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# State estimation for wastewater treatment plant with slow and fast dynamics using multiple models

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# Purposes and problematics

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### **Objectives**

- 1. Propose a state estimation method for two-time scale systems using Multiple Models (MM)
- 2. Apply it to an activated sludge model reactor

### Interests

1. Ability to use the convexity properties of the MM in order to design observers and control laws for system diagnosis purpose

### Motivation

- 1. Difficulty to deal with the **modeling complexity** of nonlinear systems
- 2. Difficulty to model a process under the singularly perturbed systems
- 3. Existence of multiple **time scale dynamics** : identification and separation

# Outline

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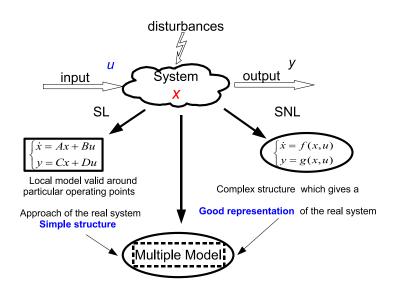
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What is the Multiple Model?

# Dynamical system described by a Multiple Model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t), u(t)) \left[A_i x(t) + B_i u(t)\right] \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t), u(t)) \left[C_i x(t) + D_i u(t)\right] \end{cases}$$

$$\sum\limits_{i=1}^r \mu_i(x,u) = 1$$
 and  $\mu_i(x,u) \geq 0$ 

Interest : this form is particularly attractive for

- stability
- stabilization
- observability
- state estimation
- diagnosis

studies

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# State estimation method

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## Identification of slow and fast dynamics

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### Homotopy method

Linearization :  $\dot{x}(t) = f(x(t), u(t)) \implies \dot{x}(t) = A_0 x(t) + B_0 u(t)$ 

$$A_0 = \frac{\partial f(x,u)}{\partial x} \Big|_{(x_0,u_0)}, \qquad B_0 = \frac{\partial f(x,u)}{\partial u} \Big|_{(x_0,u_0)}$$

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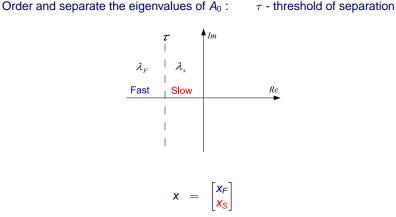
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# Singularly perturbed systems

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### Standard form

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# $\begin{aligned} \epsilon \dot{x}_{F}(t) &= f_{F}(x_{S}(t), x_{F}(t), u(t), \epsilon) \end{aligned} \tag{1a} \\ \dot{x}_{S}(t) &= f_{S}(x_{S}(t), x_{F}(t), u(t), \epsilon) \end{aligned} \tag{1b}$

where  $\epsilon$  - singular perturbed parameter

Reduced form :  $\epsilon \longrightarrow 0$ 

$$0 = f_F(x_S(t), x_F(t), u(t), 0)$$
(2a)  
$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), 0)$$
(2b)

## Difficulties :

- transform a NL system into the singularly perturbed form
- obtain  $\epsilon$

### If possible (for particular cases of SNL), then :

▶ resolution of the algebraic system (2a) extract  $x_F$  and replace it in (2b)

#### Singularly perturbed systems

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# Singularly perturbed systems

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Singularly perturbed systems

### Two-time scales

$$\begin{bmatrix} \dot{x}_{F}(t) \\ \dot{x}_{S}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\epsilon} f_{F}(x_{S}(t), x_{F}(t), u(t), \epsilon) \\ f_{S}(x_{S}(t), x_{F}(t), u(t), \epsilon) \end{bmatrix}$$
$$y(t) = C \begin{bmatrix} x_{F}(t) \\ x_{S}(t) \end{bmatrix}$$

equivalent transformation ↕ sector nonlinearity approach Multiple model

$$\begin{bmatrix} \dot{x}_{F}(t) \\ \dot{x}_{S}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_{i}(x_{S}, x_{F}, u) \left\{ \begin{bmatrix} A_{FF}^{i} & A_{FS}^{i} \\ A_{SF}^{i} & A_{SS}^{i} \end{bmatrix} \cdot \begin{bmatrix} x_{F}(t) \\ x_{S}(t) \end{bmatrix} + \begin{bmatrix} B_{F}^{i} \\ B_{S}^{i} \end{bmatrix} u \right\}$$
$$y(t) = C \begin{bmatrix} x_{F}(t) \\ x_{S}(t) \end{bmatrix}$$

