# Modeling and fleet effect for the diagnosis of a system behavior

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## Definition and goal

- Fleet of machines: a collection of machines a priori identical
- Estimating a generic model for a fleet of identical machines
- Deduce a generic strategy for the diagnosis of a fleet of machines

## Motivations

- Reducing the cost of estimating the model of each machine
- Facility to construct the model of a new machine
- Facility to replace a machine by another one
- Reducing the cost of system maintenance

# Plan

# Introduction

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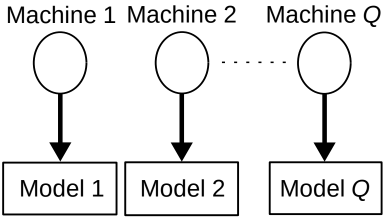
Modeling the behavior of $Q$ machines is done:

**Classical approach**



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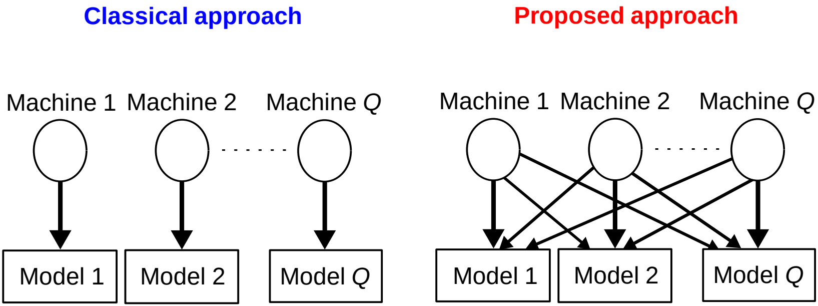
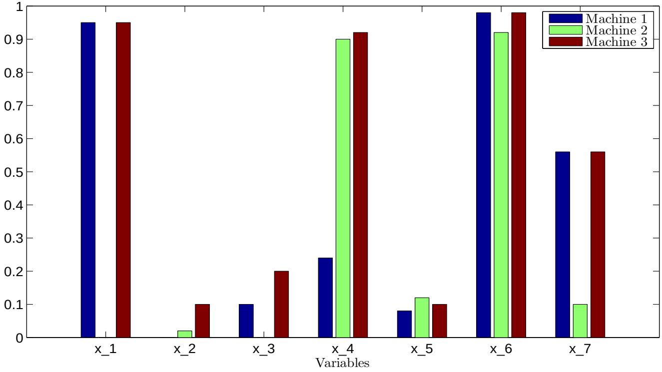


- The problem consists in determining if a generic model representing the normal behavior of each machine of the fleet can be established.

- A generic model is composed of two parts:
  - a common part made up of the variables of the machine itself,
  - a distinct part related to the environmental variables.

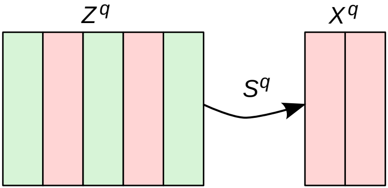


# Identification of models with their common parts

# Preliminary step: identify the structure of the models

For the $q^{th}$ machine, denote:

- $y^q$ variable to explain,
- $Z^q$ matrix of variables possibly explaining $y^q$,
- $X^q$ matrix of variables selected for explaining $y^q$,
- $\hat{\theta}^q$ estimated vector of the model parameters,
- $\hat{y}^q$ estimate of $y^q$.




$S^q$ is a selection matrix.

