Observer design for state and clinker hardness estimation in cement mill

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Abstract: This paper addresses a solution to state and clinker hardness estimation in a cement mill process. A Takagi-Sugeno model with unmeasurable premise variables is developed for the nonlinear model of a cement mill. Based on this model, a nonlinear observer is proposed in order to estimate the state variables and also the clinker hardness, which is an unknown input of the process. The convergence of the estimation error is studied using the Lyapunov theory and the input-to-state stability (ISS) approach. An optimization problem with LMI constraints is then provided for the synthesis of this observer. Finally, simulation results and some discussions about the effectiveness of the observer are given.

Keywords: Cement mill process, nonlinear systems, Takagi-Sugeno modeling, unknown premise variables, state and unknown input estimation.

1. INTRODUCTION

The general objective of process control in the mineral industry is to optimize the recovery of the valuable minerals, while maintaining the quality of the concentrates delivered to the metal extraction plants. Surveys on the current status and future trends in the automation and control of mineral and metal processing are proposed by Jamsa-Jounela (2001) and Hodouin et al. (2001). As explained in Hodouin (2011), because the processes are strongly disturbed, poorly modeled and difficult to measure, the peripheral tools of the control loop (fault detection and isolation system, data reconciliation procedure, observers, soft sensors, optimizers, model parameter tuners) are as important as the controller itself. In the present paper, we focus on system state and unknown input estimation, which is often a necessary step for automatic control since neither all the state variables nor all the inputs can be measured.

Estimation is often achieved with the use of observers or Kalman filters. Andersen et al. (2006) proposes to estimate the coal flow in pulverized coal mills with a Kalman filter using the measurements of combustion air flow led into the furnace and oxygen concentration in the flue gas. The estimation robustness is enhanced with an extended Kalman filter in Cuevas and Cipriano (2008). In Niemczyk and J.D. (2011), the state of a coal pulverizer process are estimated from the grinding power consumption and the amount of coal accumulated in the mill by employing a variant of a Luenberger observer for bilinear systems.

Alternative approaches for state estimation consist in designing appropriate data driven models, such as neural networks (NN) and support vector machine (SVM) (Curilem et al., 2011). In Ko and Shang (2011), a time delay NN model is developed to predict the feed particle size of a semi-autogenous grinding mill. Estimation based on NN is also proposed in Makokha and Moys (2012). In McElroy et al. (2009), it is shown how the unique insights provided by a discrete element method model of a rotating drum can be used to create soft-sensor models detecting flow regime. In Acuña and Curilem (2009), a comparison is made between a dynamic neural network (NN) model and a support vector machine (SVM) model for estimating the filling level of the mill for a semi autogenous ore grinding process.

Acoustic methods can also be employed. In Aldrich and Theron (2000), digital acoustic signals are transformed to power spectral densities that can be related to particle size distributions in the mill. Acoustic signal analysis is also used in Andrade-Romero et al. (2011) to characterize the resistance percentage of small-size coarse aggregate in a ball mill. In Tang et al. (2012), a novel soft sensor approach is proposed, based on spectral analysis for mill load estimation.

Complex grinding mill circuits are hard to control due to poor plant models, large external disturbances, uncertainties from internal couplings, and process variables that are difficult to measure. To cope with this difficulty, Olivier et al. (2012) proposes a novel fractional order disturbance observer for a run-of-mine ore milling circuit. Ball mill grinding circuits considered in Chen et al. (2009) encompass lumped disturbances including external disturbances, such as the variations of ore hardness and feed particle size, and internal disturbances, such as model mismatches and coupling effects. A disturbance observer based multivariable control scheme is developed to control a two-input-two-output ball mill grinding circuit. Such an observer is introduced to estimate the disturbances in grinding circuit in Yang et al. (2010) and also for feed-forward control, in Yang et al. (2011). In Lepore et al. (2007), a multivariable controller is developed for a mill. As the particle size distribution inside the mill is not directly measurable, a receding-horizon observer is designed, using measurements at the mill exit only.

In this paper, it is proposed to model the cement mill with a Takagi-Sugeno (T-S) system with unmeasurable premise variable. The system is described by three state variables, where only two are accessible to the measure. Moreover, only one input out of two is known. Thus, there is a need to reconstruct, not only the whole state vector, but also an unknown input. For that purpose, a state and unknown input observer for T-S system is designed in order that the state and unknown input estimation errors are bounded. More precisely the input-to-state (ISS) stability of the system describing the estimation errors is established.

