2014 IEEE Conference on Control Applications (CCA)
Part of 2014 IEEE Multi-conference on Systems and Control
October 8-10, 2014. Antibes, France

State tracking control for Takagi-Sugeno models

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Abstract

This work addresses the model reference tracking control problem. It aims to highlight the encoutered difficulties and the proposed solutions to achieve the tracking objective for nonlinear systems described by Takagi-Sugeno (T-S) models.
Different control strategies are exposed. Exact state tracking is proposed and structural conditions for it are given. Approximate state tracking is also studied. The choice of the reference model to be tracked is discussed, as well as the criterion to be minimized to achieve given tracking objectives.

1. Introduction and paper outline

The main objective of this work is to deal with reference model tracking for nonlinear systems described by Takagi-Sugeno (T-S) models. The T-S models are known to be an efficient way to deal with the problems of estimation and control of nonlinear systems by writing them in a polytopic form. Originally introduced by [1], the T-S representation allows to exactly describe nonlinear systems, provided that the nonlinearities are bounded. This is reasonable since state variables as well as parameters of physical systems are bounded, and so is the input of the system which may be considered naturally stable, or equipped with a stabilizing control (see for example [2] and the references therein).
Despite an abundant literature on stability conditions of T-S models, few authors have dealt with the tracking problem for all the state of the system; indeed most of the works deal with the tracking of the system output, which generally reduces to some combinations of these variables. One can refer to some works concerned with state or output feedback with H_∞ performances [3], [4] and [5]. The nonlinear tracking control problem is expressed in terms of Linear Matrix Inequality (LMI) and is based on the T-S and Parallel Distributed Compensation (PDC) structures (an $\mathscr{L}_2$ tracking performance related to the tracking error is formulated and a PDC state feedback control (or output feedback control) is designed. See [5], [4] for examples). However, in the cited references, a referred "suitable" choice for the reference model is made without any explanations nor details.
The last remark motivated the present study. In fact, either for the linear case or the nonlinear one, few works detail the influence of the reference model choice, which is not a trivial task. In [6] for example, the authors referred to the Erzberger's conditions, but with no further explanations. For these reasons, in the proposed work, a focus is made not only on the design control procedure, but also on the tracking (matching) conditions.
The paper is organized as follows. In section II, the structural conditions to achieve state tracking are introduced in the T-S case. These conditions are an extension of the well known Erzberger's conditions. Two procedures for state tracking are considered: for the first one, the controller structure is firstly fixed and then the structural conditions and the appropriate gains of the controller are deduced. For the second one, no prerequisited control law is considered, with the objective to achieve a null tracking error for both strategies. In section III, the quadratic optimal control with an extension to the Model Predictive Control for T-S models are presented. Simulation examples are given in section IV. Finally, section V summarizes the obtained results.

2. Structural conditions for exact state tracking for T-S systems

2.1. Model and objective

Let us consider the following T-S model [1]:

$$x_{k+1} = A_k\, x_k + B_k\, u_k \tag{1}$$

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s.t. $x_k \in R^{n_x}$ and $u_k \in R^{n_u}$ with:

$$A_k = \sum_{i=1}^{r} \mu_{i,k}(\xi_k) A_i, \quad B_k = \sum_{i=1}^{r} \mu_{i,k}(\xi_k) B_i \tag{2}$$

where B_k is supposed to be a full column rank matrix and where the weighting functions $\mu_{i,k}(\xi_k)$ depend on the so-called premise variable ξ_k which may be a state, input, or output combination. These weighting functions satisfy the following convex sum property:

$$0 \leq \mu_{i,k}(\xi_k) \leq 1, \sum_{i=1}^{r} \mu_{i,k}(\xi_k) = 1 \tag{3}$$

The considered linear reference model is the following:

$$x_{r,k+1} = A_r\, x_{r,k} + B_r\, u_{r,k} \tag{4}$$

s.t. $x_{r,k} \in R^{n_x}$, $u_{r,k} \in R^{n_u}$ and where the desired performances are defined by the choice of the matrices A_r and B_r.

