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Finite Memory State Observer Design for Polytopic Systems. Application to Actuator Fault Diagnosis

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Abstract

This paper addresses the Finite Memory Observer (FMO) design applied to polytopic models. After a brief introduction on FMO for linear systems, the nonlinear models represented in a Takagi-Sugeno (T-S) or Polytopic form are then considered. The considered observer design will be applied to investigate the fault diagnosis for nonlinear discrete-time systems subject to unknown input where joint system states and unknown inputs estimation is proposed.

1. Introduction

In order to detect and isolate a sensor fault through the estimation of system outputs using measurable signals and the model of the system, fault detection and isolation (FDI) techniques based on the time-evolution of the residual signals obtained by the comparison between the measured outputs and the estimated outputs [1], [2] are commonly considered. The procedure is performed by defining and generating some residual signal in order to detect the occurring fault(s). The residual signals often consist in output estimation error, provided by classical or unknown input observers. Then the residual analysis and / or structuration may lead to fault isolation. A way to do so is to establish the theoretical influence of each fault on each residual, namely the signature table. Then, a decision logic is used to generate fault indicators based on these residuals.
System states or outputs estimation is the basis for the FDI methods. Among estimation techniques, those using Kalman filters or a Luenberger observer are widely used. These estimators are said to be infinite memory and hence the state estimation error converges to zero in infinite time. In contrast, the Finite Memory Observer (FMO) has the advantage to ensure the convergence of the state estimation in a finite time, at least in the absence of disturbances.
Despite the interest mentioned above, few studies have been published on the FMO compared to those on infinite memory observers. The pioneering works are due to Jazwinski [3], [4] and [5] where a state estimation formulation from a discrete or continuous integral form of the inputs-outputs has been proposed. This form was also considered by Medvedev [6], [7] and Byrski [8] [9]. This filtering technique applied for the continuous case [10], [11] as well as for the discrete one [12], [13], offers a generic aspect in the sense that it is applied for state estimation, parameter estimation and control with sliding horizon. Note that the nonlinear case has been less discussed, however one can refer to the following works [14], [15], [16].
Several works based on sliding horizon for an exact state reconstruction in a finite time (without measurement noise nor model uncertainties) may be found in the literature with different terminologies like exact observers, FMO, integral observers and ideal observers. Most of these studies are academic, nevertheless some of them are applied to electric power transmission networks [17], diesel engines diagnosis [18], fuel cell estimation [19], state converters estimation [20] or general applications in the diagnosis framework [21], [22].
Given the advantage of the FMO, it seems interesting to extend its scope to nonlinear systems. As it was mentioned previously, few works deal with the nonlinear case. This is why in the present paper a particular attention is given to nonlinear systems represented in a polytopic or Takagi-Sugeno (T-S) form. The polytopic model may have different names, such as fuzzy model (Takagi-Sugeno model), multi-model, local model networks, etc. It allows the representation of nonlinear behaviors by the interpolation of a set of linear submodels. Each submodel contributes to the global behavior of

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the nonlinear system through a weighting function [23]. The T-S structure may be obtained by transforming the original system into a polytopic linear model based on the sector nonlinearity approach and the convex polytopic transformation. This transformation has the major interest to exactly represent the system without any loss of informations since the considered nonlinearities are bounded (each parameter varies between two known values).
In the present work, finite memory observer for nonlinear systems represented in a T-S form are proposed. The paper is organized as follows. Section II introduces the state and unknown input estimation with finite memory observer for linear systems. In section III the T-S systems are considered for both measurable and unmeasurable premise variables. Illustrative examples are presented in Section IV and conclusion results are detailed in section V.

2. Preliminaries: Finite Memory Observer for linear systems

The FMO is designed on a finite sliding horizon of length $r+1$. From available measurements at time $k+r$ in a time interval $[k:k+r]$, the system states are then estimated in finite time. The horizon is moved by one step forward $[k+1:k+r+1]$ which allows to estimate the state at the instant $k+r+1$. The next section details the above procedure and expands it to the unknown inputs estimation.