The paper is organized as follows. The section 2 is devoted to the description of the studied process. In section 3, the Takagi-Sugeno approach is presented and applied to the process. The state and unknown input observer design is detailed in section 4. Before concluding, simulation results are provided.

2. PROCESS DESCRIPTION

The cement mill, represented in figure 1, consists of a ball mill in closed-loop with a separator. The separator is driven by its rotational speed v (rpm). The rotating ball mill is fed with cement clinker at feeding rate u (tons/h), in which balls perform the breakage of the material particles by fracture and/or attrition. The output material of the mill is transfered to the separator which separates the material into the finished product flow y_f (tons/h) and the recycled flow y_r (tons/h), which is recirculated to the mill inlet.

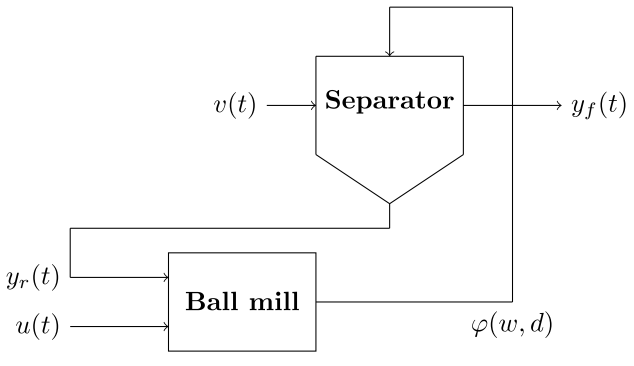

Fig. 1. Cement mill process

The mathematical model of the process is described by three differential equations (see Grognard et al. (2001)). The first two equations (1) and (2) describe how the input flow rate of the separator is divided into the overflow and the underflow depending on the separation function. Equation (3) corresponds to the material conservation in the ball mill while expressing the time evolution of the load inside the ball mill.

$$T_f \dot{y}_f(t) = -y_f(t) + (1-\alpha(v))\varphi(w(t), d(t)) \tag{1}$$
$$T_r \dot{y}_r(t) = -y_r(t) + \alpha(v)\varphi(w(t), d(t)) \tag{2}$$
$$\dot{w}(t) = -\varphi(w(t), d(t)) + y_r(t) + u(t) \tag{3}$$

where

$$\varphi(w(t), d(t)) = p_1 w(t) \exp\left(-p_2 d(t) w(t)\right) \tag{4}$$
$$\alpha(v(t)) = p_3 v^3(t) + p_4 v^4(t) + p_5 v^5(t) \tag{5}$$

and T_f, T_r (h) are time constants, $w(t)$ is the amount of material in the mill (mill load) and $d(t)$ is the clinker hardness. Here, the separation function defined by $\alpha(v)$ depends on the rotation speed of the separator and ball mill outflow rate is defined by the function $\varphi(w(t), d(t))$ which is related to its load and the hardness of the material. The system state equation is defined by

$$x(t) = [y_f(t) \ y_r(t) \ w(t)]^T$$

The measured outputs are the finished product $y_f(t)$ and the recycled flow $y_r(t)$. Then, the system is described by the following state equations

$$\dot{x}(t) = f(x(t), d(t)) + Bu(t) \tag{6}$$
$$y(t) = Cx(t) \tag{7}$$

where

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$f(x(t), d(t)) = \begin{pmatrix} \frac{1}{T_f}\left(-x_1(t) + (1-\alpha(v))\varphi(x_3(t), d(t))\right) \\ \frac{1}{T_r}\left(-x_2(t) + \alpha(v)\varphi(x_3(t), d(t))\right) \\ x_2(t) - \varphi(x_3(t), d(t)) \end{pmatrix}$$

in which the hardness $d(t)$ is unknown.

