# "DISTINGUISHABILITY" OF SWITCHING SYSTEMS FOR DIAGNOSIS

**E. A. Domlan, J. Ragot, D. Maquin**

*Centre de Recherche en Automatique de Nancy, UMR CNRS* 7039
*Institut National Polytechnique de Lorraine,*
2, *avenue de la forêt de Haye,*
54516 *Vandœuvre-Lès-Nancy, France*
*{eadomlan, jragot, dmaquin} @ensem.inpl-nancy.fr*

Abstract
This article tackles the problem of the diagnosis of switching systems. These systems are described by several regimes, each of them being active under certain particular operating conditions which can be controlled or not. If the operating conditions are unknown, the diagnosis of such systems is complex. It is even more complicated in the situation where the various models of the operating regimes of the switching system tend to give very close outputs when they are excited with the same inputs. Moreover the disturbing effect of the measurement noise can make the diagnosis unrealizable. It is then necessary to specify the conditions under which one can find the active operating regime of a switching system from the only knowledge of the inputs applied to the system and its outputs.

Keywords: bounded noise, diagnosis, "distinguishability", switching system.

## 1. INTRODUCTION

The major part of real life processes presents a typically hybrid behavior, in the sense that these processes are subjected at the same time to a continuous and a discrete dynamics, with discrete events which intervene punctually or continuously to change the system's continuous dynamics. Multiple-models approach [1, 5, 8, 9] or hybrid approach [16, 17] allow a good modeling of this kind of process. A significant class of hybrid systems, switching systems, is obtained by formulating the assumption that the transition from one continuous dynamics to the other is governed by an abrupt function like a step function. The system obtained is thus represented by a number of *a priori* known models of operating regime. The system can switch from one operating regime to the other and the switching instant is not necessarily known. The diagnosis of such a system starts with the implementation of methods allowing to know, at any moment, the operating regime of the system. Many recent publications [2–4, 6, 7, 10–15] are related to this delicate problem. In all these publications, the inputs and the outputs of the considered system are used to recognize the active operating regime or to detect faults on the system. Unfortunately, it can happen that two operating regimes of the system give similar outputs when they are excited with the same inputs. Under the influence of the measurement noise, it becomes delicate, even impossible, to distinguish the two operating regimes based on the knowledge of the measurements of the inputs and the outputs of the system. To our knowledge, this point of view was not approached in the previous publications. This article aims to determine if it is always possible to recognize the operating regime of a switching system based on the measurements of its inputs and its outputs. Thus, one has to specify the conditions of "distinguishability" of the various operating regimes of the system. The article is limited to the case of static systems, the case of dynamic systems being at the present time our subject of interest.

## 2. POSITION OF THE PROBLEM

Let us consider a static SISO system with input $u(k)$ and output $y(k)$ disturbed by a bounded noise $\varepsilon_y(k)$. The gain $K$ of the system can take various beforehand indexed values ($K_i,\ i = 1, \ldots, p$). From the measurements of the input $u(k)$ and the output $y(k)$, can one find, at every moment, the active gain of the system? Moreover, is it possible to specify the conditions of "distinguishability" of the gains according to the measurement noise? The considered system is represented by the equation (1):

$$\begin{cases} y(k) = K_i u(k), \quad u \in \mathbb{R}, \quad y \in \mathbb{R} \\ y_m(k) = y(k) + \varepsilon_y(k) \\ K_i \in \{K_1, K_2, \cdots, K_p\}, \quad p \in \mathbb{N}^* \setminus \{1\} \\ \varepsilon_y(k) \in [-\delta, \delta], \ \delta > 0 \end{cases} \tag{1}$$

The variable $y_m(\cdot)$ is the measured output of the system. We suppose thereafter that $p = 2$, this for reasons of clearness in the drafting. The extension of the following developments to values of $p$ higher than two is commonplace.

