On the Unknown Input Observer Design : a Decoupling Class Approach

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Plan

Introduction

Study goal:

- State estimation for linear discrete-time systems
- Take into account unknown inputs (UIs).

Technical goals:

- Relax the structural and rank constraints on the system matrices
- Characterize a class of unknown inputs from which the estimation error is decoupled
- Ensure an $\mathscr{L}_2$ attenuation of the UI to the state estimation error.

Unknown Input Class for Exact Decoupling

UI Class for Exact Decoupling

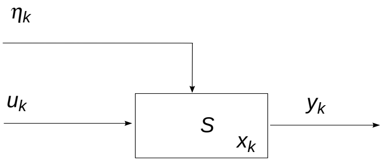


FIGURE: System subject to UI

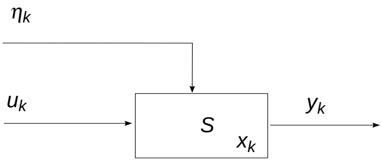


FIGURE: System subject to UI

System under UI

$$S : \begin{cases} x_{k+1} = Ax_k + Bu_k + D\eta_{k-1} \\ y_k = Cx_k + E\eta_{k-1} \end{cases}$$

$x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $\eta_k \in \mathbb{R}$ and $y_k \in \mathbb{R}^p$.

$A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $D \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{p\times n}$ and $E \in \mathbb{R}^{p\times 1}$.

UI Class for Exact Decoupling

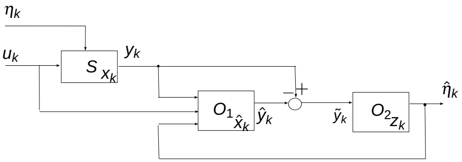


FIGURE: System state+ UI Observers

UI Class for Exact Decoupling

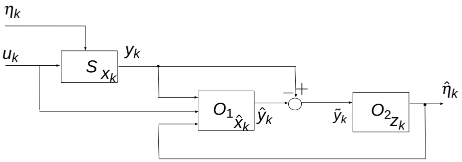


FIGURE: System state+ UI Observers

UI and State Observers

$$O_1 : \begin{cases} \hat{x}_{k+1} = A\hat{x}_k + Bu_k + D\hat{\eta}_{k-1} + K\tilde{y}_k \\ \hat{y}_k = C\hat{x}_k + E\hat{\eta}_{k-1} \\ \tilde{y}_k = y_k - \hat{y}_k \end{cases}$$

$$O_2 : \begin{cases} z_{k+1} = \Lambda z_k + \Gamma\tilde{y}_k \\ \hat{\eta}_{k+1} = \lambda\hat{\eta}_k + \gamma z_k \end{cases}$$

$z_k \in \mathbb{R}^q$, $K \in \mathbb{R}^{n\times p}$

$\Gamma \in \mathbb{R}^{q\times p}$, $\gamma \in \mathbb{R}^{1\times q}$, $\Lambda \in \mathbb{R}^{q\times q}$ and $\lambda \in \mathbb{R}$.

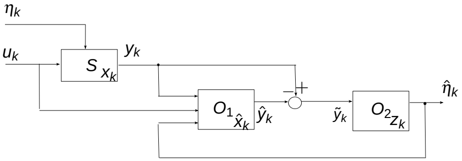


FIGURE: System state+ UI Observers

State and UI reconstruction errors

$$\begin{cases} \tilde{\eta}_k = \eta_k - \hat{\eta}_k \\ \tilde{x}_k = x_k - \hat{x}_k \end{cases}$$

$$\begin{cases} \tilde{\eta}_k = \left[1-(q-\lambda)^{-1}\gamma\overline{Z}^{-1}\overline{\Lambda}\right]\eta_k \\ \tilde{x}_k = (qI_n-(A-KC))^{-1}(D-KE)q^{-1}\left[1-(q-\lambda)^{-1}\gamma\overline{Z}^{-1}\overline{\Lambda}\right]\eta_k \end{cases}$$

with q being the forward shift operator.

$$\begin{aligned} \overline{\Lambda} &= \Gamma C(qI_n-(A-KC))^{-1}(D-KE)+\Gamma E \\ \overline{Z} &= q(qI_q-\Lambda)+\overline{\Lambda}(q-\lambda)^{-1}\gamma \end{aligned}$$

UI Class for Exact Decoupling

Exact decoupling conditions

In order to:

- decouple the state estimation from the UI
- have an exact estimation of the UI.

