Operating mode recognition: Application in continuous casting

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***Abstract*— System often has several operating modes that are controlled by the operator to follow a desired change or not in the case of fault occurrence or change due to the environment of the system. The challenge is to be able to know the current operating mode in order to apply the appropriate controls. The aim of this work is to recognise the active mode at any time, and to estimate the switching time between modes. The proposed method is able to detect mode changes without the knowledge of the model parameters characterising each mode. An application in continuous casting illustrates the ability to detect a variation of a friction coefficient.**

I. INTRODUCTION

This communication addresses the problem of operating mode recognition for systems represented by switched regression models. These different models can either characterise normal or abnormal (e.g. when the system is subject to parametric fault) situations. Indeed, to get rid of the model complexity, a widely used modeling strategy consists to represent the system behaviour using a set of models with a simple structure, each model describing the behaviour of the system in a particular operating zone. Switched regression models characterise systems governed by continuous differential (continuous time) or difference (discrete time) equations and discrete variables. The system is described by several operating regimes, called modes, and the transition from one mode to another is governed by a discrete event which occurrence depends on the system variables (input, output, state) or external variables (human operator, fault).

If the operating mode change results from a controlled or measured event, at any time, the operating mode is directly known. If the models are not *a priori* known, based on the knowledge of the inputs/outputs of the system acquired when it operates in a given mode, it is then possible to implement some identification algorithm [1], [2], [3] for identifying the parameter of the corresponding model. Assuming the different models are therefore known as well as their number, the later mode recognition is then easy and can be done by the analysis of the residuals computed as the difference between the measured and predicted outputs by the different models. The reader is referred to [4], [5] for techniques based on bank of observers or [6], [7] and [8] for the use of multi-model approach.

The problem becomes more difficult when the event responsible for the mode change is not known. Indeed, for this unsupervised classification problem, if the different models are unknown, it is necessary to estimate simultaneously model parameters and data partitioning in order to associate to each model data that will allow its identification. To this end, in [9] and [10] the authors used Principal Component Analysis while [11] and [12] use a least squares method.

The main contribution of the proposed method is to detect mode changes without knowing the model parameters characterising each mode. The number of operating modes (described by so-called local models) as well as the model structures describing each of these modes are known *a priori*. The method relies on the estimation of the parameters of a "global" model of the system, resulting from a multiplicative combination of local models. The sensitivity analysis of the global model with regard the input/output variables then provides an indicator to detect changes in the operating mode.

At first, the method is presented using a simple static model without noise. Then, the noise affecting the output measurements is taken into account. Finally, the proposed method is applied on a simplified simulated model of a continuous casting mold in order to detect the variation of a friction coefficient. This detection is of prime interest as the friction coefficient characterises a sticking phenomenon between the solidified steel and the mold.

II. METHOD PRINCIPLE

A. System model

Let us consider the three following models:

$$\begin{cases} M_1 : y(k) - a_1 u_1(k) - b_1 u_2(k) = 0 \\ M_2 : y(k) - a_2 u_1(k) - b_2 u_2(k) = 0 \\ M_3 : y(k) - a_3 u_1(k) - b_3 u_2(k) = 0 \end{cases} \tag{1}$$

Depending on the operating conditions, the system behaviour is described by one of the three models M_1, M_2 or M_3, i. e. at every moment, the data triplet $(u_1(k), u_2(k), y(k))$ checks one of the three models M_1, M_2 or M_3. A general model, with decoupled operating modes, can then be written:

$$\begin{aligned} M : r(k) = &(y(k) - a_1 u_1(k) - b_1 u_2(k)) \times \\ &(y(k) - a_2 u_1(k) - b_2 u_2(k)) \times \\ &(y(k) - a_3 u_1(k) - b_3 u_2(k)) = 0 \end{aligned} \tag{2}$$