The ideal tracking objective is to adjust, at each instant k, the control u_k in such a way that the system state x_k follows the reference model state $x_{r,k}$ with a null tracking error. For this purpose, two procedures may be considered. The first strategy consists in setting the controller structure and then deduce the appropriate structural conditions that must satisfy the reference model and controller gains. The second strategy is not based on a prerequisited control law structure. The idea is to find, as for the first strategy, the appropriate structural conditions, but also an analytical expression for the control law. If the ideal tracking is not reachable, some compromises need to be defined such as, for example, tracking of a subset of the states, instead of all of them.

2.2. Prerequisited control law

In order to achieve the tracking objective, the following control law is considered:

$$u_k = K_k\, x_k + K_{r,k}\, u_{r,k} \tag{5}$$

Substituting (5) into (1), the closed-loop system is:

$$x_{k+1} = (A_k + B_k\, K_k)\, x_k + B_k\, K_{r,k}\, u_{r,k} \tag{6}$$

The matching conditions for the reference model and the system are then obtained by comparing the closed-loop system (6) and the reference model (4). They are given by:

$$\begin{cases} A_k + B_k\, K_k &= A_r \\ B_k\, K_{r,k} &= B_r \end{cases} \tag{7}$$

From (7), in order to have a solution in respect to the gain K_k and $K_{r,k}$, the following rank conditions have to be fullfiled:

$$\begin{cases} rank[B_k] &= rank[B_k | A_r - A_k] \\ rank[B_k] &= rank[B_k | B_r] \end{cases} \tag{8}$$

where B_k and A_k are defined in (2). If conditions (8) are fulfilled, then at each sampling time, the gains K_k and $K_{r,k}$ are given by:

$$K_{r,k} = B_k^+ B_r, \quad K_k = B_k^+ (A_r - A_k) \tag{9}$$

with B_k^+ a suitable pseudo-inverse matrix of the full column rank B_k matrix.

Note that in order to satisfy the matching conditions (8), from definitions (2), since the system matrices A_k and B_k depend on the time, one sufficient, but not unique, condition is to consider the matrices A_i, B_i and A_r, B_r in the following canonical form:

$$\begin{array}{c} A_i = \begin{pmatrix} A_0 \\ \hline \overline{A}_i \end{pmatrix}, A_r = \begin{pmatrix} A_0 \\ \hline \overline{A}_r \end{pmatrix} \\ B_i = \begin{pmatrix} 0_{n_x - n_u} \\ \hline b_i \end{pmatrix}, B_r = \begin{pmatrix} 0_{n_x - n_u} \\ \hline b_r \end{pmatrix} \end{array} \tag{10}$$

with A_0 a matrix of dimension $(n_x - n_u) \times n_x$, $\overline{A}_i$ and $\overline{A}_r$ matrices of dimensions $n_u \times n_x$. b_i and b_r are of dimension $n_u \times n_u$. The structure (10) means that:

1. the $(n_x - n_u)$ first rows of the matrices A_i are equal to the $(n_x - n_u)$ first rows of the matrix A_r
2. the $(n_x - n_u)$ first rows of the matrices B_i are null
3. the $(n_x - n_u)$ first rows of the matrix B_r are null

allowing to fully satisfy the rank conditions (8). It is important to note that the matching conditions (8) between the reference model and the system depend on the choice of the control law, u_k given by (5). It means that these conditions have to be adapted when changing the structure of the control law.

2.3. Numerical example

To illustrate the above conditions, let us consider the following academic example:

$$A_r = \begin{pmatrix} 0.2 & 0.5 & 0 \\ -0.2 & 0.99 & -0.1 \\ 0 & 0 & 0.2 \end{pmatrix}, B_r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0.2 & 0.5 & 0 \\ -0.2 & 0.99 & -0.1 \\ 0 & 0 & 0.1 \end{pmatrix}, B_1 = \begin{pmatrix} 0 \\ 0 \\ 1.5 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.2 & 0.5 & 0 \\ -0.2 & 0.99 & -0.1 \\ 0 & 0 & 1.1 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ 0 \\ -0.5 \end{pmatrix} \quad (11)$$

The weighting functions are taken as:

$$\mu_{1,k} = \frac{2 - \sin(x_{1,k}) - \tanh(x_{2,k})}{4}, \ \mu_{2,k} = 1 - \mu_{1,k} \quad (12)$$

Applying the tracking control law (5) with (9), the system and model reference states are depicted in figure 1 (respectively noted x_i and x_{ir}, $i = 1, \ldots, 3$). In figure 2, the control inputs $u_{r,k}$ and u_k are represented.