The following condition has to be verified:

$$\left[1-(q-\lambda)^{-1}\gamma\overline{Z}^{-1}\overline{\Lambda}\right]\eta_k = 0 \tag{1}$$

$$\begin{cases} \overline{\Lambda} & = & \Gamma C(qI_n-(A-KC))^{-1}(D-KE)+\Gamma E \\ \overline{Z} & = & q(qI_q-\Lambda)+\overline{\Lambda}(q-\lambda)^{-1}\gamma \end{cases}$$

The structure of the UI ensuring an exact decoupling of the estimation error to the UI:

$$\eta_k = \sum_i A_i \lambda_i^k, \quad \forall A_i, i = 1:n$$

with this class of UI, we obtain $\| \tilde{x}_k \|_{k\to\infty} \quad \to \quad 0$

Partial decoupling and $\mathscr{L}_2$ attenuation of the UI effect on the system

Partial Decoupling Observer

- Decoupling Strategy:

  for any UI, it is always possible to write:

$$\eta_k = \eta_k^d + \eta_k^a$$

  - Exact decoupling of $\tilde{x}_k$ from η_k^d
  - Attenuation: $\mathscr{L}_2$ approach $\| \tilde{x}_k \|_2 < \mu \| \eta_k^a \|_2$

- State observer gain K design:

find K, such that
$$\begin{cases} \| \tilde{x}_k \|_{k\to\infty} \to 0 & \text{If} \quad \eta_k^a = 0 \\ \| \tilde{x}_k \|_2 < \mu \| \eta_k^a \|_2 & \text{If} \quad \eta_k^a \neq 0 \end{cases}$$

- Reconstruction error

$$\begin{cases} \tilde{x}_{k+1} = (A - KC)\tilde{x}_k + (D - KE)\tilde{\eta}_{k-1} \\ \tilde{\eta}_k = \eta_k - \lambda \eta_{k-1} - \gamma z_{k-1} + \lambda \tilde{\eta}_{k-1} \\ z_{k+1} = \Gamma C \tilde{x}_k + \Gamma E \tilde{\eta}_{k-1} + \Lambda z_k \end{cases}$$

K is the state observer gain and $\Gamma, \gamma, \Lambda, \lambda$ the UI observer parameters.

Partial Decoupling Observer

- The augmented state: system state, UI and observer state:

$$\tilde{x}_k^a = \begin{bmatrix} \tilde{x}_k^T & \tilde{\eta}_{k-1}^T & z_k^T & z_{k-1}^T \end{bmatrix}^T \quad \eta_k^a = \begin{bmatrix} \eta_k^T & \eta_{k-1}^T \end{bmatrix}^T$$

- The corresponding matricial form is given by:

$$\tilde{x}_{k+1}^a = A_1 \tilde{x}_k^a + B_1 \eta_k^a \quad (2)$$

$$\tilde{x}_k = C_1 \tilde{x}_k^a \quad (3)$$

$$A_1 = \begin{pmatrix} A-KC & D-KE & 0 & 0 \\ 0 & \lambda & 0 & -\gamma \\ \Gamma C & \Gamma e & \Lambda & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & 0 \\ 1 & -\lambda \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C_1 = \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix}^T$$

- Objectives:
  1. system (2) stable
  2. bounded state estimation error: $\| \tilde{x}_k \|_2 < \mu \| \eta_k^a \|_2$

Partial Decoupling Observer : LMI linearization

These objectives are achieved if there exists $P = P^T > 0$, G, K and $\mu > 0$ such that:

- Stability and attenuation condition:

$$\begin{bmatrix} -P & A_1^T P B_1 & C_1^T & A_1^T G^T \\ * & B_1^T P B_1 - \mu^2 I & 0 & 0 \\ * & * & -\mu^2 I & 0 \\ * & * & * & -G - G^T + P \end{bmatrix} < 0$$

  where A_1 depends on K.
- LMI linearization:

$$LMI1(P,K) = \begin{bmatrix} -P & \overline{A}_1^T P B_1 & C_1^T & \overline{A}_1^T G^T & \overline{B}_1^T F^T & 0 \\ * & B_1^T P B_1 - \overline{\mu} I & 0 & 0 & 0 & B_1^T P R \\ * & * & -\overline{\mu} I & 0 & 0 & 0 \\ * & * & * & -G - G^T + P & 0 & GR \\ * & * & * & * & -\Sigma^T & 0 \\ * & * & * & * & * & -\Sigma \end{bmatrix} \lhd 0$$

  with $F = \Sigma K$. The LMI must be solved in P, G, F and the gain K is obtained by $K = \Sigma^{-1} F$.

Partial Decoupling Observer: LMI linearization

Pole assignment

- Attenuation drawback
  - Slow dynamics of the state estimation error
- Proposed solution
  - Pole assignment of the closed loop system in a specified region (disk $(q,0)$ with radius α)
  - Additive constraint condition

$$LMI2(Q,K) = \begin{bmatrix} -\alpha Q & -qQ+QA-GC \\ (-qQ+QA-GC)^T & -\alpha Q \end{bmatrix} < 0$$

This supplementary LMI must be solved regarding to Q and G with the gain $K = Q^{-1}G$, equals to the gain given by $LMI1(P,K)$.