The model (2) can be rewritten in order to show global

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system parameters:

$$M : \begin{cases} r(k) = \phi^T(k)\theta = 0 \\ \phi(k) = \big(y^3(k) \;\; y^2(k)u_1(k) \;\; y^2(k)u_2(k) \;\; y(k)u_1^2(k) \\ \qquad y(k)u_2^2(k) \;\; y(k)u_1(k)u_2(k) \;\; u_1^3(k) \;\; u_2^3(k) \\ \qquad u_1^2(k)u_2(k) \;\; u_1(k)u_2^2(k)\big)^T \\ \theta = (\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5 \;\; \theta_6 \;\; \theta_7 \;\; \theta_8 \;\; \theta_9 \;\; \theta_{10})^T \end{cases} \tag{3}$$

$\phi(k)$ is write in order of power decreasing of $y(k)$ then $u_1(k)$ and $u_2(k)$.
At any time, the triplet ($u_1(k)$, $u_2(k)$, $y(k)$) checks this global model, so $r(k) = 0$. Global parameters θ_i depend on local model parameters:

$$\begin{cases} \theta_1 = 1 \\ \theta_2 = -(a_1 + a_2 + a_3) \\ \theta_3 = -(b_1 + b_2 + b_3) \\ \theta_4 = a_1a_2 + a_2a_3 + a_1a_3 \\ \theta_5 = b_1b_2 + b_2b_3 + b_1b_3 \\ \theta_6 = a_1b_2 + a_2b_1 + a_3b_1 + b_2a_3 + a_1b_3 + a_2b_3 \\ \theta_7 = -a_1a_2a_3 \\ \theta_8 = -b_1b_2b_3 \\ \theta_9 = -(a_1b_2a_3 + a_2a_3b_1 + a_1a_2b_3) \\ \theta_{10} = -(a_1b_2b_3 + a_2b_1b_3 + b_1b_2a_3) \end{cases} \tag{4}$$

The set of six unknown parameters a_i and b_i, $i = \{1,2,3\}$ is a solution of a system of nine nonlinear equations. This system can present some problems of compatibility especially when measurements are corrupted by noise. However, here, the objective is to detect the changing of operating mode and it will be shown that the estimation of the parameters a_i and b_i is not necessary. The proposed method relies on the estimation of the global model parameters θ_i, which are estimated using a least squares method from the knowledge of the input/output signals ($u_1(k)$,$u_2(k)$,$y(k)$), and on the analysis of the time-variation of the sensitivity vector of the global model.

B. Design of a mode change indicator

Based on a remark of [10], let us evaluate the sensitivity of $r(k)$ (3) with regard to the system variables:

$$D(k) = \begin{pmatrix} \dfrac{\partial r(k)}{\partial u_1(k)} \\ \dfrac{\partial r(k)}{\partial u_2(k)} \\ \dfrac{\partial r(k)}{\partial y(k)} \end{pmatrix} = \begin{pmatrix} y^2(k)\theta_2 + 2u_1(k)y(k)\theta_4 + u_2(k)y(k)\theta_6 \\ +3u_1^2(k)\theta_7 + 2u_1(k)u_2(k)\theta_9 + u_2^2(k)\theta_{10} \\ y^2(k)\theta_3 + 2u_2(k)y(k)\theta_5 + u_1(k)y(k)\theta_6 \\ +3u_2^2(k)\theta_8 + u_1^2(k)\theta_9 + 2u_1(k)u_2(k)\theta_{10} \\ 3y^2(k)\theta_1 + 2y(k)u_1(k)\theta_2 + 2y(k)u_2(k)\theta_3 \\ +u_1^2(k)\theta_4 + u_2^2(k)\theta_5 + u_1(k)u_2(k)\theta_6 \end{pmatrix} \tag{5}$$

The components of the vector $D(k)$ can be evaluated, at every moment k, from the input and output ($u_1(k)$,$u_2(k)$,$y(k)$) of the system and the parameters θ_i without knowing the active mode at this time and without knowing the parameters a_i and b_i characterising each model.

