Nonlinear observer based sensor fault tolerant control for nonlinear systems

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Motivation and proposition

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To design a sensor fault tolerant controller for nonlinear systems

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Proposition and outline

1. To describe the nonlinear behaviour using a Takagi-Sugeno model (with measurable premise variables)
2. To design a residual generator able to detect and isolate sensor faults
3. To estimate a “fault-free” state of the system by a judicious blending (based on the residual magnitude) of the different state estimates issued from a DOS structure
4. To design an observer-based controller generating a Parallel Distributed Compensation (PDC) control law, using this “fault-free” state estimate

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Outline

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Modelling of nonlinear systems

Takagi-Sugeno model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x(t) \end{cases}$$

- Convex sum property : $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \quad \forall t, \forall i \in \{1, ..., r\}$

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Notations

r	: number of submodels
μ_i	: weighting functions
$\xi(t)$	: premise (or decision) variable, assumed, here, to be measurable

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Obtention of that kind of model

- Linearisation of an existing nonlinear model around operating points
- Direct identification of the model parameters (once the structure has been chosen) from sets of input-ouput data

Residual generator design

Faulty system (sensor faults)

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(C_i x(t) + G_i f(t)\right) \end{cases}$$

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Residual generator

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))) \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i \hat{x}(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases}$$

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Unknown gain matrices to be determined : $L_i, i = 1, \ldots, r$ and M

Residual generator design

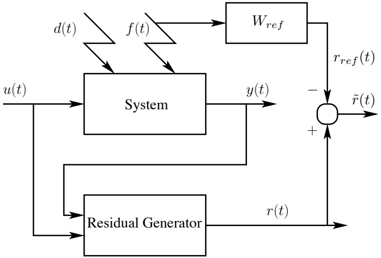


FIGURE: Residual generator

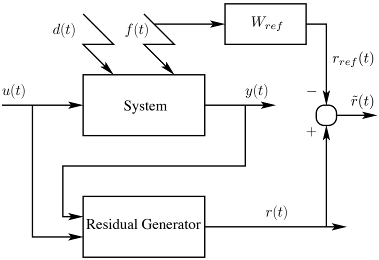
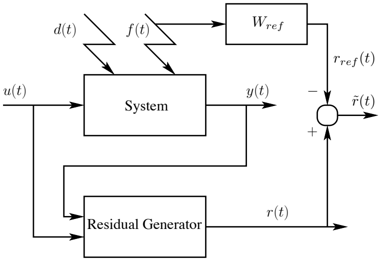


FIGURE: Residual generator

- A filter $W_{ref}(s) = \left(\begin{array}{c|c} A_{ref} & B_{ref} \\ \hline C_{ref} & D_{ref} \end{array}\right)$ is introduced to model the desired response of the residual $r(t)$ to the fault $f(t)$.
- If $W_{ref}(s)$ is diagonal each residual $r_i(t)$ is made sensitive only to the fault affecting the i^{th} output
- The gains L_i and M are computed to minimize the $\mathscr{L}_2$ gain from $f(t)$ to $\tilde{r}(t) = r_{ref}(t) - r(t)$

Residual generator design



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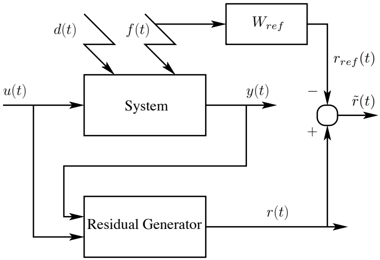

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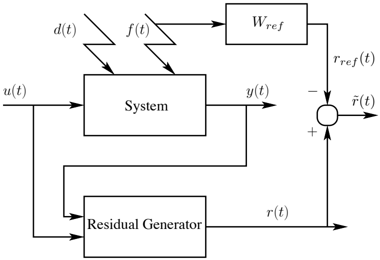

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Ichalal D., Marx B., Ragot J., Maquin D. Fault diagnosis in Takagi-Sugeno nonlinear systems. 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, SAFEPROCESS'2009, Barcelona, Spain, June 30th - July 3rd, 2009. (doi :10.3182/20090630-4-ES-2003.00084)

Theorem 1

The robust residual generator exists if there exists symmetric and positive definite matrices P_1 and P_2, and matrices K_i and M solving the following optimization problem

$$\min_{P_1,P_2,K_i,M} \gamma$$

under the following LMI constraints

$$\begin{cases} X_{ii} < 0, & i = 1,...,r \\ \frac{2}{r-1} X_{ii} + X_{ij} + X_{ji} < 0, & i,j = 1,...,r, i \neq j \end{cases}$$

where, for $(i,j) \in \{1,\dots,r\}$, X_{ij} is defined by

$$X_{ij} = \begin{pmatrix} A_i^T P_1 + P_1 A_i - C_j^T K_i^T - K_i C_j & 0 & -K_i G_j & C_i^T M^T \\ * & A_{ref}^T P_2 + P_2 A_{ref} & P_2 B_{ref} & -C_{ref} \\ * & * & -\gamma I & G_i^T M^T - D_{ref}^T \\ * & * & * & -\gamma I \end{pmatrix}$$

The residual generator gains are given by $L_i = P_1^{-1} K_i$ and M. The attenuation level from the faults $f(t)$ to the virtual residual $\tilde{r}(t) = r_{ref}(t) - r(t)$ is given by γ.