3. EXACT TAKAGI-SUGENO MODEL

The Takagi-Sugeno (T-S) modeling, introduced by Takagi and Sugeno (1985), allows to represent the behavior of a nonlinear system (*i.e.* $\dot{x}(t) = f(x(t), u(t))$) by the interpolation of a set of linear sub-models. Each sub-model contributes to the global behavior of the nonlinear system through a particular weighting function $\mu_i(z(t))$. The T-S structure is given by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))(A_i x(t) + B_i u(t)) \tag{8}$$

where $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ are known matrices, r being the number of sub-models. The weighting functions $\mu_i(z(t))$ depend on the premise variable $z(t)$ which can be measurable (as the input or the output of the system) or non measurable variables (as the state of the system). These functions verify the so-called convex sum property

$$\sum_{i=1}^{r} \mu_i(z(t)) = 1, \quad 0 \leq \mu_i(z(t)) \leq 1, \quad i = 1, ..., r \tag{9}$$

Since it is an appealing mean to tackle nonlinear systems, a considerable amount of work is devoted to the stability analysis, the state estimation, the diagnosis and the control of T-S systems, see the reference book by Tanaka and Wang (2001) or the more recent one by Lendek et al.

(2010). It is important to note that most of the previous works are dedicated to T-S systems with measurable premise variables.

It is known that if the nonlinearities of the system are bounded, the sector nonlinearity approach allows to derive an exact re-writing of any nonlinear system under a quasi-LPV (Linear Parameter-Varying) form and then under a T-S form (Tanaka and Wang, 2001). The main steps of this derivation are now presented.
Assume that the clinker hardness $d(t)$ is different from zero at all time, which is a realistic assumption, and since the function $f(x(t), d(t))$ is Lipschitz and satisfies the property $f(0, 0) = 0$, the system (6) can be re-written in quasi-LPV form as follows

$$\dot{x}(t) = \begin{pmatrix} -\frac{1}{T_f} & 0 & 0 \\ 0 & -\frac{1}{T_r} & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 & \frac{z_1(x(t), d(t))}{T_f} \\ 0 & 0 & 0 \\ 0 & 0 & -z_1(x(t), d(t)) \end{pmatrix} x(t)$$
$$+ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t) + \begin{pmatrix} -\frac{z_2(x(t), d(t))}{T_f} \\ \frac{z_2(x(t), d(t))}{T_r} \\ 0 \end{pmatrix} d(t) \quad (10)$$

where the variables $z_1(x(t), d(t))$ and $z_2(x(t), d(t))$, later selected as premise variables, are defined by

$$z_1(x(t), d(t)) = p_1 \exp(-p_2 d(t) x_3(t))$$
$$z_2(x(t), d(t)) = \alpha(v(t)) p_1 x_3(t) \frac{\exp(-p_2 d(t) x_3(t))}{d(t)}$$

One can see that the premise variables depend on the unmeasurable state variables $x_3(t)$ and $d(t)$. For the sake of brevity, the premise variable are denoted by $z(t)$ in the remaining of the paper, but it must be kept in mind that they depend on the system state. Due to their physical meaning, $d(t)$ and $z(t)$ are bounded, and so are the premise variables

$$\begin{aligned} z_1^{\min} \le z_1(x(t), d(t)) \le z_1^{\max} \\ z_2^{\min} \le z_2(x(t), d(t)) \le z_2^{\max} \end{aligned} \quad (11)$$

Since they are bounded, the premise variables can be written as

$$z_1(t) = F_1^0(z_1(t)) z_1^{\max} + F_1^1(z_1(t)) z_1^{\min} \quad (12)$$
$$z_2(t) = F_2^0(z_2(t)) z_2^{\max} + F_2^1(z_2(t)) z_2^{\min} \quad (13)$$

where the functions F_1^0, F_1^1, F_2^0 and F_2^1 are defined by

$$F_1^0(z_1(t)) = \frac{z_1(t) - z_1^{\min}}{z_1^{\max} - z_1^{\min}}, \; F_1^1(z_1(t)) = \frac{z_1^{\max} - z_1(t)}{z_1^{\max} - z_1^{\min}}$$
$$F_2^0(z_2(t)) = \frac{z_2(t) - z_2^{\min}}{z_2^{\max} - z_2^{\min}}, \; F_2^1(z_2(t)) = \frac{z_2^{\max} - z_2(t)}{z_2^{\max} - z_2^{\min}}$$