The vector $D(k)$ can be used to detect the active mode. If the system operates according to model M_1, then $y(k) = a_1u_1(k) + b_1u_2(k)$ and, due to (5), $D(k)$ takes the value:

$$D_1(k) = \begin{pmatrix} -a_1 \\ -b_1 \\ 1 \end{pmatrix} x_1(k) \tag{6}$$

with

$$\begin{aligned} x_1(k) = &((a_1 - a_2)u_1(k) + (b_1 - b_2)u_2(k)) \\ &((a_1 - a_3)u_1(k) + (b_1 - b_3)u_2(k)) \end{aligned} \tag{7}$$

Let us remark that this value cannot be calculated because the local parameters a_i and b_i are unknown. If the system operates according to model M_2, $y(k) = a_2u_1(k) + b_2u_2(k)$ and the vector $D(k)$ expresses:

$$D_2(k) = \begin{pmatrix} -a_2 \\ -b_2 \\ 1 \end{pmatrix} x_2(k) \tag{8}$$

with

$$\begin{aligned} x_2(k) = &((a_2 - a_1)u_1(k) + (b_2 - b_1)u_2(k)) \\ &((a_2 - a_3)u_1(k) + (b_2 - b_3)u_2(k)) \end{aligned} \tag{9}$$

If the system operates according to model M_3, $y(k) = a_3u_1(k) + b_3u_2(k)$ and the vector $D(k)$ expresses:

$$D_3(k) = \begin{pmatrix} -a_3 \\ -b_3 \\ 1 \end{pmatrix} x_3(k) \tag{10}$$

with

$$\begin{aligned} x_3(k) = &((a_3 - a_1)u_1(k) + (b_3 - b_1)u_2(k)) \\ &((a_3 - a_2)u_1(k) + (b_3 - b_2)u_2(k)) \end{aligned} \tag{11}$$

Consequently, the vectors $D_1(k)$, $D_2(k)$ and $D_3(k)$ are equipollent to the vectors:

$$\tilde{D}_1 = \begin{pmatrix} a_1 \\ b_1 \\ -1 \end{pmatrix}, \quad \tilde{D}_2 = \begin{pmatrix} a_2 \\ b_2 \\ -1 \end{pmatrix}, \quad \tilde{D}_3 = \begin{pmatrix} a_3 \\ b_3 \\ -1 \end{pmatrix} \tag{12}$$

and according to the operating mode, the vector $D(k)$ is collinear to the vector $\tilde{D}_1$, $\tilde{D}_2$ or $\tilde{D}_3$

To get rid of the unknown scalar $x_i(k)$ (which sign can evolve according the time instant k), each mode can be characterised by a vector issued from the ratios of the components of $D(k)$. If $d_j(k)$ denotes the j^{th} component of the vector $D(k)$, let us define the vector $\bar{D}(k)$ such that:

$$\bar{D}(k) = \begin{pmatrix} \frac{d_1(k)}{d_2(k)} & \frac{d_2(k)}{d_3(k)} & \frac{d_1(k)}{d_3(k)} \end{pmatrix}^T \tag{13}$$

From (6), (8) and (10), the direction characterising the modes M_1, M_2 and M_3 can then be expressed as:

$$\bar{D}_1 = \begin{pmatrix} \frac{a_1}{b_1} & -b_1 & -a_1 \end{pmatrix}^T \tag{14}$$

$$\bar{D}_2 = \begin{pmatrix} \frac{a_2}{b_2} & -b_2 & -a_2 \end{pmatrix}^T \tag{15}$$

$$\bar{D}_3 = \begin{pmatrix} \frac{a_3}{b_3} & -b_3 & -a_3 \end{pmatrix}^T \tag{16}$$

Vectors (14), (15) and (16) are obtained directly from $D(k)$ and are independent from $x(k)$ by construction. They have a constant amplitude and a fixed direction. Notice that the ability to compute $\bar{D}(k)$ is conditioned by $x(k) \neq 0$. This condition corresponds to the discernability of the modes. For the concerned particular case, the condition $x_i(k) \neq 0$ implies $a_1 \neq a_2 \neq a_3$ and $b_1 \neq b_2 \neq b_3$.

Therefore, the vector $\bar{D}(k)$, calculated from (13), contains the necessary information to detect, at each sample time, the changing of operating mode according to the direction in which it aligns. More precisely, the vector $\bar{D}(k)$ is equal to $\bar{D}_1$, $\bar{D}_2$ or $\bar{D}_3$ depending on the operating mode at time k.