## 3. "DISTINGUISHABILITY" IN THE RESIDUAL'S PLAN

Knowing the set of the values of the system's gain, one can define and evaluate at every instant the residuals (2) from the measurements:

$$\begin{cases} r_1(k) = y_m(k) - K_1 u(k) \\ r_2(k) = y_m(k) - K_2 u(k) \end{cases} \tag{2}$$

Now let us suppose that at an instant $k_0$, the active gain is the gain $K_1$ and that the input of the system is maintained constant at the value $u(k_0)$ $(u(k) = u(k_0), \forall k \geq k_0)$. Taking into account the definitions (1), the residuals (2) become:

$$\begin{cases} r_1(k_0) = \varepsilon_y(k_0) \\ r_2(k_0) = (K_1 - K_2)\, u(k_0) + \varepsilon_y(k_0) \end{cases} \tag{3}$$

By making the difference of the two residuals (3), one obtains:

$$r_2(k_0) - r_1(k_0) = (K_1 - K_2)\, u(k_0) \tag{4}$$

Thus, at the instant $k_0$, the residual $r_1(k_0)$ is equal to the measurement noise $\varepsilon_y(k_0)$ when the gain $K_1$ is active. The difference $r_2(k_0) - r_1(k_0)$ does not depend of the magnitude of the measurement noise but rather of the value of the input $u(k_0)$. It is then obvious that the couples of residuals $(r_1; r_2)$ characterizing the input $(u(k) = u(k_0), \forall k \geq k_0)$ describe a segment $S_1$ in the plan $\{r_1, r_2\}$. Indeed, when the gain $K_1$ is active, the residual $r_1(\cdot)$ is in the strip of the residual's plan corresponding to the boundaries of the magnitude of the noise (the strip limited by the straight lines of equation $r_1 = -\delta$ and $r_1 = \delta$). The residual $r_1(\cdot)$ belongs to the interval $[-\delta; \delta]$. Moreover, the difference between the residuals $r_2(\cdot)$ and $r_1(\cdot)$ being a constant, the place of the couples of residuals $(r_1; r_2)$ related to the input $u(k_0)$ is the intersection of the straight line of equation $r_2(k) - r_1(k) = (K_1 - K_2)\, u(k_0)$ with the strip of plan quoted previously. This intersection is obviously a segment. By making the same reasoning, when this time the gain $K_2$ is active, one shows that the difference $r_2(k_0) - r_1(k_0)$ at a given instant $k_0$ remains independent of the magnitude of the measurement noise and corresponds to the expression of equation (4). The couples of residuals $(r_1; r_2)$ depending of the input $(u(k) = u(k_0), \forall k \geq k_0)$ also describe a segment $S_2$ in the residual's plan $\{r_1, r_2\}$.

Thus for each input, it is possible to characterize the segment $S_1$ or $S_2$ (see figure 1) described by the couples of residuals $(r_1; r_2)$ according to whether one or the other of the gains is active. This characterization being made, one just has to position the residuals obtained in the plan of the residuals in order to determine which gain is active. This distinction is always effective if the two segments $S_1$ and $S_2$ do not overlap. Let us determine the co-ordinates of the end of these segments.

Let us denoted by $[P_1 Q_1]$ the segment $S_1$ described by the couple of residuals when the gain $K_1$ is active. The segment $[P_1 Q_1]$ results from the intersection between the strip of the plan of the residuals limited by the straight lines of equation $(D^-) : r_1 = -\delta$ and $(D^+) : r_1 = \delta$ with the straight line of equation $(D) : r_1 - r_2 = (K_1 - K_2)\, u$. While replacing in the equation of the difference of the two residuals, $r_1$ respectively by $-\delta$ and $\delta$, one finds easily the co-ordinates of the points $P_1$ and $Q_1$ : $P_1(-\delta; (K_1 - K_2)\, u - \delta)$ and $Q_1(\delta; (K_1 - K_2)\, U + \delta)$.

In the same way, by noting $[P_2 Q_2]$ the segment $S_2$ described by the couple of residuals when the gain $K_2$ is active, one finds the co-ordinates of the points $P_2$ and $Q_2$: $P_2((K_2 - K_1)\, u - \delta; -\delta)$ and $Q_2((K_2 - K_1)\, U + \delta; \delta)$.

So that the segments $[P_1 Q_1]$ and $[P_2 Q_2]$ do not overlap, the intersection of their projection on the axes $r_1$ or $r_2$ has to be empty. This is stated by the condition:

$$|u| > \frac{2\delta}{|K_1 - K_2|} \tag{5}$$

The inequality (5) shows that the parameters $K_1$, $K_2$ and $\delta$ being known, it is possible to distinguish the two operating regimes associated respectively with the gains $K_1$ and $K_2$ only if the system is excited with an input which is higher than twice the ratio between the maximum magnitude of the measurement noise and the absolute value of the difference of the gains $K_1$ and $K_2$.