Use of an observer bank

- The k^{th} observer is fed with the input of the system $u(t)$ and the k^{th} output $y^k(t)$ and produces the estimate $\hat{x}^k(t)$

$$\begin{cases} \dot{\hat{x}}^k(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \left(A_i \hat{x}^k(t) + B_i u(t) + L_i^k \left(y^k(t) - \hat{y}^k(t) \right) \right) \\ \hat{y}^k(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i^k \hat{x}^k(t) \end{cases}$$

where C_i^k is the k^{th} row of the matrix C_i corresponding to the k^{th} sensor.

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- The different state estimates $\hat{x}^k(t), k = 1, \dots, p$, are then blended to build a representative state estimate $\hat{x}_b(t)$ according to

$$\hat{x}_b(t) = \sum_{k=1}^{p} h_k(r(t)) \hat{x}^k(t)$$

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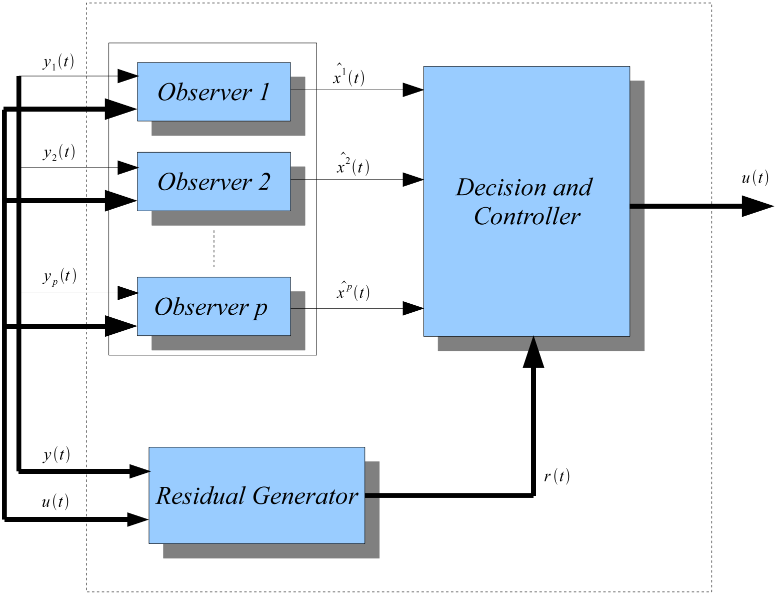
- The blending is ensured by smooth nonlinear functions $h_k(r(t))$, depending on the residual vector and satisfying the convex sum property

Fault tolerant control strategy



Fault tolerant control strategy

Computation of the blending functions $h_k(r(t))$

- If the k^{th} sensor is affected by a fault, the residual $r_k(t)$ is non zero then the function $h_k(r(t))$ must be close to zero in order to minimize the influence of $\hat{x}^k(t)$ affected by the k^{th} fault

- For example, the functions h_k, for $k = 1, \ldots, p$, can be defined as follows

$$\omega_k(r_k(t)) = \exp(-r_k^2(t)/\sigma_k)$$

$$h_k(r(t)) = \frac{\omega_k(r_k(t))}{\sum_{\ell=1}^{p} \omega_\ell(r_\ell(t))}$$

  the parameters σ_k are used to take into account the spreading of r_k around zero.

- With these definitions, a residual close to zero leads to a weight function tending to 1 when a residual significantly different from zero (in the sense of the variability σ_k) generates a weight tending to 0.