2.1. State estimation

Let us consider the following system:

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k, \quad x \in R^{n_x} \\ y_k &= Cx_k \end{cases} \tag{1}$$

where x_k is the system state at the instant k, $u_k \in R^{n_u}$ the input and y_k the output. A, B and C are the system matrices with appropriate dimensions.
Using the system equation (1), the output expression at time $k+r$ is given by:

$$y_{k+r} = CA^r x_k + CA^{r-1}Bu_k + \cdots + CBu_{k+r-1} \tag{2}$$

Gathering the outputs on the time horizon $[k:k+r]$, let us note:

$$\tilde{y}_k = M_x x_k + M_u \tilde{u}_k \tag{3}$$

with:

$$\begin{cases} \tilde{y}_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \dots \\ y_{k+r} \end{bmatrix} \quad \tilde{u}_k = \begin{bmatrix} u_k \\ u_{k+1} \\ \dots \\ u_{k+r-1} \end{bmatrix} \quad M_x = \begin{bmatrix} C \\ CA \\ \dots \\ CA^r \end{bmatrix} \\ M_u = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ CB & 0 & \ddots & \ddots & 0 \\ CAB & CB & \ddots & \ddots & \vdots \\ \vdots & & & & \\ CA^{r-1}B & CA^{r-2}B & \dots & \dots & CB \end{bmatrix} \end{cases}$$

Proposition 1 *A FMO for system (1) is given by the following structure:*

$$\begin{cases} \hat{x}_{k+r} &= A^r \hat{x}_k + T\tilde{u}_k \\ \hat{x}_k &= (M_x^T W M_x)^{-1} M_x^T W (\tilde{y}_k - M_u \tilde{u}_k) \\ T &= \begin{bmatrix} A^{r-1}B & A^{r-2}B & \dots B \end{bmatrix} \end{cases} \tag{4}$$

where W is a positive definite weighting matrix of appropriate dimension chosen accordingly to the state components for which some specific importance is given.

One can easily verify that $\hat{x}_{k+r} = x_{k+r}$ by replacing the expression of $\hat{x}_k$ and (3) in $\hat{x}_{k+r}$ (4):

$$\begin{cases} \hat{x}_{k+r} = A^r (M_x^T W M_x)^{-1} M_x^T W (\tilde{y}_k - M_u \tilde{u}_k) + T\tilde{u}_k \\ \quad = A^r (M_x^T W M_x)^{-1} M_x^T W (M_x x_k + M_u \tilde{u}_k - M_u \tilde{u}_k) \\ \qquad + T\tilde{u}_k \\ \quad = A^r x_k + T\tilde{u}_k \\ \quad = x_{k+r} \end{cases}$$

Remark 1 *Expression (4) shows that the state estimate $\hat{x}_{k+r}$ at the instant $k+r$ results from the input and outputs filtering on the time horizon $[k:k+r]$. Since the same procedure is applied one step forward $[k+1:k+r+1]$, it is possible to establish a recurrence relation between the state estimation $\hat{x}_{k+r}$ and $\hat{x}_{k+r+1}$.*

In the next subsection, an extension for joint state and unknown input FMO is considered. The proposed structure is based on the same one given in (4).

2.2. State and unknown input finite memory observer

Let us consider the following system subject to non measurable unknown input p:

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k + Pp_k, x \in \mathscr{R}^{n_x}, p \in \mathscr{R}^{n_p} \\ y_k &= Cx_k \end{cases} \tag{5}$$

