Operating mode recognition: Application in continuous casting

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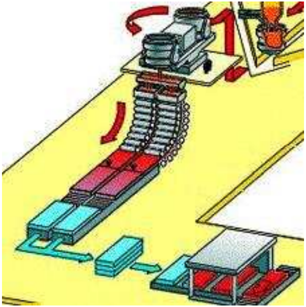
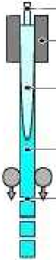
Summary

- **Introduction**
- Method principle
- Application
- Conclusion

Introduction

Industrial context: the continuous casting





Summary

Method principle

System model

Let us consider the three following models:

$$\begin{cases} M_1 : y(k) - a_1 u_1(k) - b_1 u_2(k) = 0 \\ M_2 : y(k) - a_2 u_1(k) - b_2 u_2(k) = 0 \\ M_3 : y(k) - a_3 u_1(k) - b_3 u_2(k) = 0 \end{cases} \quad (1)$$

A general model, with decoupled operating modes, can then be written:

$$\begin{aligned} M : r(k) = & (y(k) - a_1 u_1(k) - b_1 u_2(k)) \times \\ & (y(k) - a_2 u_1(k) - b_2 u_2(k)) \times \\ & (y(k) - a_3 u_1(k) - b_3 u_2(k)) = 0 \end{aligned} \quad (2)$$

The model (2) can be rewritten in order to show global system parameters:

$$M : \begin{cases} r(k) = \phi^T(k)\theta = 0 \\ \phi(k) = \big(y^3(k) \;\; y^2(k)u_1(k) \;\; y^2(k)u_2(k) \;\; y(k)u_1^2(k) \\ \quad y(k)u_2^2(k) \;\; y(k)u_1(k)u_2(k) \;\; u_1^3(k) \;\; u_2^3(k) \\ \quad u_1^2(k)u_2(k) \;\; u_1(k)u_2^2(k)\big)^T \\ \theta = (\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6 \;\; \theta_7 \;\; \theta_8 \;\; \theta_9 \;\; \theta_{10})^T \end{cases} \quad (3)$$

Method principle

System model

Global parameters θ_i depend on local model parameters:

$$\left\{\begin{array}{ll} \theta_1 = 1 & \theta_6 = a_1b_2 + a_2b_1 + a_3b_1 + b_2a_3 + a_1b_3 + a_2b_3 \\ \theta_2 = -(a_1 + a_2 + a_3) & \theta_7 = -a_1a_2a_3 \\ \theta_3 = -(b_1 + b_2 + b_3) & \theta_8 = -b_1b_2b_3 \\ \theta_4 = a_1a_2 + a_2a_3 + a_1a_3 & \theta_9 = -(a_1b_2a_3 + a_2a_3b_1 + a_1a_2b_3) \\ \theta_5 = b_1b_2 + b_2b_3 + b_1b_3 & \theta_{10} = -(a_1b_2b_3 + a_2b_1b_3 + b_1b_2a_3) \end{array}\right. \quad (4)$$

$\Rightarrow$ Ten equations for only six parameters

Proposal: using only the estimation of θ_i to detect changing oparting modes.

Method principle

Design of a mode change indicator

let us evaluate the sensitivity of $r(k)$ (3) with regard to the system variables:

$$D(k) = \begin{pmatrix} \frac{\partial r(k)}{\partial u_1(k)} \\ \frac{\partial r(k)}{\partial u_2(k)} \\ \frac{\partial r(k)}{\partial y(k)} \end{pmatrix}$$

$$= \begin{pmatrix} y^2(k)\theta_2 + 2u_1(k)y(k)\theta_4 + u_2(k)y(k)\theta_6 + 3u_1^2(k)\theta_7 + 2u_1(k)u_2(k)\theta_9 + u_2^2(k)\theta_{10} \\ y^2(k)\theta_3 + 2u_2(k)y(k)\theta_5 + u_1(k)y(k)\theta_6 + 3u_2^2(k)\theta_8 + u_1^2(k)\theta_9 + 2u_1(k)u_2(k)\theta_{10} \\ 3y^2(k)\theta_1 + 2y(k)u_1(k)\theta_2 + 2y(k)u_2(k)\theta_3 + u_1^2(k)\theta_4 + u_2^2(k)\theta_5 + u_1(k)u_2(k)\theta_6 \end{pmatrix}$$

Method principle

Design of a mode change indicator

If the system operates according to model M_1, M_2 or M_3 then $y(k) = a_i u_1(k) + b_i u_2(k)$ with $i = \{1, 2, 3\}$ and $D(k)$ takes the values:

$$D_1(k) = \begin{pmatrix} -a_1 \\ -b_1 \\ 1 \end{pmatrix} x_1(k), \; D_2(k) = \begin{pmatrix} -a_2 \\ -b_2 \\ 1 \end{pmatrix} x_2(k), \; D_3(k) = \begin{pmatrix} -a_3 \\ -b_3 \\ 1 \end{pmatrix} x_3(k) \quad (5)$$

with $x_i(k) = ((a_i - a_j)u_1(k) + (b_i - b_j)u_2(k))((a_i - a_l)u_1(k) + (b_i - b_l)u_2(k))$ and $i, j, l \in \{1, 2, 3\}$.