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Control law

The proposed control law is chosen now as a classical PDC control law, but based on the knowledge of this “fault free" state estimate $\hat{x}_b(t)$

$$u(t) = -\sum_{j=1}^{r} \mu_j(\xi(t)) K_j \hat{x}_b(t)$$

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Observer and controller gain design

k^{th} state estimation error $e^k(t) = x(t) - \hat{x}^k(t)$

$$\dot{e}^k(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) \left(A_i - L_i^k C_j^k \right) e^k(t)$$

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Closed-loop system

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{p} h_k(r(t)) \mu_i(\xi(t)) \mu_j(\xi(t)) \Big((A_i - B_i K_j) x(t) + B_i K_j e^k(t) \Big)$$

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Observer and controller gain design

Defining the augmented state vector

$$x_a^T(t) = [x^T(t)\ e^{1\,T}(t)\ \ldots\ e^{p\,T}(t)]$$

the following closed-loop system is obtained

$$\dot{x}_a(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))\left(\mathscr{A}_{ij} + \Delta\mathscr{A}_{ij}(t)\right)x_a(t)$$

where

$$\mathscr{A}_{ij} = \mathrm{diag}\left(A_i - B_iK_j,\ \ A_i - L_i^1C_j^1,\ \ \ldots,\ \ A_i - L_i^pC_j^p\right)$$

and

$$\Delta\mathscr{A}_{ij}(t) = \begin{bmatrix} 0 & h_1(r(t))B_iK_j & h_2(r(t))B_iK_j & \ldots & h_p(r(t))B_iK_j \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix}$$

Stability analysis

Stability analysis – beginning of the calculus only !

Stability analysis

- Consider the quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t) P x_a(t), \quad P = P^T = \text{diag}(X, P_1, ..., P_p) > 0$$

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- The time derivative of V is given by

$$\dot{V}(x_a(t)) = x_a^T(t) \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(t)) \mu_j(\xi(t)) (\mathscr{A}_{ij}^T P + P\mathscr{A}_{ij} + \Delta\mathscr{A}_{ij}^T(t) P + P\Delta\mathscr{A}_{ij}(t)) x_a(t)$$

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- The matrices $\Delta\mathscr{A}_{ij}(t)$ are time varying and can be reformulated as follows

$$\Delta\mathscr{A}_{ij}(t) = \underbrace{\begin{pmatrix} 0 & B_iK_j & \cdots & B_iK_j \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{pmatrix}}_{\mathscr{K}_{ij}} \underbrace{\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & h_1(r(t))I & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & h_p(r(t))I \end{pmatrix}}_{\Sigma(t)}$$

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- Knowing that the functions $h_k(r(t))$ satisfy the convex sum property, it follows that

$$\Sigma^T(t)\Sigma(t) \leq \mathrm{diag}(0, I_n, ..., I_n)$$

Stability analysis (Lyapunov)

- The derivative of the Lyapunov function is rewritten

$$\dot{V}(x_a(t)) = x_a^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))(\mathscr{A}_{ij}^T P + P\mathscr{A}_{ij} + \Sigma^T(t)\mathscr{K}_{ij}^T P + P\mathscr{K}_{ij}\Sigma(t))x_a(t)$$

Fault tolerant control strategy

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$$\dot{V}(x_a(t)) = x_a^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))(\mathscr{A}_{ij}^T P + P\mathscr{A}_{ij} + \Sigma^T(t)\mathscr{K}_{ij}^T P + P\mathscr{K}_{ij}\Sigma(t))x_a(t)$$

- It can be bounded as follows

$$\dot{V}(x_a(t)) \leq x_a^T(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))(\mathscr{A}_{ij}^T P + P\mathscr{A}_{ij} + \Sigma^T(t)\Lambda\Sigma(t) + P\mathscr{K}_{ij}\Lambda^{-1}\mathscr{K}_{ij}^T P)x_a(t)$$

where Λ is a block diagonal positive definite matrix.

Stability analysis (Lyapunov)

- The derivative of the Lyapunov function is rewritten

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The negativity of $\dot{V}(x_a(t))$ is satisfied if

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi(t))\mu_j(\xi(t))Y_{ij}(t) < 0$$

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- The paper also explain how it's possible to obtain relaxed stability conditions using Polya's theorem

System with $r = 2$ submodels

System matrices

$$A_1 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1 \\ 5 \\ 0.5 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Weighting functions

$$\mu_1(y(t)) = \frac{1 - \tanh(y_2(t))}{2}, \quad \mu_2(y(t)) = 1 - \mu_1(y(t))$$

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2 outputs $\Rightarrow$ 2 state observers

$$\omega_k(r_k(t)) = \exp(-r_k^2(t)/\sigma_k), \quad h_k(r(t)) = \frac{\omega_k(r_k(t))}{\sum_{\ell=1}^{2} \omega_\ell(r_\ell(t))}, \quad \sigma_1 = \sigma_2 = 0.01$$

First case : sensor additive time varying faults

Two additive oscillatory faults ; the first one is a low frequency fault affecting $y_2(t)$, while the second is a high frequency one affecting $y_1(t)$.