The unknown input dynamic is given by:

$$p_{k+1} = p_k + \delta_k \tag{6}$$

where δ_k is the unknown input variation at the instant k. An augmented state $x_k^a = \begin{pmatrix} x_k \\ p_k \end{pmatrix}$ is defined with the concatenation of the system state and the unknown input:

$$\begin{cases} x_{k+1}^a &= A^a x_k^a + B^a u_k + P^a \delta_k \\ y_k &= C^a x_k^a \end{cases} \tag{7}$$

with:

$$\begin{cases} x_k^a = \begin{bmatrix} x_k \\ p_k \end{bmatrix} & A^a = \begin{bmatrix} A & P \\ 0 & I \end{bmatrix} \\ B^a = \begin{bmatrix} B \\ 0 \end{bmatrix} & P^a = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad C^a = \begin{bmatrix} C & 0 \end{bmatrix} \end{cases} \tag{8}$$

Equation (3) may be extended in the form:

$$\tilde{y}_k = M_x^a x_k^a + M_u^a \tilde{u}_k + M_\delta^a \tilde{\delta}_k \tag{9}$$

with:

$$\begin{cases} \tilde{y}_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \dots \\ y_{k+r} \end{bmatrix} & \tilde{u}_k = \begin{bmatrix} u_k \\ u_{k+1} \\ \dots \\ u_{k+r-1} \end{bmatrix} \\ \tilde{\delta}_k = \begin{bmatrix} \delta_k \\ \delta_{k+1} \\ \dots \\ \delta_{k+r-1} \end{bmatrix} & M_x^a = \begin{bmatrix} C^a \\ C^a A^a \\ \vdots \\ C^a (A^a)^r \end{bmatrix} \end{cases}$$

$$\begin{cases} M_u^a = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ C^a B^a & 0 & \ddots & \ddots & 0 \\ C^a A^a B^a & C^a B^a & \ddots & \ddots & \vdots \\ \vdots & & & & \\ C^a A^{a^{r-1}} B^a & C^a A^{r-2} B^a & \dots & \dots & C^a B^a \end{bmatrix} \\ M_\delta^a = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ C^a P^a & 0 & \ddots & \ddots & 0 \\ C^a A^a P^a & C^a P^a & \ddots & \ddots & \vdots \\ \vdots & & & & \\ C^a A^{a^{r-1}} P^a & C^a A^{r-2} P^a & \dots & \dots & C^a P^a \end{bmatrix} \end{cases}$$

By analogy with the previous subsection results, the augmented state estimate $\hat{x}_k^a$ is given by a similar expression than the one given in (4). Then from $\hat{x}_k^a$ we directly deduce the estimation of $\hat{x}_k$ of the state and $\hat{p}_k$ of the unknown input. In the next section an extension of the above finite memory observer is given for nonlinear T-S systems.

3. Finite memory observers for T-S systems

The T-S representation of a nonlinear system consists in a time-varying interpolation of a set of linear submodels. Each submodel contributes to the global behavior of the nonlinear system through a weighting function $\mu_i(\xi_k)$ [23].
Let us consider the following T-S model [24]:

$$x_{k+1} = A_k\, x_k + B_k\, u_k \tag{10}$$

with:

$$A_k = \sum_{i=1}^{r} \mu_i(\xi_k) A_i, \quad B_k = \sum_{i=1}^{r} \mu_i(\xi_k) B_i \tag{11}$$

where the weighting functions $\mu_i(\xi_k)$ depend on the so-called premise variable ξ_k which may be a state, input, or output combination. These weighting functions satisfy the following convex sum property:

$$0 \le \mu_i(\xi_k) \le 1, \sum_{i=1}^{r} \mu_i(\xi_k) = 1 \tag{12}$$

s.t. $x_k \in \mathscr{R}^{n_x}$ and $u_k \in \mathscr{R}^{n_u}$.
Roughly speaking, the FMO design for T-S models is the same as for the conventional linear case. However, some difficulties occur when the premise variables are not known.