Method principle

Design of a mode change indicator

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Consequently, the vectors $D_1(k)$, $D_2(k)$ and $D_3(k)$ are equipollent to the vectors:

$$\tilde{D}_1 = \begin{pmatrix} a_1 \\ b_1 \\ -1 \end{pmatrix}, \quad \tilde{D}_2 = \begin{pmatrix} a_2 \\ b_2 \\ -1 \end{pmatrix}, \quad \tilde{D}_3 = \begin{pmatrix} a_3 \\ b_3 \\ -1 \end{pmatrix} \tag{6}$$

and according to the operating mode, the vector $D(k)$ is collinear to the vector $\tilde{D}_1$, $\tilde{D}_2$ or $\tilde{D}_3$

Method principle

Design of a mode change indicator

To get rid of the unknown scalar $x_i(k)$ (which sign can evolve according the time instant k), each mode can be characterised by a vector issued from the ratios of the components of $D(k)$. If $d_j(k)$ denotes the j^{th} component of the vector $D(k)$, let us define the vector $\bar{D}(k)$ such that:

$$\bar{D}(k) = \begin{pmatrix} \frac{d_1(k)}{d_2(k)} & \frac{d_2(k)}{d_3(k)} & \frac{d_1(k)}{d_3(k)} \end{pmatrix}^T \quad (7)$$

Method principle

Angular distance between modes

To highlights the changing operating mode we can calculate the cosine of the angle between tow vectors $D(k-1)$ and $D(k)$ by the following expression:

$$\cos(D(k-1), D(k)) = \frac{D^T(k-1)D(k)}{\|D(k-1)\| \, \|D(k)\|} \tag{8}$$

If this angle is zero, there was no mode change between times $k-1$ and k. Otherwise, the angle is equal to the angle between the two vectors that characterising the operating modes at times $k-1$ and k, this means that there has been a change in operating mode.

Method principle

Simulation

The simulation is performed with the model (1).

Simulation parameters:

$a_1 = 1 \quad a_2 = 1.3 \quad a_3 = 0.8$

$b_1 = -1 \quad b_2 = -0.8 \quad b_3 = -0.2$

Inputs $u_1(k) \in [-3.5, -0.5]$ and $u_2(k) \in [-3.5, -2.5]$

Changing of operating mode occur at random times.

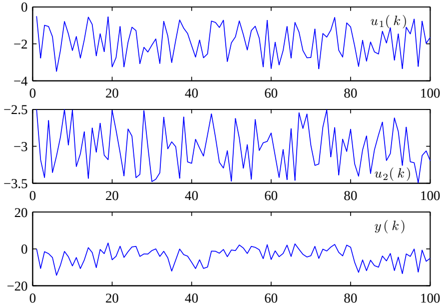

Figure: Input/output of system

Method principle

Simulation

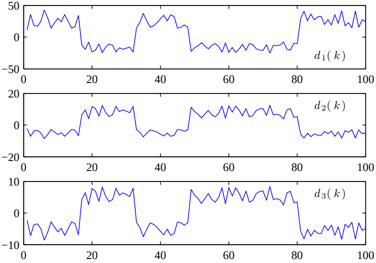

Figure: Components of vector D

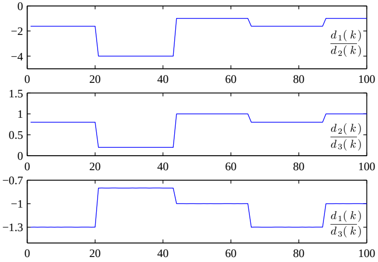

Figure: Components of vector $\bar{D}$

Method principle

Simulation

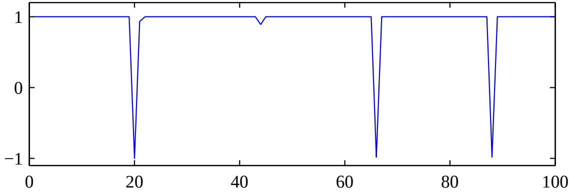

Figure: Cosine of the angle between $D(k-1)$ and $D(k)$

Method principle

Noise including

The addition of a measurement noise $e(k)$ on the output of the system modifies the local and the global models as follows:

$$\begin{cases} M_1 : y(k) - a_1 u_1(k) - b_1 u_2(k) - e(k) = 0 \\ M_2 : y(k) - a_2 u_1(k) - b_2 u_2(k) - e(k) = 0 \\ M_3 : y(k) - a_3 u_1(k) - b_3 u_2(k) - e(k) = 0 \end{cases} \tag{9}$$

$$\begin{aligned} M : r(k) = &(y(k) - a_1 u_1(k) - b_1 u_2(k) - e(k)) \times \\ &(y(k) - a_2 u_1(k) - b_2 u_2(k) - e(k)) \times \\ &(y(k) - a_3 u_1(k) - b_3 u_2(k) - e(k)) = 0 \end{aligned} \tag{10}$$

