State estimation for wastewater treatment plant with slow and fast dynamics using multiple models

A. NAGY KISS, G.MOUROT, B. MARX, J. RAGOT, G. SCHUTZ

Purposes and problematics

CRAN

A. NAGY KISS, G.MOUROT, B. MARX, J. RAGOT, G. SCHUTZ

Objectives

1. Propose a state estimation method for two-time scale systems using Multiple Models (MM)
2. Apply it to an activated sludge model reactor

Interests

1. Ability to use the convexity properties of the MM in order to design observers and control laws for system diagnosis purpose

Motivation

1. Difficulty to deal with the **modeling complexity** of nonlinear systems
2. Difficulty to model a process under the **singularly perturbed systems**
3. Existence of multiple **time scale dynamics** : identification and separation

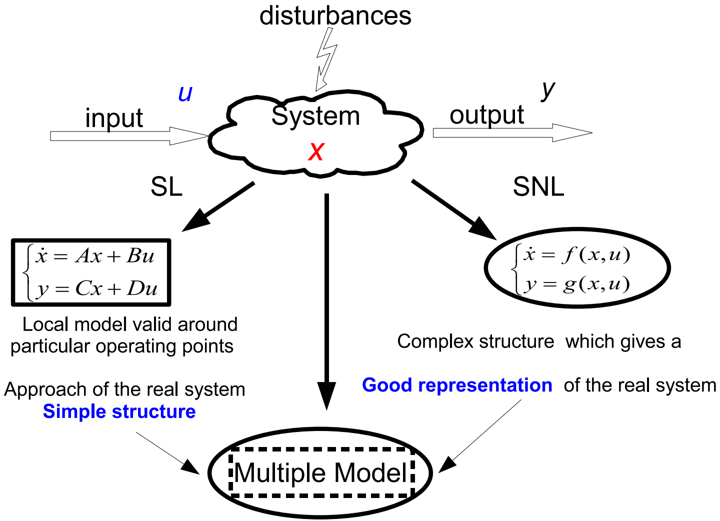
Outline

Introduction

The interest to use the Multiple Models

CRAN

A. Nagy Kiss, G. Mourot, B. Marx, J. Ragot, G. Schutz



Dynamical system described by a Multiple Model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t),u(t)) \, [A_i \, x(t) + B_i \, u(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i(x(t),u(t)) \, [C_i \, x(t) + D_i \, u(t)] \end{cases}$$

$$\sum_{i=1}^{r} \mu_i(x,u) = 1 \quad \text{and} \quad \mu_i(x,u) \geq 0$$

Interest : this form is particularly attractive for

- stability
- stabilization
- observability
- state estimation
- diagnosis

studies

CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

State estimation method

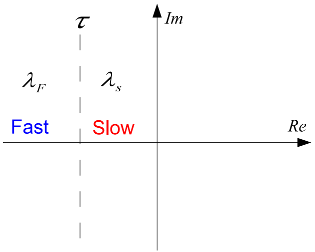
CRAN
A. NAGY KISS, G.MOUROT, B. MARX, J. RAGOT, G. SCHUTZ

Homotopy method

Linearization : $\dot{x}(t) = f(x(t), u(t)) \implies \dot{x}(t) = A_0 x(t) + B_0 u(t)$

$$A_0 = \frac{\partial f(x,u)}{\partial x}\Big|_{(x_0,u_0)}, \qquad B_0 = \frac{\partial f(x,u)}{\partial u}\Big|_{(x_0,u_0)}$$

Order and separate the eigenvalues of A_0 : τ - threshold of separation



$$x = \begin{bmatrix} x_F \\ x_S \end{bmatrix}$$

CRAN
A. NAGY KISS, G.MOUROT, B. MARX, J. RAGOT, G. SCHUTZ

Standard form

$$\epsilon \dot{x}_F(t) = f_F(x_S(t), x_F(t), u(t), \epsilon) \quad (1a)$$
$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), \epsilon) \quad (1b)$$

where ϵ - singular perturbed parameter

Reduced form : $\epsilon \longrightarrow 0$

$$0 = f_F(x_S(t), x_F(t), u(t), 0) \quad (2a)$$
$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), 0) \quad (2b)$$

Difficulties :