3.1. Known premise variables

In the present subsection, the case of known premise variables is considered. Based on the same structure as for the linear case, for the time horizon $[k : k+r]$, the output vector is given by:

$$\tilde{y}_k = M_x(\xi_k) x_k + M_u(\xi_k) \tilde{u}_k \tag{13}$$

with the following definitions:

$$\tilde{y}_k^T = \begin{bmatrix} y_k^T & \dots & y_{k+r}^T \end{bmatrix}, \tilde{u}_k^T = \begin{bmatrix} u_k^T & \dots & u_{k+r-1}^T \end{bmatrix}$$

$$M_x(\xi_k) = \begin{bmatrix} C \\ CA_k \\ CA_{k+1}A_k \\ \dots \\ CA_{k+r-1} \dots A_k \end{bmatrix}, M_u(\xi_k) = \begin{bmatrix} (M_u^1(\xi_k))^T \\ (M_u^2(\xi_k))^T \\ \vdots \\ (M_u^r(\xi_k))^T \end{bmatrix}^T$$

$$
\begin{aligned}
&(M_u^1(\xi_k))^T = \left[\begin{array}{lll} 0 & (CB_k)^T & (CA_{k+1}B_k)^T \end{array}\right. \\
&\quad \left.\ldots \quad (CA_{k+r-1}\ldots A_{k+1}B_k)^T \right] \\
&(M_u^2(\xi_k))^T = \left[\begin{array}{lll} 0 & 0 & (CB_{k+1})^T \end{array}\right. \\
&\quad \left.\ldots \quad (CA_{k+r-1}\ldots A_{k+2}B_{k+1})^T \right] \\
&(M_u^r(\xi_k))^T = \left[\begin{array}{llll} 0 & 0 & \ldots & (CB_{k+r-1})^T \end{array}\right]
\end{aligned} \tag{14}
$$

Note that the matrices A_k and B_k (11) are time dependent (depend on the premise variables ξ_k) which implies that the matrices $M_x(\xi_k)$ and $M_u(\xi_k)$ are also time dependent.
At time k, let us consider the following criterion:

$$
\Phi(x_k) = \| \tilde{y}_k - M_x(\xi_k)x_k - M_u(\xi_k)\tilde{u}_k \|_W^2 \tag{15}
$$

where W is a positive definite weighting matrix of appropriate dimension chosen accordingly to the state components for which some specific importance is given.
Supposing that M_x is full column rank, the state estimator may be given in the following form:

$$
\begin{cases}
\hat{x}_k = \left(M_x^T(\xi_k)\, W\, M_x(\xi_k)\right)^{-1} M_x^T(\xi_k)\, W\, \left(\tilde{y}_k - M_u(\xi_k)\tilde{u}_k\right) \\
\hat{x}_{k+r} = A_{k+r-1}\ldots A_k\hat{x}_k + T\tilde{u}_k \\
T = \left[A_{k+r-1}\ldots A_{k+1}B_k \quad A_{k+r-1}\ldots B_{k+1}\ldots \quad B_{k+r-1}\right]
\end{cases} \tag{16}
$$

The state estimation at time $k+r$ is then deduced using the data collected on the interval $[k : k+r]$. Then, the horizon is moved by one step forward $[k+1 : k+r+1]$ which allows to estimate the state at the instant $k+r+1$.

3.2. State filtering

The model (1) does not take into account the noise that frequently corrupts the outputs. Without a precise modeling of that noise, it is however possible to reduce its influence on the estimates by introducing, in the optimization criterion, a regularization term:

$$
\begin{aligned}
\Phi(x_k^r) = &\| \tilde{y}_k - M_x(\xi_k)x_k^r - M_u(\xi_k)\tilde{u}_k \|_W^2 \\
&+ \| x_k^r - x_{k-1} \|_F^2
\end{aligned}
$$

where F is a positive definite weighting matrix of appropriate dimension which plays the same role as W (chosen accordingly to accentuate the filtering on a considered state component).
The derivative of the above criterion with regard to the state is given by:

$$
\begin{aligned}
\frac{\partial \Phi}{\partial x_k^r} = &-2M_x^T(\xi_k)W\left(\tilde{y}_k - M_x(\xi_k)x_k^r - M_u(\xi_k)\tilde{u}_k\right) \\
&+2F\left(x_k^r - x_{k-1}\right)
\end{aligned} \tag{17}
$$