Partial Decoupling Observer : LMI linearization

Gain adjustment

- Objective: solve simultaneously the two previous conditions ($LMI1(P,K)$ and $LMI2(Q,K)$)
- Problem: interconnected conditions (same gain K to find)
- Proposed method
  - An ajustment technique allowing to set some variables and calculate others in an iterative way
  - Advantage: simplicity
  - Inconvenient: no optimality or convergence guarantee is given

Simulations

Simulations

System

$$\begin{cases} x_{k+1} = \begin{pmatrix} 0.6 & -0.2 & -0.1 & 0.1 \\ -0.1 & 0.7 & -0.1 & 0.1 \\ 0.4 & 0 & 0.9 & -0.1 \\ 0 & 0.2 & 0 & 0.8 \end{pmatrix} x_k + \begin{pmatrix} -0.3 & -0.4 \\ 0.5 & -0.4 \\ -0.1 & 0.6 \\ -0.2 & 0.7 \end{pmatrix} u_k + \begin{pmatrix} 0.2 \\ -0.3 \\ 0.1 \\ 0.1 \end{pmatrix} \eta_{k-1} \\ y_k = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} x_k - 4.5\eta_{k-1} \end{cases}$$

Unknown input

- Observer parameters: $\Lambda = 1, \lambda = 0.8$ et $\Gamma = 0.2, \gamma = -0.4$
- Unknown input

$$\eta_k = \eta_k^d + \eta_k^a$$

- Exact decoupling: $\eta_k^d = \sum A_i \lambda_i^k$

$$\lambda_1 = 1,\ \lambda_2 = 0.8,\ \lambda_3 = 0.4\,;$$
$$\lambda_4 = 0.3,\ \lambda_{5,6} = 0.4 \pm 0.2i$$

- Attenuation: $\eta_k^a = A_7\lambda_7^k + A_8\lambda_8^k$

$$\lambda_7 = 0.1, \lambda_8 = 0.1$$

Exact decoupling: $\eta_k = \eta_k^d$

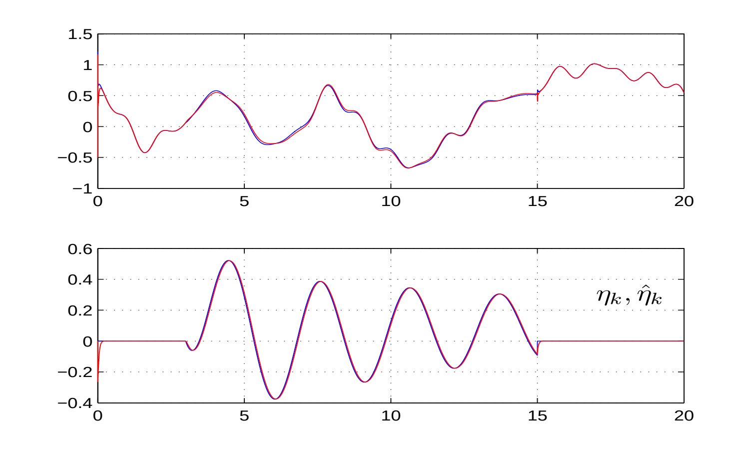
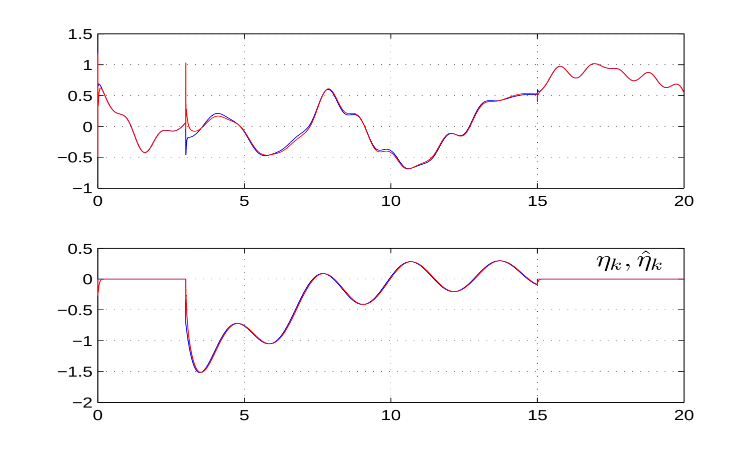


FIGURE: State 1 and its estimate (top)- UI and its estimate (bottom)

Decoupling + Attenuation: $\eta_k = \eta_k^d + \eta_k^a$



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Conclusions

- Efficient strategy:
  - for the exact decoupling of the UI
  - for the $\mathscr{L}_2$ attenuation
- Poles assignment strategy:
  - improve performances

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Perspectives

- Application for sensor fault diagnosis
  - bank of observers
- Unknown Inputs Observers for non linear systems
  - with Multi-model representation

Thanks for your attention