Another way of presenting this problem of discrimination consists in providing the upper limit $\delta$ of the measurement noise.

If the input of the systems are fixed by other considerations, then the condition (5) expresses the fact that the diagnosis is possible.

*Example*

Let us consider the system represented by the equation (1) with $p = 2$, $K_1 = 3.21$, $K_2 = 2.11$ and $\delta = 1$. One has : $\dfrac{2\delta}{|K_1 - K_2|} = 1.82$. Thus, an input of magnitude in absolute value higher than 1.82 ensures an empty intersection between the two segments $[P_1Q_1]$ and $[P_2Q_2\ ]$ and allow a perfect distinction between the two models.

The figure 1 presents the distribution of the couples of residuals in the plan of the residuals for a constant input of magnitude 2. One can see there that the two segments are disjoined ensuring a total "distinguishability" of the two models. The figure 2 shows the same example but this time for a constant input of magnitude 1.5. The intersection between the two segments appears clearly. There is a zone (the segment $[P_1Q_2]$) in which one cannot say if the couples of residuals represented must be associated to the operating regime related to the gain $K_1$ or to the one related to the gain $K_2$.

The figure 4 shows the localization of the couples of residuals in the plan of the residuals for a time-varying input. The input applied to the system is represented at the figure 3. On the figure 4, one can observe couples of residuals which are in the zone of "nondistingability". These couples of residuals correspond to the inputs of the figure 3 which magnitude is below the threshold $\dfrac{2\delta}{|K_1 - K_2|}$, threshold indicated in dotted line. Finally, on the figure 5 is presented the evolution of an indicator of diagnosis defined from the equation (5):

$$Ind_1\,(k) = \frac{1}{2}\left(1 + sign\left(|u| - \frac{2\delta}{|K_1 - K_2|}\right)\right) \tag{6}$$

This indicator takes a zero value in the zone of "nondistinguishability" and the value 1 when the diagnosis is realizable.