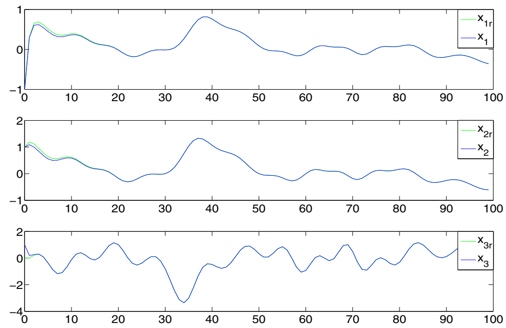

Figure 1. System and model reference states

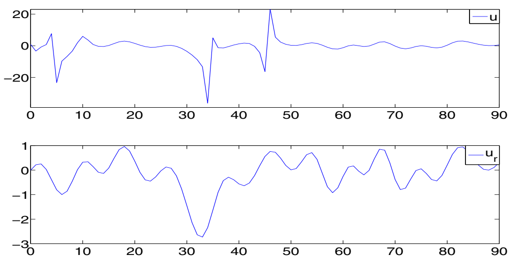

Figure 2. Control inputs $u_{r,k}$ and u_k

From the depicted figures, one can see that the control tracking is efficient for all the three states under the specified structural conditions.

2.4. No prerequisited control law

In this subsection, no prerequisited structure of the control law is considered. Then, at time k, in order to achieve the tracking objective at time $k+1$ with a null state tracking error between (4) and (1), meaning:

$$x_{r,k+1} = A_k\, x_k + B_k\, u_k \quad (13)$$

the control law u_k has to verify:

$$B_k\, u_k = x_{r,k+1} - A_k\, x_k \quad (14)$$

Note that the exact state tracking is ensured, i.e. (13) has a solution in respect to u_k, only if the following matching condition is fulfilled at each sampling time:

$$rank\,[B_k] = rank\,\left[B_k | x_{r,k+1} - A_k x_k\right] \quad (15)$$

The tracking control is deduced and given by the following equation:

$$u_k = (B_k^T B_k)^{-1} B_k^T (x_{r,k+1} - A_k\, x_k) \quad (16)$$

and the reader is invited to compare the two structures (16) and (5) with the help of (9).

Remark 1 *If the premise variables ξ_k of the weighting functions $\mu_{i,k}$ depend on the input u_k, the control law (16) will be implicit (i.e. $u_k = F(u_k)$) since A_k and B_k are input depending (2). A solution may be given by an iterative algorithm with the following recurrence:*

$$u_k^{(j+1)} = \left((B_k^{(j)})^T B_k^{(j)}\right)^{-1} \left(B_k^{(j)}\right)^T (x_{r,k+1} - A_k^{(j)}\, x_k) \quad (17)$$

with $B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) B_i$, $A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) A_i$, $j = 0, \ldots, N$ with N the number of iterations and u_k^0 the input initialization (may be taken as $u_{r,k}$ for example).
The convergence of this algorithm may be proved locally, but it will not be considered in the present work since it is not the purpose of the study (see [7], [8], [9] for more details).

As mentioned in the beginning of this section, to achieve the exact tracking, the system and model reference must be in a certain canonical form (structural conditions). Depending on the reference and system models, these conditions may be impossible to meet and thus must be relaxed. One way is to apply the tracking to some states only, but it can affect the functioning of the system since other states are left free. A better way is to use an approximate tracking of all the states.