and satisfy the following property for $i = 1, 2$ and $j = 0, 1$

$$0 \le F_i^j(z_i(t)) \le 1, \; \sum_{j=0}^{1} F_i^j(z_i(t)) = 1 \quad (14)$$

With this last property, and defining the weighting functions $\mu_i(z(t))$ by

$$\mu_1(z) = F_1^0(z_1) F_2^0(z_2), \; \mu_3(z) = F_1^1(z_1) F_2^0(z_2)$$
$$\mu_2(z) = F_1^0(z_1) F_2^1(z_2), \; \mu_4(z) = F_1^1(z_1) F_2^1(z_2)$$

the equivalent T-S model of (1) is given, without loss of information, by

$$\dot{x}(t) = \sum_{i=1}^{4} \mu_i(z(t)) \left(A_i x(t) + B u(t) + E_i d(t)\right) \quad (15)$$

where it can readily be checked that the functions $\mu_i(z(t))$ satisfy (9) and the matrices are defined by

$$A_1 = \begin{pmatrix} -\frac{1}{T_f} & 0 & -\frac{z_1^{\max}}{T_f} \\ 0 & -\frac{1}{T_r} & 0 \\ 0 & 1 & -z_1^{\max} \end{pmatrix}, \; A_3 = \begin{pmatrix} -\frac{1}{T_f} & 0 & -\frac{z_1^{\min}}{T_f} \\ 0 & -\frac{1}{T_r} & 0 \\ 0 & 1 & -z_1^{\min} \end{pmatrix},$$

$$E_1 = \begin{pmatrix} -\frac{z_2^{\max}}{T_f} \\ \frac{z_2^{\max}}{T_r} \end{pmatrix}, \; E_2 = \begin{pmatrix} -\frac{z_2^{\min}}{T_f} \\ \frac{z_2^{\min}}{T_r} \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$A_2 = A_1, \quad A_4 = A_3, \quad E_3 = E_1, \quad E_4 = E_2$$

3.1 Computation of the bounds $z_i^{\max}$ and $z_i^{\min}$

The chosen premise variables $z_i, i = 1, 2$ (11) are bounded by $z_i^{\max}$ and $z_i^{\min}$. The computation of these bounds is performed by using the sector nonlinearity approach (see chapter 14 of Tanaka and Wang (2001)). The variation analysis of each premise variable leads to obtain the corresponding bounds of the nonlinear sector, as shown in the figure 2 for the variable z_1.

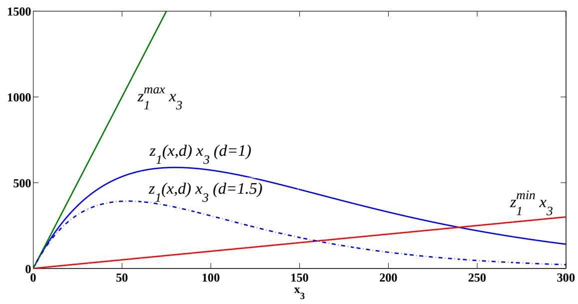

Fig. 2. Sector nonlinearity approach

The obtained exact T-S model (15) is valid in a compact set of the state space. This set can be enlarged to take into account a larger operating region by modifying the parameters $z_i^{\max}$ and $z_i^{\min}$. Furthermore, taking into account realistic situations, the values of these bounds allow satisfying observability condition of each sub-model of the T-S model.

4. OBSERVER DESIGN FOR STATE AND CLINKER HARDNESS ESTIMATION

In this section, an observer is synthesized for estimating the unmeasured system state $w(t)$ and the unknown input $d(t)$ from only the knowledge of the flow rates $y_f(t)$ and $y_r(t)$. It is known that the clinker hardness $d(t)$ may be considered as constant during limited periods of time and due to the physical meaning of the state variables, the following nonrestrictive assumption can be made

Assumption 1. In the remaining it is supposed that

- the state $x(t)$ is bounded;
- the unknown input $d(t)$ is constant (*i.e.* $\dot{d}(t) = 0$).

