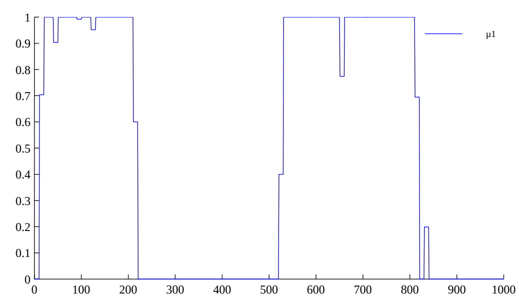

Figure 6. Activation probability of model 1 in the presence of noise

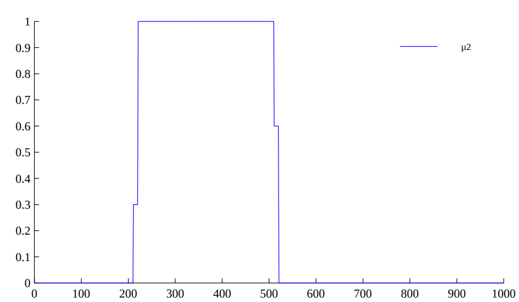

Figure 7. Activation probability of model 2 in the presence of noise

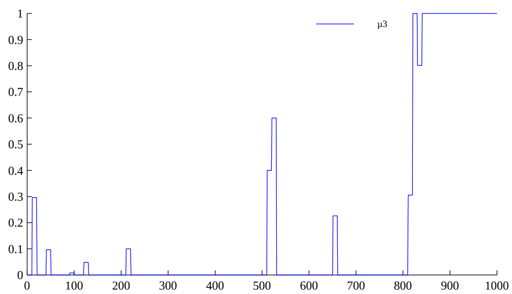

Figure 8. Activation probability of model 3 in the presence of noise

6. CONCLUSION

In this work, the structure of a Finite Memory Observer was firstly recalled. Such observer was then applied successfully for estimation of unknown input. This type of observer was used within the framework of a switching systems for which the moments of commutation between models must be detected. The comparison of the obtained results with the GPB1 method was carried out on an example. The use of a finite memory observer, based on measured outputs only, contrarily to the GPB1 method which uses estimates, gives better results, mainly in the presence of noises.

Finally, the suggested method was extended to the case of systems subjected to unknown input. In this situation, the detection of the moments of commutation is carried out simultaneously with the estimation of the unknown input.

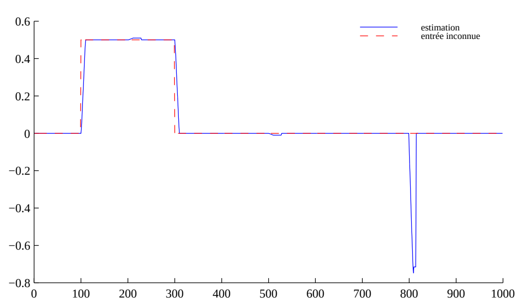

Figure 9. Unknown input estimation in the absence of noise

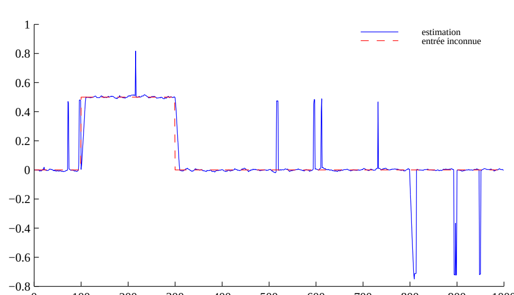

Figure 10. Unknown input estimation in the presence of noise

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