**Example :**

$$S^q = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$$

enables to select variables 2 and 4 from a set of 5 variables

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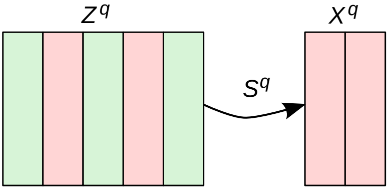
**Model :**

$$X^q = Z^q S^q$$

$$\hat{y}^q = X^q \hat{\theta}^q$$

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# Identification of models with their common parts

The method consists in minimizing:

$$\phi = \sum_{q=1}^{Q} \left(y^q - X^q \theta^q\right)^T \left(y^q - X^q \theta^q\right) + \gamma \sum_{q=1}^{Q-1} \sum_{\ell=q+1}^{Q} \left(\theta^q - \theta^\ell\right)^T W^{q,\ell} \left(\theta^q - \theta^\ell\right)$$

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$W^{q,\ell}$ is a positive diagonal matrix composed of the weights $w_i^{q,\ell}$ $(i = 1, \cdots, m)$ where :

$$\sum_{i=1}^{m} w_i^{q,\ell} - 1 = 0$$

**Unknown:** $\hat{\theta}^q$, $W^{q,\ell}$ and $\gamma$ for all $q$ and $\ell$.

The Lagrange function $\mathscr{L}$ of the problem is:

$$\begin{aligned}\mathscr{L} \;&=\; \sum_{q=1}^{Q}\left(y^q - X^q\theta^q\right)^T\left(y^q - X^q\theta^q\right) + \gamma\sum_{q=1}^{Q-1}\sum_{\ell=q+1}^{Q}\left(\theta^q-\theta^\ell\right)^T W^{q,\ell}\left(\theta^q-\theta^\ell\right)\\ &+\; \sum_{q=1}^{Q-1}\sum_{\ell=q+1}^{Q}\lambda^{q,\ell}\left(\sum_{i=1}^{m} w_i^{q,\ell} - 1\right)\end{aligned}$$

where $\lambda^{q,\ell}$ are the unknown Lagrange multipliers.

Supposing $\gamma$ is known, the first order stationarity conditions lead to expressions enabling to estimate the coefficients of the models and the weights.

$$\underbrace{\begin{pmatrix} U^1 & U^{1,2} & \dots & \dots & U^{1,Q} \\ U^{1,2} & U^2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & U^{Q-1,Q} \\ U^{1,Q} & \dots & \dots & U^{Q-1,Q} & U^Q \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} \theta^1 \\ \theta^2 \\ \vdots \\ \vdots \\ \theta^Q \end{pmatrix}}_{\Theta} = \underbrace{\begin{pmatrix} X^{1^T} y^1 \\ X^{2^T} y^2 \\ \vdots \\ \vdots \\ X^{Q^T} y^Q \end{pmatrix}}_{V}$$

- $U^q = X^{q^T} X^q + \gamma \sum_{\substack{\ell=1 \\ \ell \neq q}}^{Q} W^{q,\ell}$ $(q = 1, \dots, Q)$,
- $W^{\ell,q} = W^{q,\ell}$ $(\forall\, q = 1, \dots, Q-1$ and $\ell = q+1, \dots, Q)$,
- $U^{q,\ell} = -\gamma W^{q,\ell}$.

$$\hat{\Theta} = U^{-1} V \quad \text{if} \quad U^{-1} \exists$$

The estimated weights are :

$$\hat{w}_i^{q,\ell} = \frac{1}{1 + \sum\limits_{\substack{j=1 \\ j \neq i}}^{m} \frac{\left(\hat{\theta}_i^q - \hat{\theta}_i^\ell\right)^2}{\left(\hat{\theta}_j^q - \hat{\theta}_j^\ell\right)^2}}$$

with $\hat{\theta}_j^q \neq \hat{\theta}_j^\ell, \forall j \neq i$ whenever $\hat{\theta}_i^q = \hat{\theta}_i^\ell$.

For all $q$ and $\ell$:

- estimates $\hat{\theta}^q$ of the coefficients should be known to calculate $\hat{W}^{q,\ell}$,
- estimates $\hat{W}^{q,\ell}$ of the weights should be known to calculate $\hat{\theta}^q$.