The solution is given for:

$$
\begin{aligned}
\hat{x}_k^r = &\left(M_x^T(\xi_k)WM_x(\xi_k) + F\right)^{-1} \\
&\left(M_x^T(\xi_k)W(\tilde{y} - M_u\,\tilde{u}) + F\,\hat{x}_{k-1}\right)
\end{aligned} \tag{18}
$$

Remark 2 *By developing the inverse of the matrix $M_x^T(\xi_k)WM_x(\xi_k) + F$, the solution (18) may be expressed in terms of the non regularized solution given in (16).*

$$
\begin{aligned}
\hat{x}_k^r = &\left((M_x^T(\xi_k)\,W\,M_x(\xi_k))^{-1}F + I\right)^{-1} \\
&\left(\hat{x}_k + (M_x^T(\xi_k)\,W\,M_x(\xi_k))^{-1}F\,\hat{x}_{k-1}\right)
\end{aligned} \tag{19}
$$

where $\hat{x}_k$ correspond to the state estimation obtained without filtering.
Unsurprisingly, from (19), we notice that if $F \to 0$, then $\hat{x}_k^r \to \hat{x}_k$ and if $F \to \infty$ then $\hat{x}_k^r \to \hat{x}_{k-1}$.

3.3. Unknown premise variables

Let us now consider the case where the the weighting functions of the matrices A_k and B_k depend on the (unknown) state of the system. Instead of the analytical solution (18), an iterative solution is proposed.
For the time horizon $[k : k+r]$, let us note $\hat{x}_k^0$ the initial state estimation, which may be set equal to the previous horizon state estimate $\hat{x}_{k-1}$. The state estimate is then given by:

$$
\begin{cases}
\hat{x}_k^{(1)} = \left(M_x^T(\hat{x}_k^{(0)})\,W\,M_x(\hat{x}_k^{(0)})\right)^{-1} M_x^T(\hat{x}_k^{(0)}) \\
\qquad\qquad W\left(\tilde{y}_k - M_u(\hat{x}_k^{(0)})\tilde{u}_k\right) \\
M_x(\hat{x}_k^{(0)}) = M_x(\xi_k)\,|_{\xi_k = \hat{x}_k^{(0)}} \\
M_u(\hat{x}_k^{(0)}) = M_u(\xi_k)\,|_{\xi_k = \hat{x}_k^{(0)}}
\end{cases} \tag{20}
$$

More generally, at iteration $q+1$ we get:

$$
\begin{cases}
\hat{x}_k^{(q+1)} = \left(M_x^T(\hat{x}_k^{(q)})\,W\,M_x(\hat{x}_k^{(q)})\right)^{-1} M_x^T(\hat{x}_k^{(q)}) \\
\qquad\qquad W\left(\tilde{y}_k - M_u(\hat{x}_k^{(q)})\tilde{u}_k\right) \\
\hat{x}_{k+r}^{(q+1)} = A_{k+r-1}\ldots A_k\hat{x}_k^{(q+1)} + T(\hat{x}_k^{(q)})\tilde{u}_k \\
M_x(\hat{x}_k^{(q)}) = M_x(\xi_k)\,|_{\hat{x}_k = \hat{x}_k^{(q)}} \\
M_u(\hat{x}_k^{(q)}) = M_u(\xi_k)\,|_{\hat{x}_k = \hat{x}_k^{(q)}}
\end{cases} \tag{21}
$$

The same idea is applied when considering the noise filtering as explained in section III.B.

3.4. State and unknown input estimation

The extension of the proposed observer in section II.B to the T-S case is deduced straightforwardly by replacing A_b, B_k and C in (14) by A_k^a, B_k^a and C^a defined by (8) in which A, B and P are replaced by A_k, B_k and P_k respectively.