Method principle

Noise including

The expression (7) is still valid and the directions corresponding to the three operating modes are now given by:

$$D_1(k) = \begin{pmatrix} -a_1 \\ -b_1 \\ 1 \end{pmatrix} (x_1(k) + s_1(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{11}$$

$$D_2(k) = \begin{pmatrix} -a_2 \\ -b_2 \\ 1 \end{pmatrix} (x_2(k) + s_2(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{12}$$

$$D_3(k) = \begin{pmatrix} -a_3 \\ -b_3 \\ 1 \end{pmatrix} (x_3(k) + s_3(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{13}$$

with $s_i(k) = (2a_i - a_j - a_l)u_1(k) + (2b_i - b_j - b_l)u_2(k)$ and $i, j, l \in \{1, 2, 3\}$.

Method principle

Simulation

The simulation is performed with same model and parameters as the previous simulation.

This noise is uniform and equal in magnitude to 3 % of the maximum amplitude of the signal $y(k)$

Changing of operating mode occur at sames times than in previous simulation.

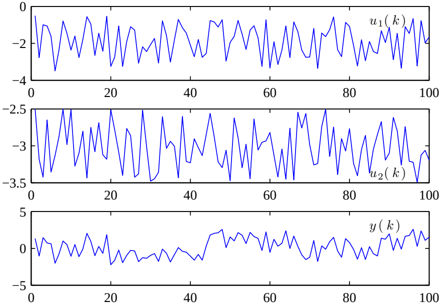

Figure: Input/output of system

Method principle

Simulation

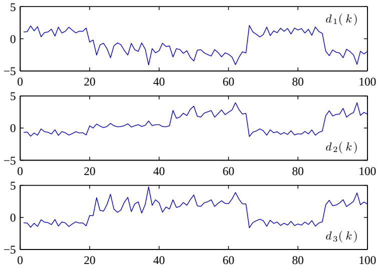

Figure: Components of vector D

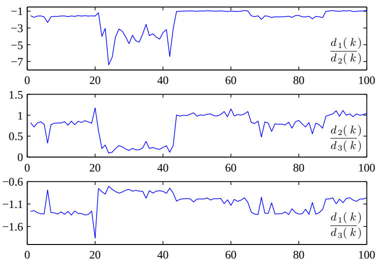

Figure: Components of vector $\bar{D}$

Method principle

Simulation

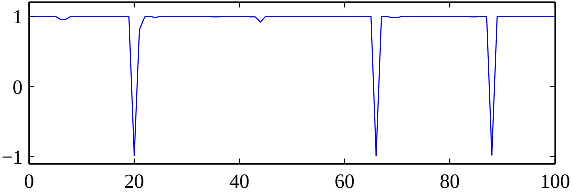

Figure: Cosine of the angle between $D(k-1)$ and $D(k)$

Method principle

Resume

The procedure for determining at each time the operating mode of the system can be sum up as:

- from previously acquired data on a system that covered all operating modes, estimate the parameters θ_i with a least squares method,
- at each time k, evaluate, from the inputs and outputs of the system, the vector $D(k)$,
- analyse the potential change in the direction of the vector $D(k)$ compared to that of $D(k-1)$ and determine if there was a change in the operating mode.

Summary

Application

Mechanical model of casting mold

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A simplified mechanical model of a continuous casting mold can be described by the following equations:

$$\begin{cases} v_p(k) = \left(1 - \frac{\tau f}{M_p}\right) v_p(k-1) + \frac{\tau f}{M_p} v_l(k-1) + \frac{\tau}{M_p} T(k-1) \\ v_l(k) = \frac{\tau f}{M_l} v_p(k-1) + \left(1 - \frac{\tau f}{M_l}\right) v_l(k-1) + \frac{\tau}{M_l} F_l(k-1) \end{cases} \quad (14)$$

- v_p : product speed
- v_l : mold speed
- f : friction coefficient between mold and product
- M_p : mass of the product
- M_l : mass of the mold
- T : the traction on the product
- F_l : the force applied to the mold
- τ : sample time