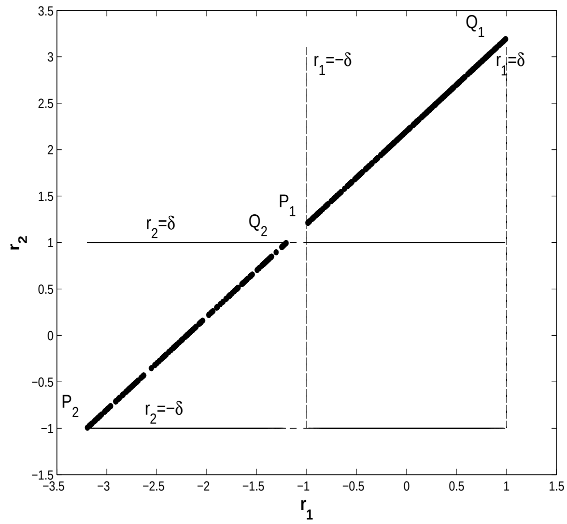


Figure 1. complete "distinguishability" in the residual's plan $\{r_1, r_2\}$

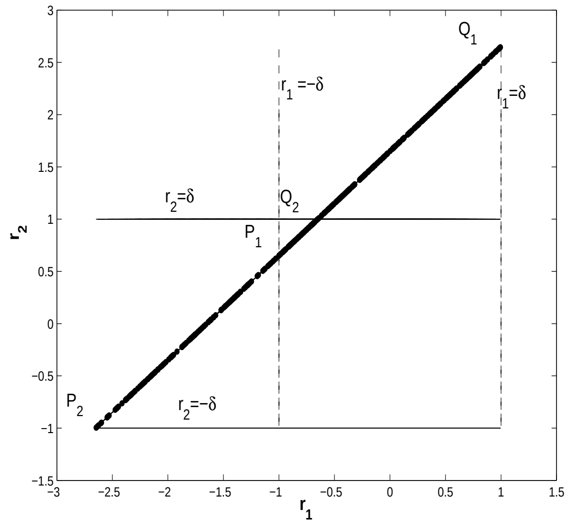


Figure 2. presence of area of "nondistinguishability" in the plan $\{r_1, r_2\}$

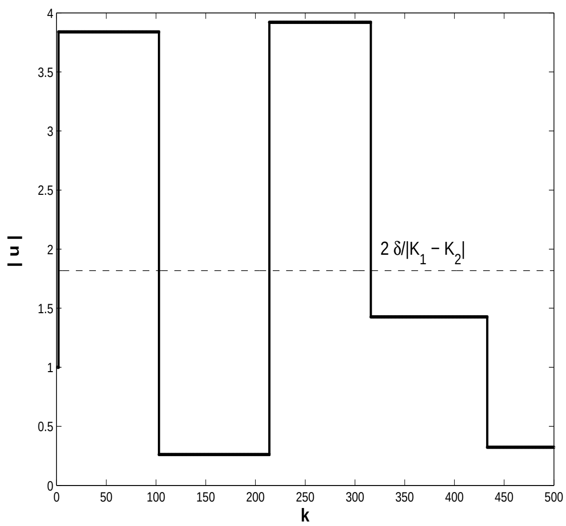


Figure 3. time-varying input

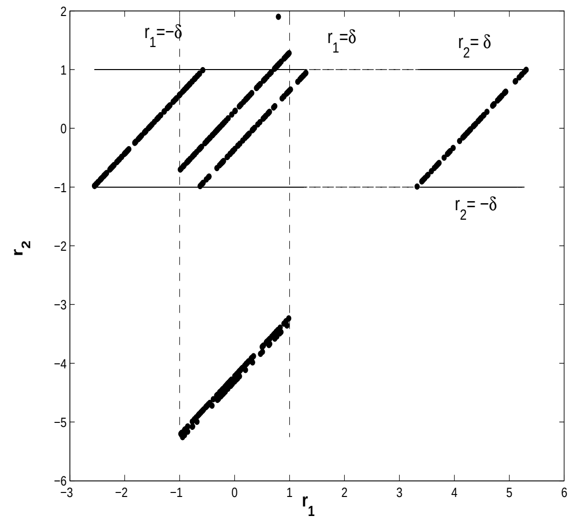


Figure 4. localization of the couples of residuals

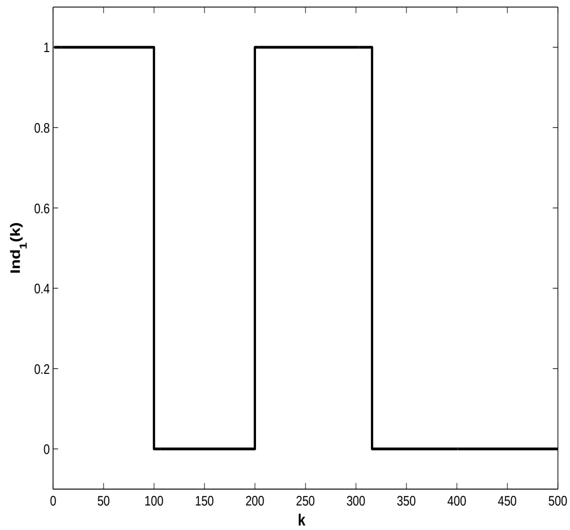


Figure 5. indicator of diagnosis

### *3.1 "Distinguishability" in the input/output plan*

A similar reasoning as previously is taken again here, but this time in the plan of the inputs and outputs of the system $\{u, y\}$. From the known input $u(k)$, one can generate at each instant the outputs associated to the two operating regimes of the system:

$$\begin{cases} y_1(k) = K_1 u(k) \\ y_2(k) = K_2 u(k) \end{cases} \tag{7}$$

The outputs of the two models generated thanks to the equation (7) define two fields to which must belong the measurements $y_m(k)$. By taking into account (1), these fields are defined by the following equations:

$$D_1 = \{y \in \mathbb{R}/y = K_1 u(k) + \varepsilon_y(k)\} \tag{8}$$

$$D_2 = \{y \in \mathbb{R}/y = K_2 u(k) + \varepsilon_y(k)\} \tag{9}$$

As $\varepsilon_y(k)$ is bounded, the fields $D_i$ are strips of the plan $\{u, y\}$ limited by the parallel straight lines:

$$D_i^+ = \{y \in \mathbb{R}/y = K_i u(k) + \delta\} \tag{10}$$

$$D_i^- = \{y \in \mathbb{R}/y = K_i u(k) - \delta\} \tag{11}$$

From a couple of measurement $(u(k_0); y_m(k_0))$, one can find the input's set which could generate the output $y_m(k_0)$, this for each model of the system. This set is obtained by projecting the intersection of the straight line of equation $y_m = y_m(k_0)$ with the fields $D_i$ on the axis of the inputs. Thus, one obtains two intervals $I_1$ and $I_2$ defined respectively for the model associated with the gain $K_1$ and for the one associated to the gain $K_2$:

$$\begin{cases} I_1 = \left[\dfrac{y_m(k_0) - \delta}{K_1}; \dfrac{y_m(k_0) + \delta}{K_1}\right] \\ I_2 = \left[\dfrac{y_m(k_0) - \delta}{K_2}; \dfrac{y_m(k_0) + \delta}{K_2}\right] \end{cases} \tag{12}$$

The gains $K_1$ and $K_2$ will be discernible at the instant $k_0$ if, for the measured output $y_m(k_0)$, the intervals $I_1$ and $I_2$ are disjoined. This is explained by the fact that the intersection of the intervals $I_1$ and $I_2$ corresponds to the set of the inputs which applied to the system give an output $y_m(k_0)$ which belong both to the strip $D_1$ and the strip $D_2$. In the case of existence of an intersection between the intervals $I_1$ and $I_2$, the gains $K_1$ and $K_2$ will not be distinguishable if the input $u(k_0)$ belongs to the interval $(I_1 \cap I_2)$. If the input $u(k_0)$ does not belong to the interval $(I_1 \cap I_2)$, the output $y_m(k_0)$ which corresponds to this input belongs to one and only one of the strips $D_i$. Thus, the "distinguishability" of the system at the instant $k_0$ is not assured if:

$$\begin{cases} (I_1 \cap I_2) \neq \emptyset \\ u(k_0) \in (I_1 \cap I_2) \end{cases} \tag{13}$$

Let us supposed that $K_1 > K_2$ and $y_m(k_0) > 0$. If $I_1 \cap I_2 \neq \emptyset$ then:

$$(I_1 \cap I_2) = \left[\frac{y_m(k_0) + \delta}{K_1}; \frac{y_m(k_0) - \delta}{K_2}\right]$$

To have $u(k_0) \in (I_1 \cap I_2)$, it is necessary that:

$$\begin{cases} \dfrac{y_m(k_0) + \delta}{K_1} - u(k_0) < 0 \\ \dfrac{y_m(k_0) - \delta}{K_2} - u(k_0) > 0 \end{cases} \tag{14}$$

That is to say:

$$\begin{cases} \dfrac{y_m(k_0)}{K_1} - u(k_0) + \dfrac{\delta}{K_1} < 0 \\ \dfrac{y_m(k_0)}{K_2} - u(k_0) - \dfrac{\delta}{K_2} > 0 \end{cases} \tag{15}$$

By supposing that $y_m(k_0) < 0$ and renewing the preceding reasoning, one obtains two other inequalities:

$$
\begin{cases}
\dfrac{y_m\left(k_0\right)}{K_1} - u\left(k_0\right) - \dfrac{\delta}{K_1} > 0 \\
\dfrac{y_m\left(k_0\right)}{K_2} - u\left(k_0\right) + \dfrac{\delta}{K_2} < 0
\end{cases}
\tag{16}
$$

In short, the field of "nondistinguishability" in the plan $\{u, y\}$ is characterized by the system of linear inequalities (17):

$$
\begin{cases}
\dfrac{y_m\left(k_0\right)}{K_1} - u\left(k_0\right) + \dfrac{\delta}{K_1} < 0 \\
\dfrac{y_m\left(k_0\right)}{K_1} - u\left(k_0\right) - \dfrac{\delta}{K_1} > 0 \\
\dfrac{y_m\left(k_0\right)}{K_2} - u\left(k_0\right) - \dfrac{\delta}{K_2} > 0 \\
\dfrac{y_m\left(k_0\right)}{K_2} - u\left(k_0\right) + \dfrac{\delta}{K_2} < 0
\end{cases}
\tag{17}
$$

The field defined by the system of inequations (17) is generally a polytope.