- transform a NL system into the singularly perturbed form
- obtain ϵ

If possible (for particular cases of SNL), then :

- resolution of the algebraic system (2a) extract x_F and replace it in (2b)

CRAN

A. NAGY KISS,
G. MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

Two-time scales

$$\begin{bmatrix} \dot{x}_F(t) \\ \dot{x}_S(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\epsilon} f_F(x_S(t), x_F(t), u(t), \epsilon) \\ f_S(x_S(t), x_F(t), u(t), \epsilon) \end{bmatrix}$$

$$y(t) = C \begin{bmatrix} x_F(t) \\ x_S(t) \end{bmatrix}$$

equivalent transformation $\Updownarrow$ sector nonlinearity approach

Multiple model

$$\begin{bmatrix} \dot{x}_F(t) \\ \dot{x}_S(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(x_S, x_F, u) \left\{ \begin{bmatrix} A^i_{FF} & A^i_{FS} \\ A^i_{SF} & A^i_{SS} \end{bmatrix} \cdot \begin{bmatrix} x_F(t) \\ x_S(t) \end{bmatrix} + \begin{bmatrix} B^i_F \\ B^i_S \end{bmatrix} u \right\}$$

$$y(t) = C \begin{bmatrix} x_F(t) \\ x_S(t) \end{bmatrix}$$

Consider x_F as unknown input : $d(t) = x_F(t)$

$$x(t) = \begin{bmatrix} d(t) \\ x_S(t) \end{bmatrix}$$

Design the matrices :

$$\bar{A}_i = \begin{bmatrix} A^i_{FF} & A^i_{FS} \\ 0 & A^i_{SS} \end{bmatrix} \quad E_i = \begin{bmatrix} 0 \\ A^i_{SF} \end{bmatrix} \quad \bar{C}_S = \begin{bmatrix} 0 & C_S \end{bmatrix}$$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) \cdot \left[\bar{A}_i\, x(t) + B_i\, u(t) + E_i\, d(t)\right] \\ y(t) = \bar{C}_S\, x(t) + C_F\, d(t) \end{cases} \tag{3}$$

- Decoupled time scales
- The estimation of x_S is made independently of x_F
- Classic structure of MM affected by unknown inputs
- Unmeasurable decision variables

State estimation method

MM with measurable decision variables :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) \cdot \left[\bar{A}_i x(t) + B_i u(t) + E_i d(t) + \omega(t)\right] \\ y(t) = \bar{C}_S x(t) + C_F d(t) \end{cases}$$

Unknown input observer :

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t), u(t)) \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \\ \hat{x}(t) = z(t) - H y(t) \end{cases} \quad (4)$$

Dynamic of the state estimation error : $\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$

Under matrix conditions $\dot{e}(t)$ reduces to :

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \left(N_i e(t) + P\omega(t)\right) \quad (5)$$

- $\mathcal{L}_2$ approach

Theorem : *$e(t) \to 0$ if $\exists$ X, M_i and S and a positive scalar λ s.t. the following conditions are respected $\forall\ i = 1, ..., r$:*

$$\begin{bmatrix} \bar{A}_i^T(X + S\bar{C}_S)^T + (X + S\bar{C}_S)\bar{A}_i - \bar{C}_S^T M_i^T - M_i\bar{C}_S + I & X + S\bar{C}_S \\ (X + S\bar{C}_S)^T & -\lambda I \end{bmatrix} < 0$$

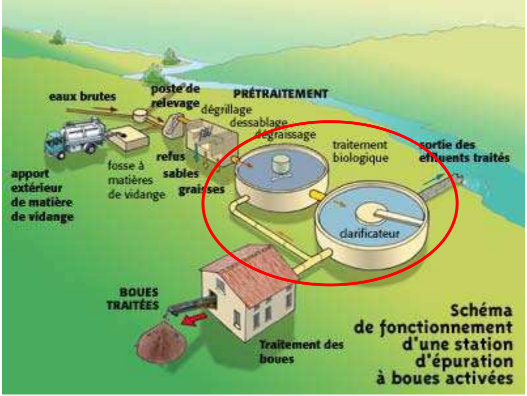
$$\begin{aligned} SC_F &= 0 \\ (X + S\bar{C}_S)E_i &= M_i C_F \end{aligned}$$