3. Approximate state tracking for T-S systems

In this section, the optimized tracking using the norm of the tracking error is addressed. This part concerns the optimal control with the introduction of the MPC for T-S models. As it was mentioned above, due to the restrictiveness of the exact tracking structural conditions, one can consider another approach to relax these structural conditions. This approach is the so-called quadratic optimal control for T-S models and aims at minimizing the tracking error. This is a natural way to deal with the problem by optimizing the tracking error. Indeed, it is less difficult and conservative to deal with a minimization problem than an equality constraint such as expressed by the structural conditions of the previous section.

3.1. Control law design

At each time instant k, the objective is to minimize the following criterion which is the norm of the tracking error:

$$\Phi_k(u_k) = \| B_k\, u_k - x_{r,k+1} + A_k\, x_k \|_W^2 \tag{18}$$

where W is a positive definite weighting matrix chosen accordingly to the state components for which some specific tracking is desired.
The control tracking law is then given by:

$$u_k = (B_k^T W B_k)^{-1} B_k^T W (x_{r,k+1} - A_k\, x_k) \tag{19}$$

where the matrices A_k and B_k have been already defined in (2).

Remark 2 *When the premise variables ξ_k depend on the control as explained previously (remark 1), the same procedure goes for the control law (19) with the iterative resolution, it becomes for $j = 0, \ldots, N$:*

$$u_k^{(j+1)} = ((B_k^{(j)})^T W B_k^{(j)})^{-1} (B_k^{(j)})^T W\, (x_{r,k+1} - A_k^{(j)}\, x_k) \tag{20}$$

with $B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) B_i$, $A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) A_i$.

The proposed control tracking law u_k (19) (i.e. (20) for input dependent premise variables) aims to ensure the tracking of $x_{r,k+1}$ by x_{k+1} at each time instant k. Since the controller has no ability to anticipate future events, it can not take control actions accordingly and the tracking performances may degrade. Based on this statement, the MPC tracking is introduced for the T-S models. In fact, the main advantage of MPC is to allow the current time slot to be optimized, while keeping future time slot in account. This is achieved by optimizing a finite time horizon, but only implementing the current time slot.
As for linear and nonlinear model reference tracking control, the MPC requires iterative solution and aims to ensure that the tracking error is minimized on a finite sliding horizon. Roughly speaking, the procedure for T-S models is the same as for the conventional MPC. However, some difficulties occur when the premise variables depend on the control.

3.2. Premise variables independent of the input

Considering a finite horizon of $p+1$ steps, using the state equation

$$x_{k+1} = A_k x_k + B_k u_k \tag{21}$$

it follows the state expression at time $k+p+1$

$$\begin{aligned} x_{k+p+1} &= A_{k+p} x_{k+p} + B_{k+p} u_{k+p} \\ &= \Phi_{k+p,k} x_k + \sum_{l=k}^{k+p-1} \Phi_{k+p,l+1} B_l u_l \end{aligned} \tag{22}$$

where

$$\Phi_{k,l} = \begin{cases} A_{k-1} \ldots A_l \text{ if } k > l \geq 0 \\ I \text{ if } k = l \end{cases} \tag{23}$$

with A_k and B_k defined by (2).
Gathering the states on the time horizon $[k+1 : k+p+1]$, let us note:

$$\bar{x}_{k+1,p} = \begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{k+p+1} \end{bmatrix}, \bar{u}_{k,p} = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+p} \end{bmatrix}, \mathscr{A}_{k,p} = \begin{bmatrix} A_k \\ A_{k+1} A_k \\ \vdots \\ \prod_{i=0}^{p} A_{k+p-i} \end{bmatrix} \tag{24}$$

$$\mathscr{B}_{k,p} = \begin{bmatrix} B_k & 0 & \ldots & 0 \\ A_{k+1} B_k & B_{k+1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=0}^{p-1} A_{k+p-i} B_k & \prod_{i=0}^{p-2} A_{k+p-i} B_{k+1} & \ldots & B_{k+p} \end{bmatrix}$$

Using (22) and (24), the state $\bar{x}$ is written as:

$$\bar{x}_{k+1,p} = \mathscr{A}_{k,p} x_k + \mathscr{B}_{k,p} \bar{u}_{k,p}, \quad \bar{x}_{k,p} \in R^{n(p+1)} \tag{25}$$