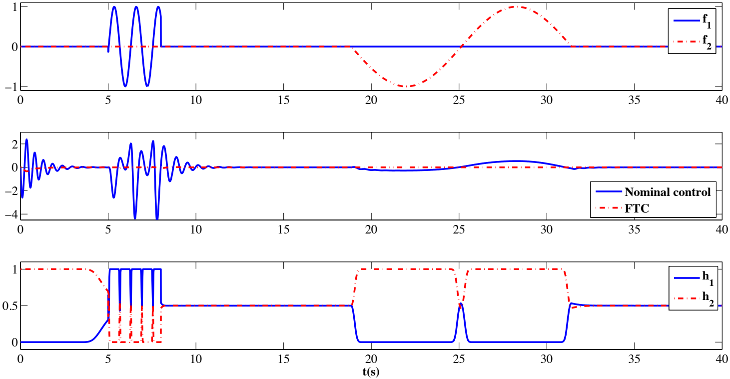

FIGURE: Faults, control signals and weighting functions

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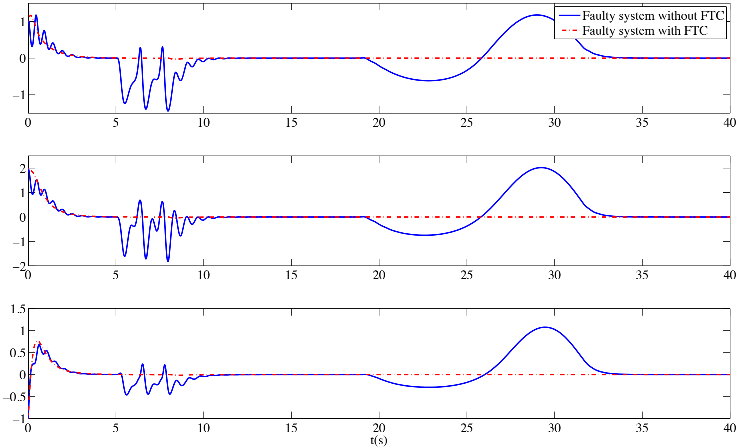
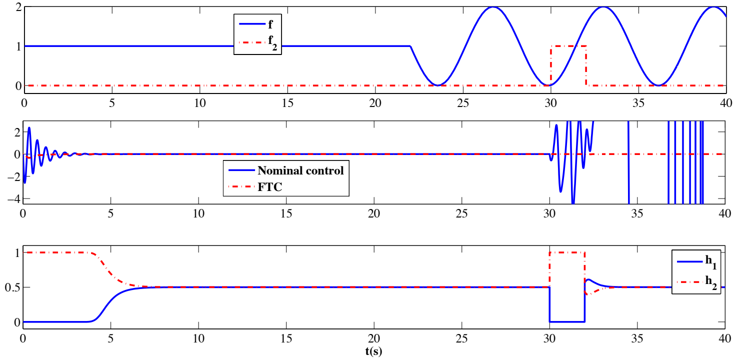


FIGURE: State comparison (system without FTC and with FTC)

$$y_1(t) = f(t)C^1x(t) + f_2(t)$$

$f(t)$: multiplicative fault $\qquad$ $f_2(t)$: additive fault



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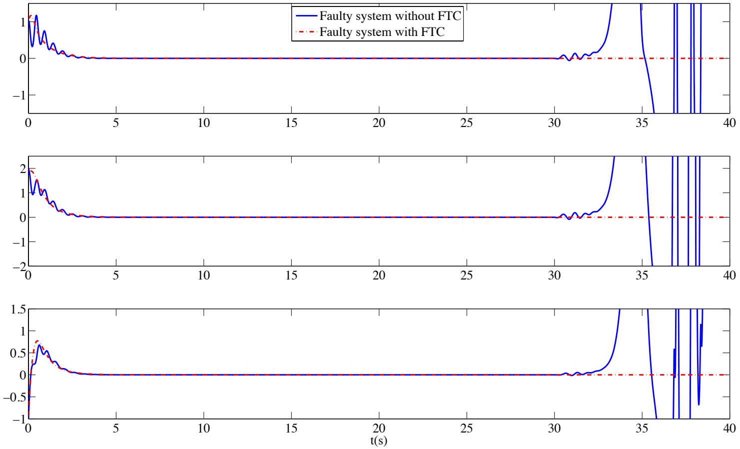
Second case : sensor parametric fault



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Conclusions

- The design of a sensor fault tolerant controller has been proposed for nonlinear systems described by a T-S Model
- The approach is based on a bank of state observers, a residual generator for diagnosis and a smooth selecting mechanism to choose an adequate state estimate to compensate the effects of the faults on the system.
- The stability of the closed-loop system is studied by Lyapunov theory and LMI constraints are provided to design the gain matrices of the different blocks of the proposed FTC scheme

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Get in touch

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