C. Angular distance between modes

To highlights the changing operating mode we can calculate the cosine of the angle between tow vectors $D(k-1)$ and $D(k)$ by the following expression:

$$\cos(D(k-1), D(k)) = \frac{D^T(k-1)D(k)}{\|D(k-1)\| \, \|D(k)\|} \tag{17}$$

If this angle is zero, there was no mode change between times $k-1$ and k. Otherwise, the angle is equal to the angle between the two vectors that characterising the operating modes at times $k-1$ and k, this means that there has been a change in operating mode.

The procedure for determining at each time the operating mode of the system can be sum up as:

- from previously acquired data on a system that covered all operating modes, estimate the parameters θ_i with a least squares method,
- at each time k, evaluate, from the inputs and outputs of the system, the vector $D(k)$ using (5),
- analyse the potential change in the direction of the vector $D(k)$ compared to that of $D(k-1)$ and determine if there was a change in the operating mode.

The addition of a measurement noise $e(k)$ on the output of the system modifies the local and the global models as follows:

$$\begin{cases} M_1 : y(k) - a_1u_1(k) - b_1u_2(k) - e(k) = 0 \\ M_2 : y(k) - a_2u_1(k) - b_2u_2(k) - e(k) = 0 \\ M_3 : y(k) - a_3u_1(k) - b_3u_2(k) - e(k) = 0 \end{cases} \tag{18}$$

$$\begin{aligned} M : r(k) = &(y(k) - a_1u_1(k) - b_1u_2(k) - e(k))\times \\ &(y(k) - a_2u_1(k) - b_2u_2(k) - e(k))\times \\ &(y(k) - a_3u_1(k) - b_3u_2(k) - e(k)) = 0 \end{aligned} \tag{19}$$

The expression (5) is still valid and the directions corresponding to the three operating modes are now given by:

$$D_1(k) = \begin{pmatrix} -a_1 \\ -b_1 \\ 1 \end{pmatrix} (x_1(k) + s_1(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{20}$$

$$D_2(k) = \begin{pmatrix} -a_2 \\ -b_2 \\ 1 \end{pmatrix} (x_2(k) + s_2(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{21}$$

$$D_3(k) = \begin{pmatrix} -a_3 \\ -b_3 \\ 1 \end{pmatrix} (x_3(k) + s_3(k)e(k)) + \begin{pmatrix} \theta_2 \\ \theta_3 \\ 2\theta_1 \end{pmatrix} e^2(k) \tag{22}$$

with $s_i(k) = (2a_i - a_j - a_l)u_1(k) + (2b_i - b_j - b_l)u_2(k)$ and $i, j, l \in \{1, 2, 3\}$. Unlike the noise-free case, vectors $D_1(k)$, $D_2(k)$ and $D_3(k)$ are not collinear to vectors $\tilde{D}_1$, $\tilde{D}_2$ and $\tilde{D}_3$, but describe a wrap around directions $\tilde{D}_1$, $\tilde{D}_2$ and $\tilde{D}_3$, the extent of this envelope is directly related to the amplitude of the noise.

III. NUMERICAL EXAMPLES

A. First example

A first simulation is performed with the model (1) with $a_1 = 1$, $b_1 = -1$, $a_2 = 1.3$, $b_2 = -0.8$, $a_3 = 0.8$ and $b_3 = -0.2$ on a time horizon of 100 samples. Inputs $u_1(k)$ and $u_2(k)$ are bounded random inputs of respective averages -2 and -3 and bounds $[-3.5, -0.5]$ and $[-3.5, -2.5]$. Changing of operating mode occur at random times. With the output $y(k)$ (Fig. 1), it is difficult to determine the moments of mode change. Fig. 2 and 3 respectively show the time evolution of the three components of the vector $D(k)$ and $\bar{D}(k)$. The changing of operating mode of the system then appears. Fig. 4 shows that the moments of mode change can easily be located.