To ensure the reference model tracking on the time horizon $[k+1 : k+p+1]$, the control $\bar{u}_{k,p}$ is ajusted in order to minimize the criterion:

$$\Phi_{k,p}(\bar{u}_{k,p}) = \| \bar{x}_{r,k,p} - \mathscr{A}_{k,p} x_k - \mathscr{B}_{k,p} \bar{u}_{k,p} \|_W^2 \tag{26}$$

with $\bar{x}_{r,k,p} = \begin{bmatrix} x_{r,k+1} & \ldots & x_{r,k+p+1} \end{bmatrix}^T \in R^{n(p+1)}$.
This leads to:

$$\bar{u}_{k,p} = (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} \mathscr{B}_{k,p}^T W (\bar{x}_{r,k,p} - \mathscr{A}_{k,p} x_k) \tag{27}$$

where the input at the step k defined by:

$$u_k = \begin{bmatrix} I_{n_u} & 0 & \ldots & 0 \end{bmatrix} \bar{u}_{k,p} \tag{28}$$

is then applied to the system. For the next step, the horizon is moved and the criterion $\Phi_{k+1,p}$ is optimized in order to obtain and apply the control u_{k+1}. As explained before, the control $\bar{u}_{k,p}$ (in particular u_k) is calculated while keeping future time slot in account, which explains the anticipative character of the MPC control.

3.3. Extension to premise variables dependent of the input

Since the weighting functions of the matrices $\mathscr{A}_{k,p}$ and $\mathscr{B}_{k,p}$ (24) may depend on the control $\bar{u}_{k,p}$, instead of the analytical solution (27), the following iterative algorithm is proposed:

1. define a threshold δ
2. for $j=0$, define $\overline{u}_{k,p}^{(j)}$ and $\overline{u}_{k,p}^{(j-1)}$
3. compute $\mathscr{A}_{k,p}^{(j)}$ and $\mathscr{B}_{k,p}^{(j)}$
4. while $||\overline{u}_{k,p}^{(j)} - \overline{u}_{k,p}^{(j-1)}|| > \delta$

$$\overline{u}_{k,p}^{(j+1)} = \left((\mathscr{B}_{k,p}^{(j)})^T W \mathscr{B}_{k,p}^{(j)}\right)^{-1} (\mathscr{B}_{k,p}^{(j)})^T W (\overline{x}_r - \mathscr{A}_{k,p}^{(j)} x_k) \tag{29}$$

$j \leftarrow j+1$, compute $\mathscr{A}_{k,p}^{(j+1)}$ and $\mathscr{B}_{k,p}^{(j+1)}$.
with:

$$\begin{cases} \mathscr{A}_{k,p}^{(j)} = \begin{bmatrix} A_k^{(j)} \\ A_{k+1}^{(j)} A_k^{(j)} \\ \vdots \\ \prod_{i=0}^{p} A_{k+p-i}^{(j)} \end{bmatrix}, A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u^{(j)}(k)) A_i \\ \mathscr{B}_{k,p}^{(j)} = \begin{bmatrix} B_k^{(j)} & 0 & \dots & 0 \\ A_{k+1}^{(j)} B_k^{(j)} & B_{k+1}^{(j)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=0}^{p-1} A_{k+p-i}^{(j)} B_k^{(j)} & \prod_{i=0}^{p-2} A_{k+p-i}^{(j)} B_{k+1}^{(j)} & \dots & B_{k+p}^{(j)} \end{bmatrix} \\ B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u^{(j)}(k)) B_i \end{cases} \tag{30}$$

After the algorithm convergence, the control input at the step k is defined by:

$$u_k = \begin{bmatrix} I_{n_u} & 0 & \dots & 0 \end{bmatrix} \overline{u}_{k,p} \tag{31}$$

and is applied to the system. For the next step, the horizon is moved and the criterion $\Phi_{k+1,p}$ is optimized in order to obtain u_{k+1}.