Under this realistic assumption, the proposed proportional-integral observer with gains L_i and K_i takes the following form

$$\dot{\hat{x}}(t) = \sum_{i=1}^{4} \mu_i(\hat{z}(t))(A_i\hat{x}(t)+Bu(t)+E_i\hat{d}(t)+L_i(y(t)-\hat{y}(t)))$$
$$\dot{\hat{d}}(t) = \sum_{i=1}^{4} \mu_i(\hat{z}(t))K_i(y(t)-\hat{y}(t)) \tag{16}$$
$$\hat{y}(t) = C\hat{x}(t)$$

where $\hat{x}$, $\hat{d}$ and $\hat{z}$ respectively denote the estimates of x, d and z. Let us define the augmented state by $x_a(t) = [x^T(t) \;\; d^T(t)]^T$ and denote its estimate by $\hat{x}_a$. The state and unknown input estimation error, defined by $e_a(t) = x_a(t) - \hat{x}_a(t)$, obeys the following differential equation

$$\dot{e}_a(t) = \sum_{i=1}^{4} \mu_i(\hat{z}(t))(\mathcal{A}_i - \mathcal{M}_i\mathcal{C})e_a(t) + \Delta(t) \tag{17}$$

where

$$\mathcal{A}_i = \begin{pmatrix} A_i & E_i \\ 0 & 0 \end{pmatrix}, \quad \mathcal{M}_i = \begin{pmatrix} L_i \\ K_i \end{pmatrix}, \quad \mathcal{C} = (C \;\; 0)\,,$$
$$\Delta(t) = \sum_{i=1}^{4} (\mu_i(z(t)) - \mu_i(\hat{z}(t)))\mathcal{A}_i x_a(t) \tag{18}$$

The aim is to design an observer (16) for the system (6)-(7), such that the state and unknown input estimation errors remain bounded. Thus the definition of input to state stability (ISS) is needed.

Definition 1. (Sontag and Wang (1995)) The system (17) is said to be ISS if there exists a $\mathcal{KL}$ function $\beta : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and a $\mathcal{K}$ function α such that, for each input $\Delta(t)$ satisfying $\|\Delta(t)\|_\infty < \infty$ and each initial condition $e_a(0) \in \mathbb{R}^3$, the trajectory of (17) associated with $e_a(0)$ and $\Delta(t)$ satisfies

$$\|e_a(t)\| \leq \beta\left(\|e_a(0)\|, t\right) + \alpha\left(\|\Delta(t)\|_\infty\right), \forall t \tag{19}$$

From Assumption 1 and the fact that the functions μ_i are bounded, the term $\Delta(t)$ is bounded. Indeed, the system is stable which provides bounded states for bounded input δ_f. Under bounded perturbation term $\Delta(t)$, the observer (16) is synthesized by solving the optimization problem under LMI constraints given in the Theorem 1.

Theorem 1. Under the Assumption 1 and for a given parameter $\sigma \in [0, 1]$, if there exists a symmetric and positive definite matrix P, gain matrices $\mathcal{K}_i$ and positive scalars $\bar{\gamma}$ and $\bar{\alpha}$ solution to the following optimization problem, $i = 1, ..., 4$

$$\min_{P,\mathcal{K}_i,\overline{\gamma},\overline{\alpha}} \sigma\bar{\alpha} + (1-\sigma)\bar{\gamma}$$
$$\begin{pmatrix} \mathcal{A}_i^T P + P\mathcal{A}_i - \mathcal{K}_i C - C^T\mathcal{K}_i^T + I & P \\ P & -\overline{\gamma}I \end{pmatrix} < 0 \tag{20}$$
$$\begin{pmatrix} -\bar{\alpha}I & P \\ P & -I \end{pmatrix} \leq 0 \tag{21}$$
$$P \geq I \tag{22}$$

then the error dynamics (17) is ISS with respect to $\Delta(t)$ and $e_a(t)$ satisfies the following inequality

$$\|e_a(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\left(e^{-\frac{1}{2\lambda_{\max}(P)}t}e_a(0) + \gamma\Delta_\infty\right) \tag{23}$$

The gains of the observer are computed by $\mathcal{M}_i = P^{-1}\mathcal{K}_i$. The attenuation level of the transfer from $\Delta(t)$ to the estimation error $e_a(t)$ is $\gamma = \sqrt{\overline{\gamma}}$ and the $\lambda_{\max}(P) \leq \alpha$ where $\alpha = \sqrt{\overline{\alpha}}$. Hence, the size of the convergence set D is obtained by computing the quantity $\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\gamma\Delta_\infty$.