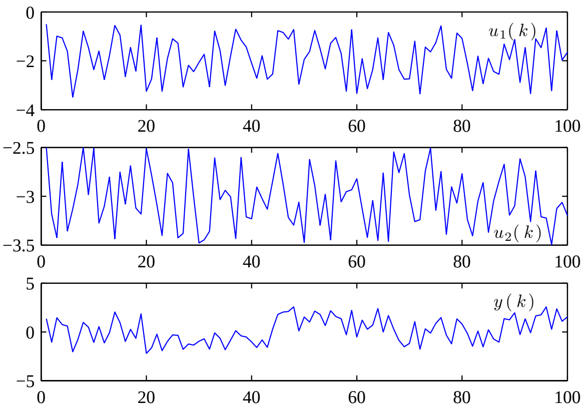

Fig. 1. Inputs/outputs of the system

B. Second example

For the second simulation, noise has been added to the output. This noise is uniform and equal in magnitude to 3 % of the maximum amplitude of the signal $y(k)$. The simulation is performed on 100 samples. Mode changes occur at same time as in previous example. Fig. 5 shows the time evolution of the inputs and the noised output of the system while Fig. 6 presents the three components of the vector $D(k)$.

Fig. 7 shows the components of the vector $\bar{D}(k)$ and Fig. 8 shows the cosine of the angle between $D(k-1)$ and $D(k)$, highlighting the mode changes. We ca remark that the second operating mode change is less evident than the others because