# Singularly perturbed systems

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dynamics Estimation results Consider  $x_F$  as unknown input :  $d(t) = x_F(t)$  $x(t) = \begin{bmatrix} d(t) \\ x_S(t) \end{bmatrix}$ 

Design the matrices :

$$\begin{split} \bar{A}_{i} &= \begin{bmatrix} A_{FF}^{i} & A_{FS}^{i} \\ 0 & A_{SS}^{i} \end{bmatrix} \quad E_{i} &= \begin{bmatrix} 0 \\ A_{SF}^{i} \end{bmatrix} \quad \bar{C}_{S} &= \begin{bmatrix} 0 & C_{S} \end{bmatrix} \\ \begin{cases} \dot{x}(t) &= \sum_{i=1}^{r} \mu_{i}(\mathbf{x}, u) \cdot \begin{bmatrix} \bar{A}_{i} x(t) + B_{i} u(t) + E_{i} d(t) \end{bmatrix} \\ y(t) &= \bar{C}_{S} x(t) + C_{F} d(t) \end{split}$$

- Decoupled time scales
- The estimation of x<sub>S</sub> is made independently of x<sub>F</sub>
- Classic structure of MM affected by unknown inputs
- Unmeasurable decision variables

(3)

### State estimation method

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### MM with measurable decision variables :

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{\mathbf{x}}, u) \cdot \left[\bar{\mathbf{A}}_i \mathbf{x}(t) + \mathbf{B}_i u(t) + \mathbf{E}_i \mathbf{d}(t) + \omega(t)\right] \\ \mathbf{y}(t) = \bar{\mathbf{C}}_{\mathbf{S}} \mathbf{x}(t) + \mathbf{C}_{\mathbf{F}} \mathbf{d}(t) \end{cases}$$

Unknown input observer :

<

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t), u(t)) [N_i z(t) + G_i u(t) + L_i y(t)] \\ \hat{x}(t) = z(t) - H y(t) \end{cases}$$
(4)

Dynamic of the state estimation error :  $\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$ Under matrix conditions  $\dot{e}(t)$  reduces to :

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{r} \mu_i(\hat{\boldsymbol{x}}(t)) \left( N_i \boldsymbol{e}(t) + \boldsymbol{P}\omega(t) \right)$$
(5)

### *L*<sub>2</sub> approach

### Unknown input observer

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**Theorem :**  $e(t) \rightarrow 0$  if  $\exists X, M_i$  and S and a positive scalar  $\lambda$  s.t. the following conditions are respected  $\forall i = 1, ..., r$  :

$$\begin{array}{c} \bar{A}_{i}^{T}(X+S\bar{C}_{S})^{T}+(X+S\bar{C}_{S})\bar{A}_{i}-\bar{C}_{S}^{T}M_{i}^{T}-M_{i}\bar{C}_{S}+I \qquad X+S\bar{C}_{S} \\ (X+S\bar{C}_{S})^{T} \qquad -\lambda I \end{array} \right] < 0$$

 $\begin{array}{rcl} & SC_F & = & 0 \\ (X+S\bar{C}_S)E_i & = & M_iC_F \end{array}$ 

The observer matrices

$$H = X^{-1}S$$

$$N_i = (I + H\bar{C}_S)\bar{A}_i - X^{-1}M_i\bar{C}_S$$

$$L_i = X^{-1}M_i - N_iH$$

$$G_i = (I + H\bar{C}_S)B_i$$
(6)

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# Wastewater treatment

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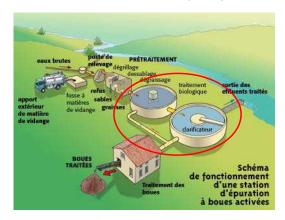
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### The diagram of the wastewater treatment plant process