4. Illustrative examples

In the following section, numerical examples are given in order to illustrate the effectiveness of the proposed observers.
Let us consider the T-S model with two submodels and state dependent premise variables:

$$\begin{cases} A_1 = \begin{bmatrix} 0.392 & 0.040 & 0 \\ 0.080 & 0.200 & 0.040 \\ -0.200 & -0.040 & 0.160 \end{bmatrix} B_1 = \begin{bmatrix} 0.10 \\ 0.40 \\ -1.00 \end{bmatrix} \\ A_2 = \begin{bmatrix} 0.931 & 0.095 & 0 \\ 0.190 & 0.475 & 0.095 \\ -0.475 & -0.095 & 0.380 \end{bmatrix} B_2 = \begin{bmatrix} 0.10 \\ 0.40 \\ -1.00 \end{bmatrix} \\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\ \mu_1(x_k) = \frac{1}{2}(1+\tanh(x_1(k)/0.5)),\ \mu_2(x_k) = 1-\mu_1(k) \end{cases} \quad (22)$$

4.1. State estimation

In the considered example, the weighting functions are state dependent ($x_1(k)$). The formulation given by (20) and (21) of section III.C is then applied.

In figure 1 are depicted the system states x_i, $i = 1,2,3$ as well as their estimates. Only one state is measured and the considered time horizon length is equal to 3. Figure 2 depicts the system input $u(k)$, output $y(k)$ and its estimate $\hat{y}(k)$ as well as the weighting function $\mu_1(x_k)$ which covers the two modes (submodels).
As seen on these figures, the states are well estimated (but no measurement noise was considered).

4.2. Estimation with noise and unknown inputs

In this subsection, unknown inputs are also considered. In order to illustrate the efficiency of proposed filtering algorithm, two simulations results are presented. The first case is about state and unknown input estimation, the measurement are subject to an additive measurement noise but the filtering algorithm is not applied. In the second case, the state filtering given in section III.B is applied.

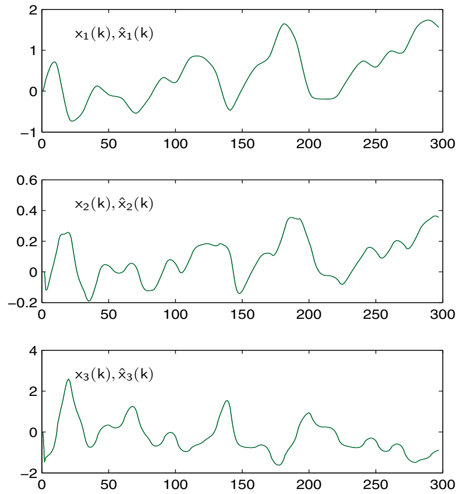

Figure 1. System states x_k and their estimates $\hat{x}_k$

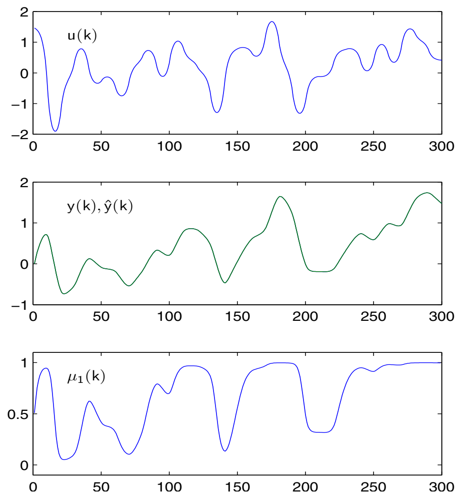

Figure 2. Inputs u_k, measured and estimated outputs y_k, $\hat{y}_k$ and weighting function $\mu_1(x_k)$

The unknown input matrix P is defined as:

$$P = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \quad (23)$$

In this second example, the matrix C is defined by:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (24)$$

In figure 3 are depicted the system states and their estimates. The joint estimation state/unknown input was done with measurement noise but without any filtering. As the figure shows, the third state estimate $\hat{x}_3(k)$ is

greatly affected by the noise. This result may be explained by the fact that since only the states x_1 and x_2 are measured, the estimation is made to the detriment of the third one.
In order to improve the estimation, the filtering proposed in section III.B is then applied. The figure 4 shows the unknown input δ_k and its estimate $\hat{\delta}_k$ as well as the input u_k. The filtering effect is clearly illustrated in figures 5 and 6 where the improvement is clearly shown for the third state.