# Resolution algorithm

For a given $\gamma$:

1. Initialization: Set $l = 1$. $\theta^{(l),q}$, $q = 1,\ldots,Q$, are estimated using:

$$\hat{\theta}^{(l),q} = \left(X^{q^T} X^q\right)^{-1} X^{q^T} y^q.$$

2. The weights are estimated using:

$$\hat{w}_i^{(l),q,\ell} = \frac{1}{1 + \sum\limits_{\substack{j=1 \\ j\neq i}}^{m} \frac{\left(\hat{\theta}_i^{(l),q} - \hat{\theta}_i^{(l),\ell}\right)^2}{\left(\hat{\theta}_j^{(l),q} - \hat{\theta}_j^{(l),\ell}\right)^2}}$$

for all $i = 1,\ldots,m, q = 1,\ldots,Q-1$ and $\ell = q+1,\ldots,Q$.

3. New estimates of the coefficients are obtained:

$$\hat{\Theta}^{(l+1)} = U^{(l)^{-1}} V.$$

4. Set $l = l+1$. Repeat steps 2 and 4 until the convergence of the solution.

$$\triangle \hat{w}_i^{q,\ell} = \frac{1}{1 + \sum\limits_{\substack{j=1 \\ j \neq i}}^{m} \frac{\left(\hat{\theta}_i^q - \hat{\theta}_i^\ell\right)^2}{\left(\hat{\theta}_j^q - \hat{\theta}_j^\ell\right)^2}} \Longrightarrow \hat{w}_i^{q,\ell} \approx 1 \text{ whenever } \left|\hat{\theta}_i^q - \hat{\theta}_i^\ell\right| \underset{j \neq i}{\ll} \left|\hat{\theta}_j^q - \hat{\theta}_j^\ell\right|:$$

$\hat{\theta}_i^q$ and $\hat{\theta}_i^\ell$ will be considered identical even if they are not so close.

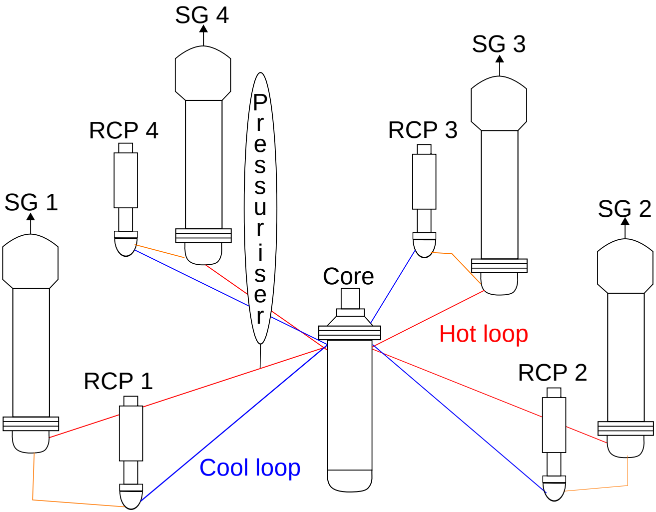
$$\hat{w}_i^{q,\ell} = \frac{1}{2}\Big(\tanh\left(\alpha(\hat{\theta}_i^q - \hat{\theta}_i^\ell) + \delta_i\right) - \tanh\left(\alpha(\hat{\theta}_i^q - \hat{\theta}_i^\ell) - \delta_i\right)\Big)$$

$\alpha$ is the speed variation of the hyperbolic function and $\delta_i$ is the threshold below which $\hat{\theta}_i^q$ and $\hat{\theta}_i^\ell$ are considered identical.

- $\hat{w}_i^{q,\ell}$ is close to 1 if $\hat{\theta}_i^q$ and $\hat{\theta}_i^\ell$ have relatively similar values,
- $\hat{w}_i^{q,\ell}$ is nearly null if $\hat{\theta}_i^q$ and $\hat{\theta}_i^\ell$ have different values.