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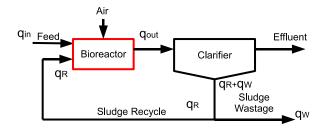
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## The diagram of the set biological reactor + clarifier



1. The operating mode : constant volume

 $q_{out} = q_{in} + q_R$ 

2. Model : a part of ASM1  $\longrightarrow$  carbonated pollution

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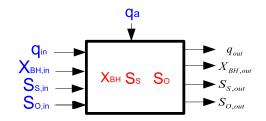
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# The reduced ASM1 model : biological reactor



Simplification hypothesis :

 $\begin{cases} X_{BH,out}(t) = X_{BH}(t) \\ S_{S,out}(t) = S_{S}(t) \\ S_{O,out}(t) = S_{O}(t) \\ S_{O,in}(t) = 0 \end{cases}$ 

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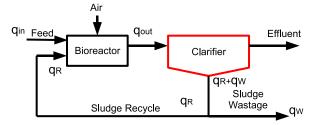
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## The diagram of the set biological reactor + clarifier



1. Clarifier

$$\begin{aligned} (q_{\textit{in}}+q_{\textit{R}})X_{\textit{BH}} &= (q_{\textit{W}}+q_{\textit{R}})X_{\textit{BH},\textit{R}} \\ S_{\textit{S},\textit{R}} &= S_{\textit{S}} \end{aligned}$$

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### The reduced ASM1 model

$$\begin{split} \dot{S}_{S} &= \frac{q_{in}}{V} \left( S_{S,in} - S_{S} \right) + (1 - f) b_{H} X_{BH} - \frac{\mu_{H}}{Y_{H}} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BH} \\ \dot{S}_{O} &= -\frac{q_{in}}{V} S_{O} + K q_{a} \left( S_{O,sat} - S_{O} \right) - \frac{1 - Y_{H}}{Y_{H}} \mu_{H} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BH} \\ \dot{X}_{BH} &= \frac{q_{in}}{V} X_{BH,in} - \frac{q_{W}}{V} \frac{q_{in} + q_{R}}{q_{W} + q_{R}} X_{BH} + \mu_{H} \frac{S_{S}}{K_{S} + S_{S}} \frac{S_{O}}{K_{OH} + S_{O}} X_{BH} - b_{H} X_{BH} \\ x &= \begin{bmatrix} S_{S} \\ S_{O} \\ X_{BH} \end{bmatrix} \qquad u = \begin{bmatrix} S_{S,in} \\ q_{a} \\ X_{BH,in} \end{bmatrix} \end{split}$$

Constants parameters :  $\theta = (\mu_H, b_H, f, Y_H, S_{O,sat}, K_S, K_{OH}, K)$ 

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### Slow and fast dynamics



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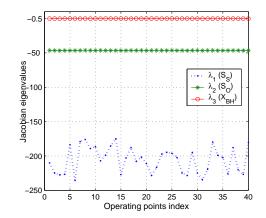
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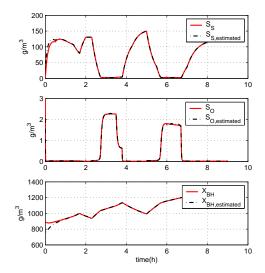
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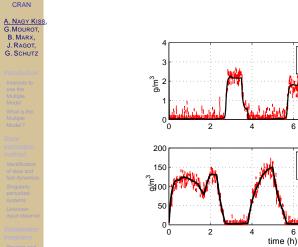
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### Output estimation results



Estimation results

8

У<sub>1</sub>

6

6

y ₁ estimated

8

 $y_{2}^{y}$  estimated

10

10

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# **Conclusions and Future prospects**

# Conclusions and Future prospects

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### Conclusions

- 1. Identification of slow and fast dynamics
- 2. Usage of MM with two time scales
- 3. State estimation using an unknown input observer and MM
- 4. Application to a part of the ASM1 model of a wastewater treatment plant

### **Future prospects**

### Using the Multiple Model form

- 1. System Diagnosis :
  - detect
  - isolate

faults

- identify
- 2. Apply to wastewater treatment plant model

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# Thank you for your attention