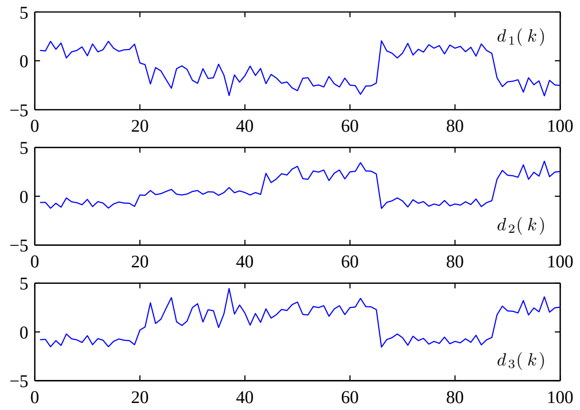

Fig. 2. Components of the vector $D(k)$

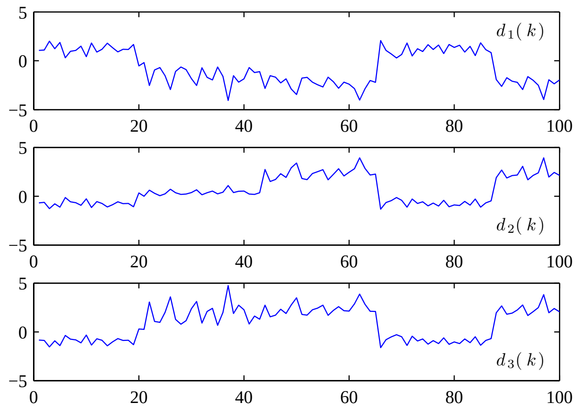

Fig. 6. Components of the vector $D(k)$ with noise

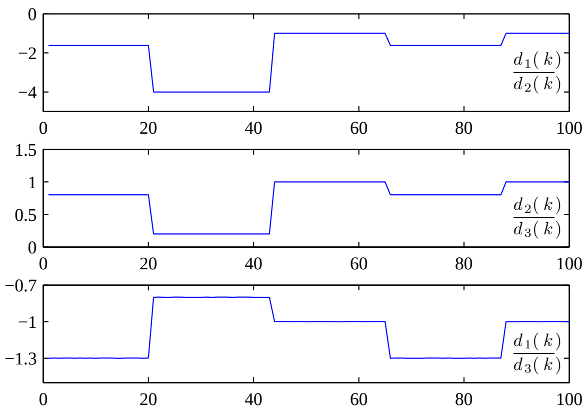

Fig. 3. Components of $\bar{D}(k)$

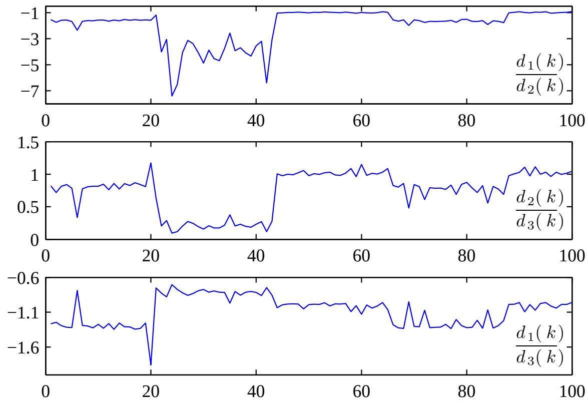

Fig. 7. Components of $\bar{D}(k)$

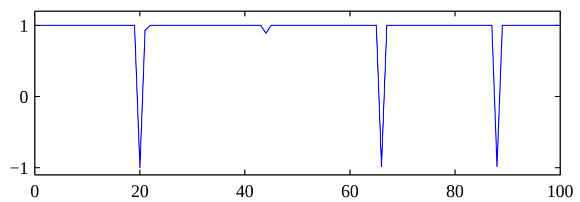

Fig. 4. Cosine of the angle between $D(k-1)$ and $D(k)$

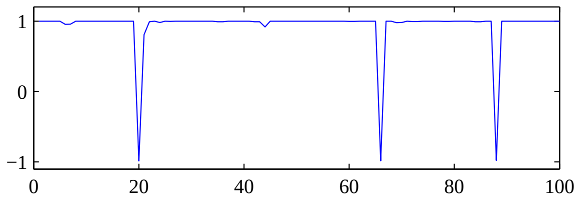

Fig. 8. Cosine of the angle between $D(k-1)$ and $D(k)$

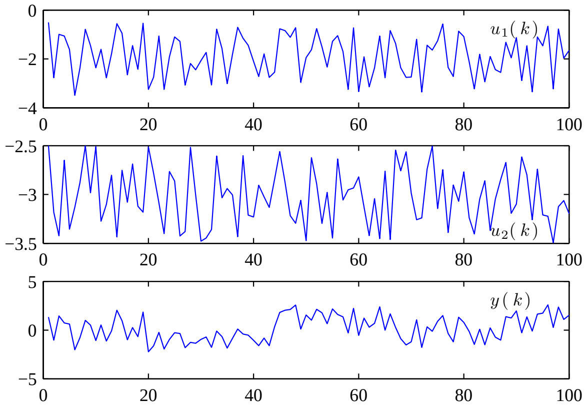

Fig. 5. Input and output of the system

the angle between vector characterising this modes is small. The analysis of the components of $\bar{D}(k)$ using an abrupt change detection method e.g. the Page-Hinkley test, allows the changing of mode to be detected.

IV. APPLICATION ON CONTINUOUS CASTING

The previous method can be easily extended to the case of dynamic systems. In the following, it is applied on a simplified mechanical model of a continuous casting mold.

A. Mechanical model of casting mold

A simplified mechanical model of a continuous casting mold can be described by the following equations:

$$\begin{cases} v_p(k) = \left(1-\frac{\tau f}{M_p}\right) v_p(k-1) + \frac{\tau f}{M_p} v_l(k-1) + \frac{\tau}{M_p} T(k-1) \\ v_l(k) = \frac{\tau f}{M_l} v_p(k-1) + \left(1-\frac{\tau f}{M_l}\right) v_l(k-1) + \frac{\tau}{M_l} F_l(k-1) \end{cases} \tag{23}$$

where M_p is the mass of the product, M_l the mass of the mold and τ the sample time chosen for the discrete representation of the system. The controls of this system are the traction T on the product and the force F_l applied to the mold. The product speed v_p and the mold speed v_l are measured. The variation of the friction coefficient f characterises a "sticker" phenomenon between the solidified steel and the mold which generated poor quality on product structure. The objective is to detect as quickly as possible this variation of friction. Here, we consider three friction coefficients f_1, f_2 and f_3. We obtain the following models M_i with $i = \{1,2,3\}$:

$$M_1 \ : \ \begin{cases} \frac{M_p}{\tau}(v_p(k) - v_p(k-1) - T(k-1)) \\ \quad + f_1(v_p(k-1) - v_l(k-1)) = 0 \\ \frac{M_l}{\tau}(v_l(k) - v_l(k-1) - F_l(k-1)) \\ \quad - f_1(v_p(k-1) - v_l(k-1)) = 0 \end{cases} \tag{24}$$