Proof. Assume that the LMIs (20) are feasible. Let us consider the vector

$$\xi(t) = \left[e_a^T(t) \;\; \Delta^T(t)\right]^T \tag{24}$$

Pre- and post multiplying (20) by $\xi(t)$ and $\xi^T(t)$ respectively and with $\overline{\gamma} = \gamma^2$ and $\Phi_i = \mathcal{A}_i - \mathcal{M}_i C$, the following is obtained

$$e_a^T(t)\left(\Phi_i^T P + P\Phi_i\right)e_a(t) + e_a^T(t)P\Delta(t)$$
$$+\Delta^T(t)Pe_a(t) + e_a^T(t)e_a(t) - \gamma^2\Delta^T(t)\Delta(t) < 0 \tag{25}$$

Since $0 \leq \mu_i(.) \leq 1$, multiplying (25) by $\mu_i(\hat{z}(t))$, and summing the obtained four equations, one obtains

$$\sum_{i=1}^{4} \mu_i(\hat{z}(t))(e_a^T(t)\left(\Phi_i^T P + P\Phi_i\right)e_a(t) + e_a^T(t)P\Delta(t)$$
$$+ \Delta^T(t)Pe_a(t)) < -e_a^T(t)e_a(t) + \gamma^2\Delta^T(t)\Delta(t) \tag{26}$$

which is equivalent to

$$\dot{V}(t) < -e_a^T(t)e_a(t) + \gamma^2\Delta^T(t)\Delta(t) \tag{27}$$

where

$$V(t) = e_a^T(t)Pe_a(t), \quad P = P^T > 0 \tag{28}$$

From (28), it obviously follows that

$$\lambda_{\min}(P)\left\|e_a(t)\right\|^2 \leq V(t) \leq \lambda_{\max}(P)\left\|e_a(t)\right\|^2 \tag{29}$$

Consequently, if (20) holds, the time derivative of $V(t)$ is bounded as follows

$$\dot{V}(t) \leq -\frac{1}{\lambda_{\max}(P)}V(t) + \gamma^2\left\|\Delta(t)\right\|^2 \tag{30}$$

By using the Gronwall-lemma, it follows

$$V(t) \leq V(0)e^{-\frac{t}{\lambda_{\max}(P)}} + \gamma^2\int_0^t e^{-\frac{t-s}{\lambda_{\max}(P)}}\left\|\Delta(s)\right\|^2 ds \tag{31}$$

Defining Δ_∞ the upper bound of the euclidean norm of $\Delta(t)$ (*i.e.* $\|\Delta(t)\| \leq \Delta_\infty, \forall t$), it follows

$$V(t) \leq V(0)e^{-\frac{t}{\lambda_{\max}(P)}} + \gamma^2\Delta_\infty^2\lambda_{\max}(P) \tag{32}$$

Finally, using (29) with the square root, one obtains

$$\|e_a(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\left(e^{-\frac{1}{2\lambda_{\max}(P)}t}e_a(0) + \gamma\Delta_\infty\right) \tag{33}$$

From this inequality, we conclude that if $\Delta(t) \equiv 0$ then $e(t) \to 0$ when $t \to \infty$. Moreover, in the presence of the perturbation $\Delta(t)$, the error $\|e_a(t)\|$ is bounded by $\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\gamma\Delta_\infty$ when $t \to \infty$. The inequality (33) establishes the ISS of (17).

Note that the size of the convergence set D depends on the selected matrix P and the parameter γ. The set D should be made as small as possible to ensure a good accuracy of convergence. The choice of γ and P providing a small set of convergence is not obvious because the problem is not convex. In the next, a technique is proposed to transform the non convex problem to a convex one under LMI constraints. Let us consider the following inequality

$$\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \leq \sqrt{\alpha} \tag{34}$$

where α is a positive scalar to minimize. Since $\lambda_{\max}(P) > \lambda_{\min}(P)$, the minimal value of $\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ which can be obtained is equal to 1. From this, one can impose $P \geq I$ which leads to $\lambda_{\min}(P) \geq 1$. It follows

$$\lambda_{\max}(P) \leq \alpha \Leftrightarrow P^T P - \alpha^2 I \leq 0 \quad (35)$$

Using Schur's complement lemma

$$\begin{pmatrix} -\alpha^2 I & P \\ P & -I \end{pmatrix} \leq 0 \quad (36)$$

defining $\bar{\alpha} = \alpha^2$ and considering the objective function

$$\min \quad \sigma\bar{\alpha} + (1-\sigma)\bar{\gamma} \quad (37)$$

where $\sigma \in [0, 1]$, the optimization problem (37) under LMI constraints (20)-(22) is then obtained which ends the proof.