# Application on reactor coolant pumps

# Description of the system



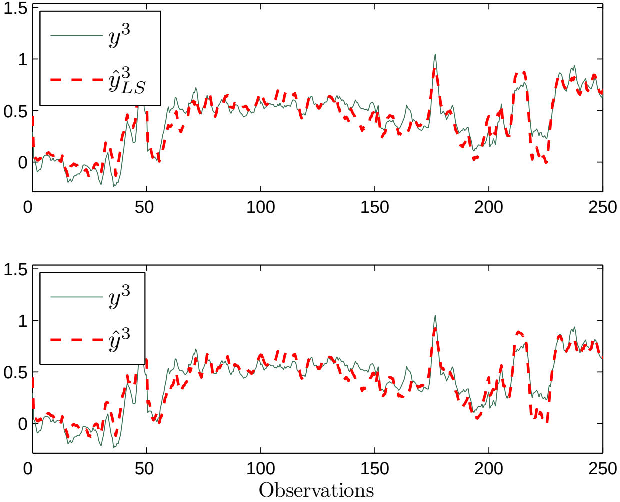

RCP: Reactor Coolant Pump SG: Steam Generator

$y^q = LF_1^q$, $q = 1,\ldots,4$. The least squares method applied on the data set of each power plant gives:

| q | cte | $HT^q$ | $IT^q$ | $CT^q$ | $LT_1^q$ | $LF_4^q$ |
|---|---|---|---|---|---|---|
| 1 | -211.86 | 1.49 | 4.18 | -1.43 | 1.25 | 0.87 |
| 2 | -2823.50 | 2.38 | 2.59 | 7.03 | -3.15 | 0.99 |
| 3 | -1149.24 | 0.87 | 9.29 | 3.37 | -9.60 | 0.95 |
| 4 | 17203.61 | -12.02 | 5.73 | -45.86 | -3.07 | 0.90 |

The coefficients of the models taking into account the fleet effect are:

| q | cte | $HT^q$ | $IT^q$ | $CT^q$ | $LT_1^q$ | $LF_4^q$ |
|---|---|---|---|---|---|---|
| 1 | -283.25 | 1.51 | 4.17 | -1.22 | 1.05 | 0.90 |
| 2 | -2272.87 | 1.93 | 2.75 | 5.63 | -3.10 | 0.97 |
| 3 | -1535.86 | 1.18 | 9.24 | 4.36 | -9.47 | 0.94 |
| 4 | 16983.65 | -11.87 | 5.58 | -45.29 | -2.93 | 0.92 |




$\hat{y}^3$ estimates $y^3$ as good as $\hat{y}^3_{LS}$. The same phenomenon is observed for the models estimating $y^1$, $y^2$ and $y^4$.

Non-zero weights reflects the proximity between coefficients.

| $q,\ell$ | cte | $HT^q$ | $IT^q$ | $CT^q$ | $LT_1^q$ | $LF_4^q$ |
|---|---|---|---|---|---|---|
| 1,2 | 0.00 | 0.94 | 0.19 | 0.00 | 0.00 | 0.82 |
| 1,3 | 0.00 | 0.96 | 0.00 | 0.03 | 0.00 | 0.92 |
| 1,4 | 0.00 | 0.00 | 0.20 | 0.00 | 0.00 | 0.97 |
| 2,3 | 0.00 | 0.79 | 0.00 | 0.94 | 0.00 | 0.97 |
| 2,4 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 | 0.91 |
| 3,4 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.96 |

- The coefficient of $LF_4^q$ is common for the 4 power plants,
- the coefficient of $HT^q$ is common for plants number 1, 2 and 3,
- the coefficient of $CT^q$ is common to plants number 2 and 3,
- the coefficient of $LT_1^q$ is common to plants number 2 and 4.

# Conclusions and perspectives

## Conclusions

The method allows simultaneously to:

- identify the common part to each couple of the models of the machines of a fleet,
- identify the coefficients of these models considering the shared common parts.

## Perspectives

- study the use of the proposed approach in constructing the model of a new machine,
- deduce a generic strategy for the diagnosis of a fleet of machines.

**Thank you for your attention**