4. Numerical example

In order to illustrate the proposed approach, let us consider the following numerical example:

$$A_r = \begin{pmatrix} 0.2 & 0.5 & 0 & 0 & 0 \\ -0.2 & 0.19 & -0.1 & 0 & 0 \\ 0 & -1 & 0.2 & 0 & 0 \\ 0 & 0.2 & -0.2 & 0.9 & -0.2 \\ 0.1 & 0.3 & 0 & 0 & 0.7 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0.3 & 0.5 & 0 & 0 & 0.1 \\ -0.2 & 0.69 & -0.1 & 0 & 0 \\ 0 & -1.1 & 0.5 & 0 & 0.1 \\ -0.51 & -0.1 & 0 & 0.9 & -0.2 \\ 0.1 & 0.3 & 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.6 & 0.5 & 0 & 0 & 0 \\ -0.2 & 0.39 & -0.1 & 0 & 0 \\ 0 & -1 & 0.5 & 0 & 0 \\ 0 & 0.2 & -0.2 & 1 & -0.2 \\ 0.1 & 0.3 & 0 & 0 & 0.8 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} -0.2 & 0.3 \\ 0.4 & 0.5 \\ 0.3 & 1.3 \\ 0.55 & 0.45 \\ -0.7 & -0.3 \end{pmatrix}, B_2 = \begin{pmatrix} -0.8 & -0.3 \\ -0.2 & -0.1 \\ -0.3 & 0.7 \\ -0.05 & -0.15 \\ -1.3 & -0.9 \end{pmatrix}$$

$$B_r = \begin{pmatrix} -0.5 & 0 \\ 0.1 & 0.2 \\ 0 & 1 \\ 0.25 & 0.15 \\ -1 & -0.6 \end{pmatrix}$$

The weighting functions are input dependent and given by:

$$\begin{cases} \mu_{1,k} = \dfrac{1+2\tanh(u_{1,k})}{2} \\ \mu_{2,k} = 1 - \mu_{1,k} \end{cases} \tag{32}$$

One can verify that the exact tracking conditions (8) are not fulfilled. For this reason, the MPC is performed for three steps forward ($p=2$) with iterative resolution (30).
First, the objective is to ensure a good tracking of the fourth and fifth state components. Consequently, the weighting matrix is chosen as $W = \text{diag}(1,1,1,2,2)$, implying a relaxation of the tracking of the first three state components. The system and reference model states are depicted in figure 3 (respectively noted x_i

and $x_{ir}, i = 1, \ldots, 5$). In figure 4, the control inputs $u_{r,k}$ and u_k are represented. From the depicted figures, one can see that the control tracking is efficient (especially for the fourth and fifth states) although the structural conditions are not fulfilled.
Secondly, if the main objective is an accurate tracking

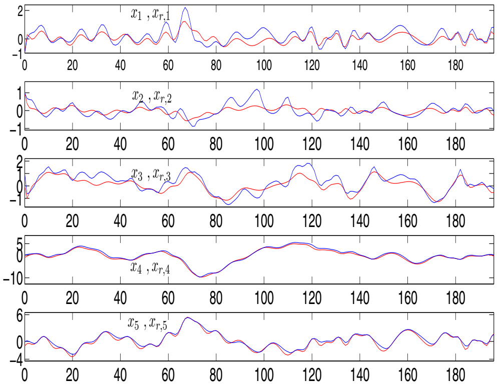

Figure 3. System and reference model states for $W = \mathrm{diag}(1,1,1,2,2)$

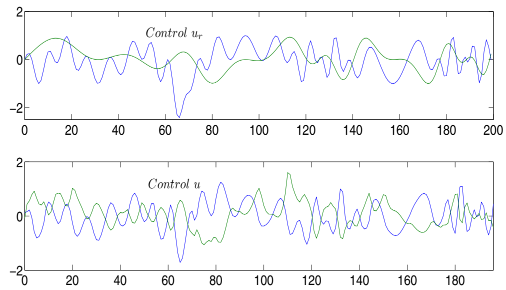

Figure 4. Control inputs $u_r(k)$ **and** $u(k)$

of x_{3r} by x_3, one should set the weighting matrix as $W = \mathrm{diag}(1,1,2,1,1)$. The obtained results are displayed in figure 5 for the system and model reference states and in figure 6 for the control inputs $u_{r,k}$ and u_k.