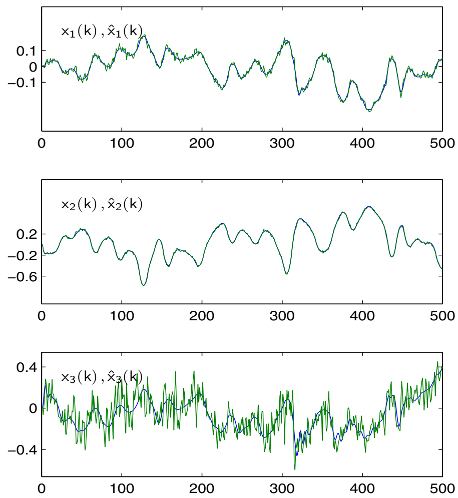

Figure 3. System states x_k and their estimates $\hat{x}_k$: with noise measurement and without filtering

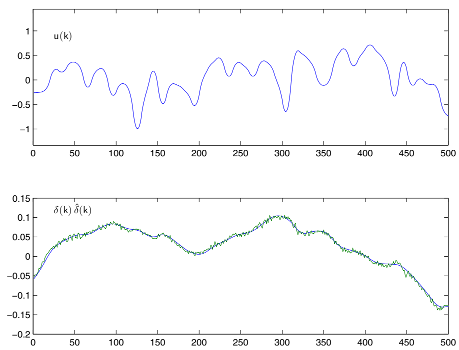

Figure 4. The input u_k and unknown input δ_k and its estimate: with noise measurement and without filtering

From the depicted figures, one can observe the efficiency of the proposed algorithms.

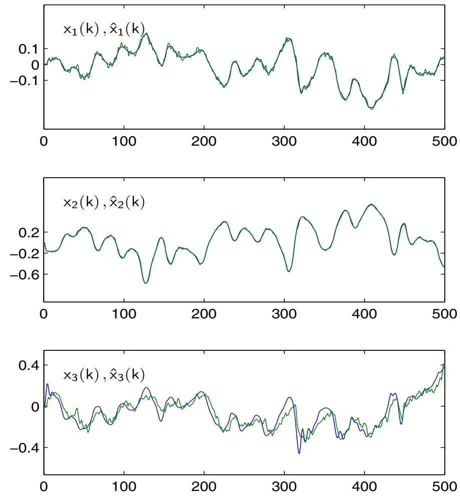

Figure 5. System states x_k and their estimates $\hat{x}_k$: with noise measurement and filtering

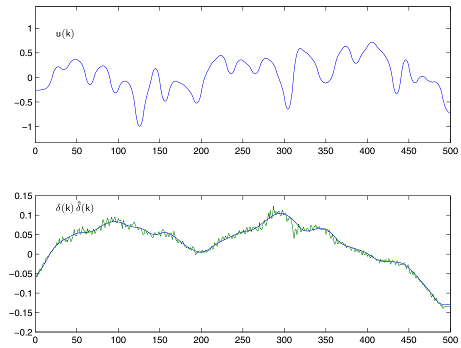

Figure 6. The input u_k and unknown input δ_k and its estimate: with noise measurement and filtering

5. Conclusion

In this paper, a Finite Memory Observer design for nonlinear T-S model was considered. A joint state and unknown input reconstruction algorithm was proposed for both measurable and unmeasurable premise variables. The case of measurement noise was also studied with the proposition of a filtering algorithm. Numerical examples were presented in order to highlight the approach efficiency.
As futur work, it turns possible to to extend the proposed approach to sensor fault detection and isolation with the help of a bank of finite memory observers.

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