5. SIMULATION RESULTS

In this section, obtained simulation results are provided with some discussions about the proposed observer performances. The observer is designed by using the matrices A_i, E_i and C, where the bounds of the premise variables are $z_1^{\min} = 2$, $z_1^{\max} = 20$ and $z_2^{\min} = 90$, $z_2^{\max} = 440$. Using an iterative approach on the parameter $\sigma \in [0, 1]$ and solving the optimization problem given in the Theorem 1, the minimal size of the convergence region is $0.5055\Delta_\infty$ obtained with $\sigma = 0.05$. The corresponding gains are

$$L_1 = \begin{pmatrix} 23.22 & -3.12 \\ -1.98 & 12.40 \\ 123.58 & 6.25 \end{pmatrix}, L_2 = \begin{pmatrix} 22.29 & 0.35 \\ 2.75 & -6.08 \\ 124.18 & 5.97 \end{pmatrix},$$

$$L_3 = \begin{pmatrix} 5.84 & -3.82 \\ -6.38 & 12.07 \\ 11.28 & 1.61 \end{pmatrix}, L_4 = \begin{pmatrix} 4.81 & -0.47 \\ -1.79 & -6.40 \\ 11.30 & 1.38 \end{pmatrix}$$

$$K_1 = 10^3 \times \begin{pmatrix} -2.32 & 9.31 \end{pmatrix}, K_2 = 10^3 \times \begin{pmatrix} -0.47 & 1.90 \end{pmatrix}$$

$$K_3 = 10^3 \times \begin{pmatrix} -2.32 & 9.31 \end{pmatrix}, K_4 = 10^3 \times \begin{pmatrix} -0.47 & 1.90 \end{pmatrix}$$

The norm of the state estimation error $\|e(t)\|$ converges to the region with a size defined by $\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\gamma\Delta_\infty = 0.5055 \times \Delta_\infty$. In the steady state, the computation of Δ_∞ gives values less than 0.1, so the obtained bound of the error $\|e(t)\|$ is $0.5055 \times \Delta_\infty = 0.05055$, in the other hand the computation of $\|e(t)\|$ gives values less than $10^{-4} < 0.05055$ which confirms the Theorem 1. The initial conditions of the system are $x(0) = [50 \ \ 50 \ \ 50]^T$ and those of the observer are $\hat{x}(0) = [30 \ \ 30 \ \ 30]^T$ and $\hat{d}(0) = 0.5$. The feeding rate (control input) u (tons/h), computed from a PI controller as follows

$$u(t) = -x_{2ref} + k_1(x_{3ref} - x_3(t)) + x_c(t) \quad (38)$$

$$\dot{x}_c(t) = k_2(x_{3ref}(t) - x_3(t)) \quad (39)$$

where the variables $x_2(t)$ and $x_3(t)$ are controlled in order to track the reference trajectories x_{2ref} and x_{3ref}. The parameters of the controller are fixed as follows $k_1 = 15$ and $k_2 = 30$. The obtained control input is shown in the figure 3 (bottom). The weighting functions $\mu_i(.)$ of the T-S model are depicted in the figure 3 (top) and one can see that, since the system is nonlinear, all the sub-models are activated at each time.

The estimated states are depicted in the figure 4 while the state estimation errors are shown in the figure 5 (top) and the clinker hardness estimation is given in the figure 5 (bottom). Given this figure, it should be noted that when the hardness of the ore is increasing, then the output rate

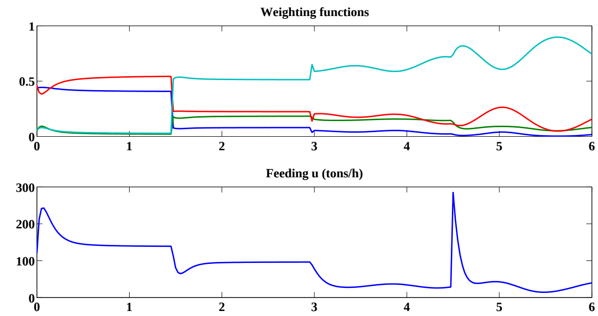

Fig. 3. Time evolution of the weighting functions μ_i (top) Feeding rate (control input) (bottom)

decreases, which preserves the physical meaning of such a process. In addition, in the case of noised measurements