The figure 6 illustrates the recognition of the active gain in the plan $\{u, y\}$ for the same system as the one used previously. The zone of "nondistinguishability" is given by the system of inequations (17) and corresponds to the parallelogram $(P_1P_2P_3P_4)$. By not taking into account the relative position of the input $u(k_0)$ compared to the intersection of the intervals $I_1$ and $I_2$, one raises the real field of uncertainty $(P_1P_2P_3P_4)$. The field of uncertainty corresponds in this case to the parallelogram $(P_1P_2P_3P_4)$ increased with the grayed zones on the figure 6. This is due to the fact that, for a measurement $y_m(k_0)$ of the output, the obtained intervals $I_1$ and $I_2$ remain the same ones, independently from the input $u(k_0)$ of the system. If the input does not belong to the field of intersection of the projections, the diagnosis remains realizable although the projections are not disjoined. One then defines in the equation (18) a new indicator of diagnosis:

$$
\begin{gathered}
Ind_2\left(k\right) = \frac{1}{2}\left(1 + sign\left(|u| - \frac{2\delta}{|K_1 - K_2|}\right)\right) \\
\psi(u\left(k\right); y_m\left(k\right))
\end{gathered}
\tag{18}
$$

$$
\psi(u\left(k\right); y_m\left(k\right)) = \begin{cases}
0 \text{ if } (I = I_1 \cap I_2 \neq \emptyset) \\
\text{and} \quad (u\left(k\right) \in I) \\
1 \text{ otherwise}
\end{cases}
$$

As in the case of the representation in the plan of the residuals, this indicator is equal to 1 when the diagnosis is feasible from the couple of measurement $(u(k_0); y_m(k_0))$ and to 0 in the contrary case.

*Remark* 1

The preceding developments were made with the aim to characterize the set of the inputs guaranteeing the "distinguishability" of the models as-

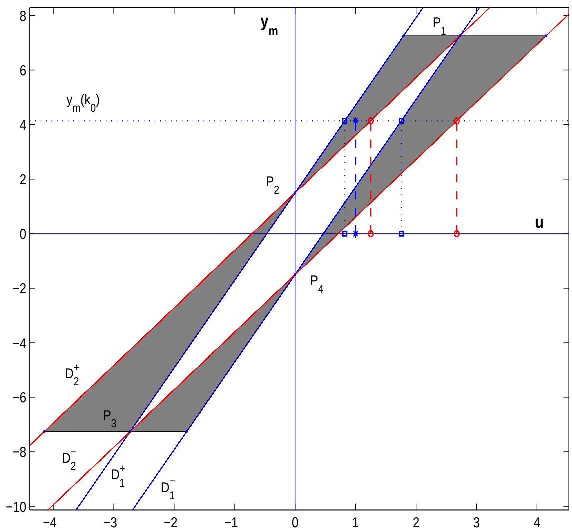


Figure 6. field of the solutions in the plan $\{u, y\}$

sociated to the various operating regimes of the switching system. It is possible to make the reverse reasoning i.e. to suppose that one knows the field of variation of the input and to characterize the maximum error $\delta$ authorized on the measurement.

*Remark* 2

Up to now, it was only taken account of the presence of a measurement noise $\varepsilon_y(\cdot)$ on the output of the system. It is also possible to take into account the presence of a measurement noise $\varepsilon_u(\cdot)$ on the input applied to the system. In this configuration, the uncertainties generated on the residuals will be larger because they will take into account as well as the measurement noise on the input of the system as the one on its output.

## 4. CONCLUSION

This article presents an analysis of the conditions under which it is possible to find the model associated to the operating regime in progress for a switching system from the only knowledge of the inputs and the outputs of the system. The adopted approach for the analysis of this problem rests on the concepts resulting from the interval methods applied to the generated residuals from the various models of the system. It was put forward conditions on the inputs and the outputs of the system according to the measurement noise. Other situations require a thorough study, in particular the case of coupled errors, the presence of disturbing parameters or errors on all variables. An extension to uncertain systems with bounded parameters will be considered in our future work.

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