$$M_2 \ : \ \begin{cases} \frac{M_p}{\tau}(v_p(k) - v_p(k-1) - T(k-1)) \\ \quad + f_2(v_p(k-1) - v_l(k-1)) = 0 \\ \frac{M_l}{\tau}(v_l(k) - v_l(k-1) - F_l(k-1)) \\ \quad - f_2(v_p(k-1) - v_l(k-1)) = 0 \end{cases} \tag{25}$$

$$M_3 \ : \ \begin{cases} \frac{M_p}{\tau}(v_p(k) - v_p(k-1) - T(k-1)) \\ \quad + f_3(v_p(k-1) - v_l(k-1)) = 0 \\ \frac{M_l}{\tau}(v_l(k) - v_l(k-1) - F_l(k-1)) \\ \quad - f_3(v_p(k-1) - v_l(k-1)) = 0 \end{cases} \tag{26}$$

The system 23 presented two equations so we have two residual $r_q(k)$ (with $q = I, II$) depending on system equations.

Defining:

$$\begin{aligned} w_1(k) &= \tfrac{M_p}{\tau}(v_p(k) - v_p(k-1)) - T(k-1) \\ w_2(k) &= \tfrac{M_l}{\tau}(v_l(k) - v_l(k-1)) - F_l(k-1) \\ w_3(k-1) &= v_p(k-1) - v_l(k-1) \end{aligned} \tag{27}$$

we obtain the global model decoupled from the two operating modes :

$$\begin{aligned} r_I(k) =& (w_1(k) + f_1 w_3(k-1))(w_1(k) + f_2 w_3(k-1)) \\ & (w_1(k) + f_3 w_3(k-1)) \\ =& \phi_1(k)^T \theta \\ r_{II}(k) =& (w_2(k) - f_1 w_3(k-1))(w_2(k) - f_2 w_3(k-1)) \\ & (w_2(k) - f_3 w_3(k-1)) \\ =& \phi_2(k)^T \theta \end{aligned} \tag{28}$$

with:

$$\theta = \begin{pmatrix} 1 \\ f + f_1 + f_2 \\ f f_1 + f f_2 + f_1 f_2 \\ f f_1 f_2 \end{pmatrix}$$
$$\phi_1(k) = \begin{pmatrix} w_1^3 \\ w_1^2 w_3 \\ w_1 w_3^2 \\ w_3^3 \end{pmatrix}, \ \phi_2(k) = \begin{pmatrix} w_2^3 \\ -w_2^2 w_3 \\ w_2 w_3^2 \\ -w_3^3 \end{pmatrix} \tag{29}$$

We evaluate the sensitivity of $r_q(k)$ with regard to the variables $w_1(k)$, $w_2(k)$ and $w_3(k)$:

$$D_I(k) = \begin{pmatrix} \dfrac{\partial r_I(k)}{\partial w_1(k)} \\ \\ \dfrac{\partial r_I(k)}{\partial w_3(k-1)} \end{pmatrix} \tag{30}$$

$$D_{II}(k) = \begin{pmatrix} \dfrac{\partial r_{II}(k)}{\partial w_2(k)} \\ \\ \dfrac{\partial r_{II}(k)}{\partial w_3(k-1)} \end{pmatrix} \tag{31}$$

So we obtain a sensitivity vector $D_q(k)$ for each system equation, $q = I, II$. We note $d_{q,j}$ the j^{th} component of the vector $D_q(k)$.

B. Simulation and results

The simulation was performed with the model (23). The parameter values are $M_l = 30t$, $M_p = 239t$, $f_1 = 173$, $f_2 = 198$, $f_3 = 147$ and $\tau = 0.1s$. The simulation is performed on 150 seconds. The switching operating modes are generated by a function $h(k)$ that is presents in Fig.12. A centred and uniformly distributed noise with an amplitude equal to 5 % of the maximum amplitude of each of the output signals has been added to the measurements. θ_i are obtained by applying a less square method.