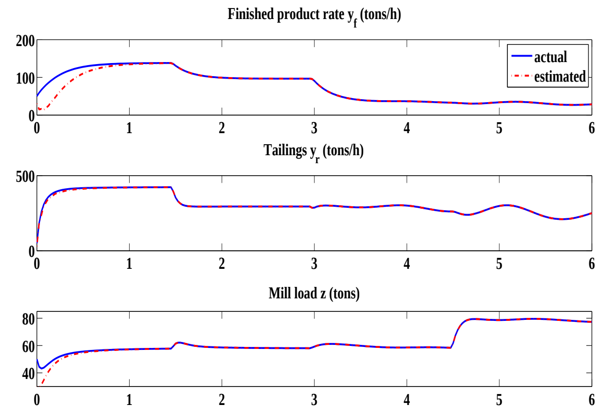

Fig. 4. System states and their estimates

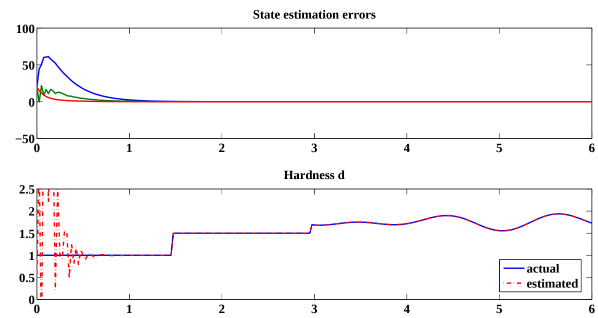

Fig. 5. State estimation errors (top) and clinker hardness and its estimate (bottom)

with centered noise signal in the range $[-10, 10]$, the observer is, also, able to provide an acceptable estimations of the state vector $x(t)$ and the hardness $d(t)$ as shown in the figure 6. One can note that even if the hardness $d(t)$ is time varying, an acceptable estimation is provided by the observer. The transient phenomenon in estimation of $d(t)$ is due to the fact that at $t = 0$ the values $d(0)$ and $\hat{d}(0)$ are different and the imaginary parts of observer poles are large. The magnitude of the overshoot in this transient can be reduced by pole clustering in LMI region which allows to reduce the imaginary part of the eigenvalues of the observer.

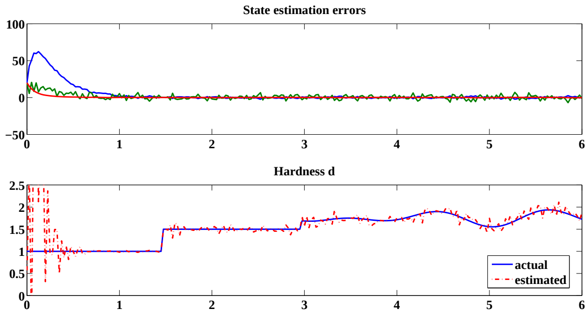

Fig. 6. State estimation errors (top) and clinker hardness and its estimate with noised measurements

6. CONCLUSION

The goal of this paper is to design an observer for mill load and clinker hardness estimation in cement mill process with only the knowledge of the feeding u (tons/h), the tailings y_r (tons/h) and the finished product rate y_f (tons/h). A nonlinear mathematical model of the process dynamics is considered and transformed with sector nonlinearity transformation under a Takagi-Sugeno model with unmeasurable premise variables. By using the Lyapunov theory and the input-to-state stability (ISS), optimization problem with LMI constraints is established which ensures ISS stability with a good accuracy as shown in the simulation results. Simulations results are provided in order to illustrate the proposed approach. The observer is also tested with noised measurements. It follows that the proposed observer gives accurate estimations of the states of the system and the clinker hardness even if .

Work is underway to extend the proposed state reconstruction when one takes into account, in addition to flow rates, particle size distributions of the product throughout the separation-grinding loop.

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