Application

Mechanical model of casting mold

The objective is to detect friction variation.
We consider three friction coefficients f_1, f_2 and f_3
The system 14 presented two equations so we have two residual $r_q(k)$ (with $q = I, II$) depending on system equations.
Defining:

$$
\begin{aligned}
w_1(k) &= \frac{M_p}{\tau}(v_p(k) - v_p(k-1)) - T(k-1) \\
w_2(k) &= \frac{M_l}{\tau}(v_l(k) - v_l(k-1)) - F_l(k-1) \\
w_3(k-1) &= v_p(k-1) - v_l(k-1)
\end{aligned}
\qquad (15)
$$

Application

Mechanical model of casting mold

We obtain the global model decoupled from the two operating modes :

$$
\begin{aligned}
r_I(k) =& (w_1(k) + f_1 w_3(k-1))(w_1(k) + f_2 w_3(k-1)) \\
& (w_1(k) + f_3 w_3(k-1)) \\
=& \phi_1(k)^T \theta \\
r_{II}(k) =& (w_2(k) - f_1 w_3(k-1))(w_2(k) - f_2 w_3(k-1)) \\
& (w_2(k) - f_3 w_3(k-1)) \\
=& \phi_2(k)^T \theta
\end{aligned}
\tag{16}
$$

with:

$$
\theta = \begin{pmatrix} 1 \\ f_1 + f_2 + f_3 \\ f_1 f_2 + f_1 f_3 + f_2 f_3 \\ f_1 f_2 f_3 \end{pmatrix}, \quad \phi_1(k) = \begin{pmatrix} w_1^3 \\ w_1^2 w_3 \\ w_1 w_3^2 \\ w_3^3 \end{pmatrix}, \quad \phi_2(k) = \begin{pmatrix} w_2^3 \\ -w_2^2 w_3 \\ w_2 w_3^2 \\ -w_3^3 \end{pmatrix} \tag{17}
$$

Application

Mechanical model of casting mold

We evaluate the sensitivity of $r_q(k)$ with regard to the variables $w_1(k)$, $w_2(k)$ and $w_3(k)$:

$$D_I(k) = \begin{pmatrix} \dfrac{\partial r_I(k)}{\partial w_1(k)} \\ \\ \dfrac{\partial r_I(k)}{\partial w_3(k-1)} \end{pmatrix} \quad (18)$$

$$D_{II}(k) = \begin{pmatrix} \dfrac{\partial r_{II}(k)}{\partial w_2(k)} \\ \\ \dfrac{\partial r_{II}(k)}{\partial w_3(k-1)} \end{pmatrix} \quad (19)$$

So we obtain a sensitivity vector $D_q(k)$ for each system equation, $q = I, II$. We note $d_{q,j}$ the j^{th} component of the vector $D_q(k)$.

Application

Simulation

The simulation is performed with the model (14).

Simulation parameters: $M_l = 30t \quad M_p = 239t \quad \tau = 0.1s$
$f_1 = 173 \quad f_2 = 198 \quad f_3 = 147$

The switching operating modes are generated by a function $h(k)$

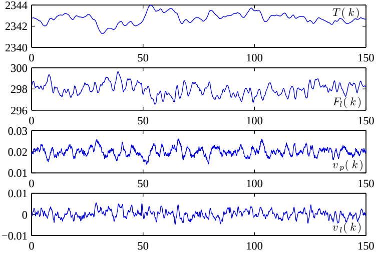

Figure: Input/output of the system

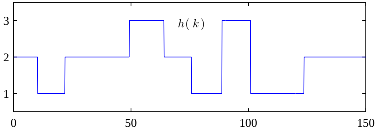

Figure: $h(k)$

Application

Simulation

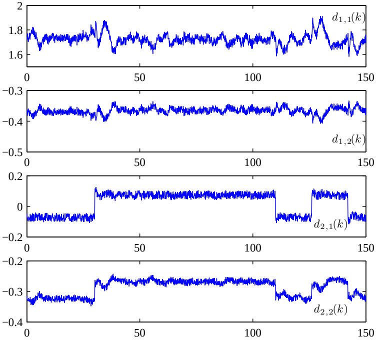

Figure: Components of vector D

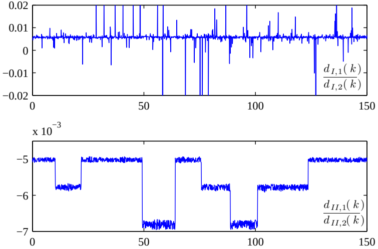

Figure: Components of vector $\bar{D}$

Summary

- Introduction
- Method principle
- Application
- **Conclusion**

Conclusion

- study of the occurence of an a priori unknown operating mode
- study of the influence of uncertainty on the parameters
- application to real data from continuous casting

Thank you for your attention