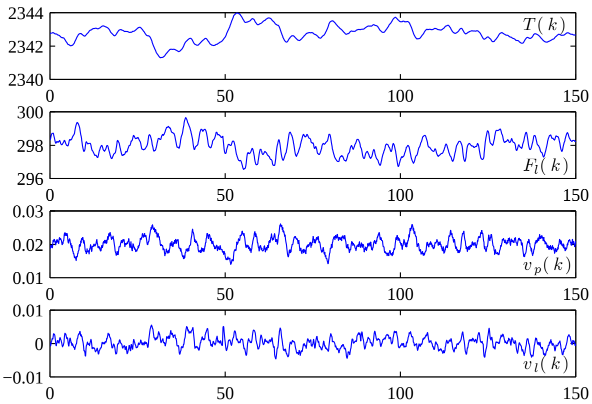

Fig. 9. Inputs/outputs of the system

Fig. 9 does not allow detection of mode changes. Fig. 10 presents $d_{q,j}$. Fig. 11 presents the ratio of each component of vector $D_q(k)$. We can see the second equation which permit to obtain $D_{II}(k)$ is more relevant of mode changes. That is due to parameters values.

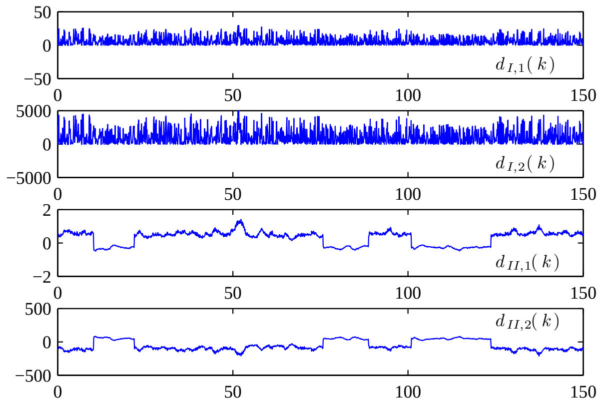

Fig. 10. Components of $D(k)$

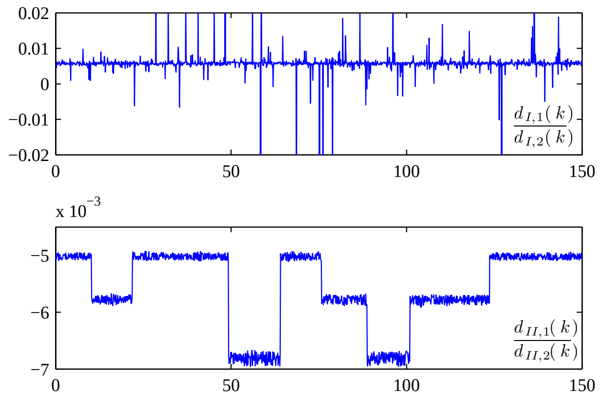

Fig. 11. Components of $\bar{D}(k)$

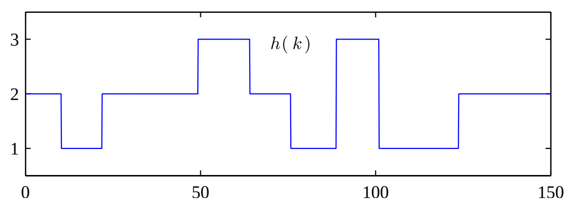

Fig. 12. $h(k)$

V. CONCLUSIONS

The proposed method for the detection of the active system at each instant and for the estimation of the time of mode change was applied for different types of model static, dynamic, mono-output and multi-outputs. Its main interest is that knowledge of the model parameters of each mode is not required, only the estimation of global parameters of the system is used.

This presentation was made under a procedure supervised. For this, we have a data set to estimate the parameters of the overall system. It is easy to extend this method to the case unsupervised. The global parameters of the system are then estimated at each time using a recursive algorithm for example and analysis of the gradient vector to discover the emergence of new modes. The drawback in this case will be the delay for the characterisation of the new mode. We must have enough data in the new mode to have a correct estimation of global parameters. In addition, the calculation of the angle between the vectors related to each mode gradient must be extended in the case of a system to noisy measurements. In this case, it would be possible to determine the maximum permissible noise amplitude for the detection of mode changes. Future work is to take in account the uncertainty about global parameters and their possible variations and determined the minimum distance between mode that permit the detection of changing operating mode.

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