One can observe that for this case, the third state tracking has been improved when the fourth and fifth states tracking have been slightly deteriorated.

In order to quantify the improvement due to the predictive control, let us consider the following criterion:

$$\phi_i = \sum_{k=0}^{N} |x_{r,k,i} - x_{k,i}|$$

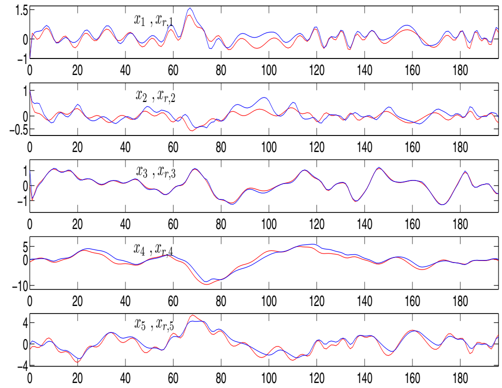

Figure 5. System and reference model states for $W = \mathrm{diag}(1,1,2,1,1)$

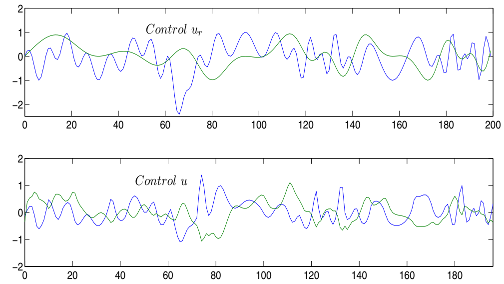

Figure 6. Control inputs $u_r(k)$ **and** $u(k)$

where i the component number of a vector ($i = 1, \ldots, 5$), N is the simulation horizon and $x_{k,i}$ is obtained from non predictive control (16).
ϕ_{ip} is analogously defined with $x_{k,i}$ obtained with MPC:

$$\phi_{ip} = \sum_{k=0}^{N} |x_{r,k,i} - x_{k,i}^{p}|$$

Finally, the performance gain τ_i due to MPC is obtained from

$$\tau_i = 100\frac{\phi_i - \phi_{ip}}{\phi_i}$$

For the considered example ($W = \mathrm{diag}(1,1,1,2,2)$), we obtain the following improvement (for each state): $\tau_1 = 2.54\%$, $\tau_2 = 12.99\%$, $\tau_3 = 5.21\%$, $\tau_4 = 64.72\%$, and $\tau_5 = 14.56\%$.
In order to highlight the influence of the time horizon length $p+1$ on the tracking performances, the improvement of ϕ_i, namely τ_i, is computed for time horizon characterized by $p \in \{1,2,3,4\}$. The results are gathered in table I.
One can conclude, for the presented example, that a

	$p=1$	$p=2$	$p=3$	$p=4$
τ_1	1.76%	2.54%	8.99%	14.02%
τ_2	4.80%	12.99%	22.28%	28.85%
τ_3	3.81%	5.21%	6.21%	7.67%
τ_4	46.90%	64.72%	77.52%	81.1%
τ_5	10.62%	14.56%	13.64%	14.81%

Table 1.

horizon of length $p = 4$ gives the best results.
It is also important to highlight that depending on the dynamic characteristic of the reference on a time horizon, a too short, as well as a too long horizon may not give the best expected results. A compromise is then needed. To quantify the best horizon length, a comparative study as the one presented may be a good solution.

5. Conclusion

In this paper, the tracking objective for nonlinear T-S model was considered. Structural conditions for perfect tracking were established, as well as the quadratic optimal control. A Model Predictive Control for the T-S case with finite time horizon was also developed and the prediction influence was highlighted via a numerical example.
A first perspective for the present work is to generalize the matching conditions and the structure proposed in (10) for a general control structure law and establish the relation between the reference model (A_r, B_r) and the system matrices (A_i, B_i). During the study, a strong correlation between the time horizon length and the model reference dynamics was pointed, in fact, a second interesting perspective will be to present a choice criterion that optimizes the time horizon length according to the model reference dynamics.

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