The observer matrices

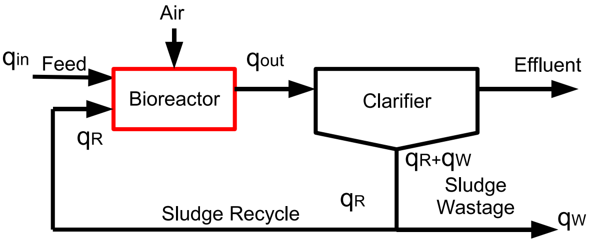
$$\begin{aligned} H &= X^{-1}S \\ N_i &= (I + H\bar{C}_S)\bar{A}_i - X^{-1}M_i\bar{C}_S \\ L_i &= X^{-1}M_i - N_i H \\ G_i &= (I + H\bar{C}_S)B_i \end{aligned} \tag{6}$$

Wastewater treatment

The diagram of the wastewater treatment plant process



The diagram of the set biological reactor + clarifier

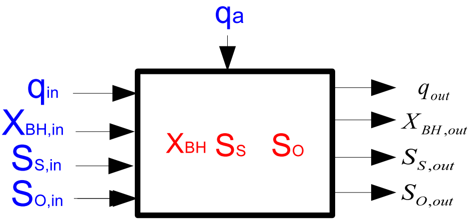


1. The operating mode : constant volume

$$q_{out} = q_{in} + q_R$$

2. Model : a part of ASM1 $\longrightarrow$ carbonated pollution

The reduced ASM1 model : biological reactor

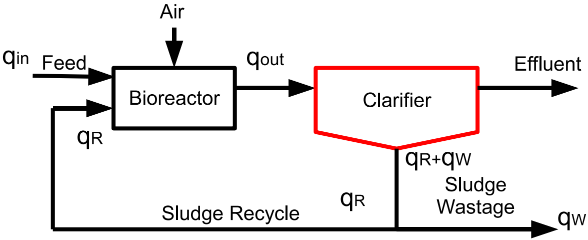


Simplification hypothesis :

$$
\begin{cases}
X_{BH,out}(t) = X_{BH}(t) \\
S_{S,out}(t) = S_{S}(t) \\
S_{O,out}(t) = S_{O}(t) \\
S_{O,in}(t) = 0
\end{cases}
$$

CRAN
A. NAGY KISS, G. MOUROT, B. MARX, J. RAGOT, G. SCHUTZ

The diagram of the set biological reactor + clarifier



1. Clarifier

$$(q_{in} + q_R)X_{BH} = (q_W + q_R)X_{BH,R}$$

$$S_{S,R} = S_S$$

CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

The reduced ASM1 model

$$\dot{S}_S = \frac{q_{in}}{V}(S_{S,in} - S_S) + (1-f)b_H X_{BH} - \frac{\mu_H}{Y_H}\frac{S_S}{K_S + S_S}\frac{S_O}{K_{OH} + S_O}X_{BH}$$

$$\dot{S}_O = -\frac{q_{in}}{V}S_O + K q_a (S_{O,sat} - S_O) - \frac{1 - Y_H}{Y_H}\mu_H \frac{S_S}{K_S + S_S}\frac{S_O}{K_{OH} + S_O}X_{BH}$$

$$\dot{X}_{BH} = \frac{q_{in}}{V}X_{BH,in} - \frac{q_W}{V}\frac{q_{in} + q_R}{q_W + q_R}X_{BH} + \mu_H \frac{S_S}{K_S + S_S}\frac{S_O}{K_{OH} + S_O}X_{BH} - b_H X_{BH}$$

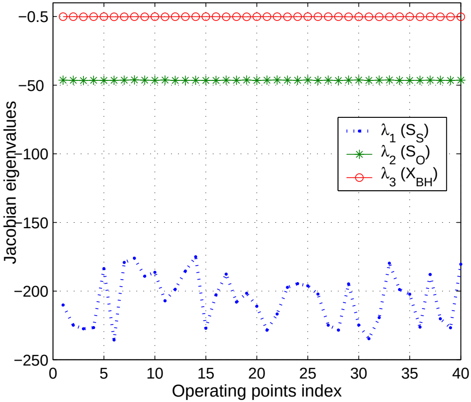
$$x = \begin{bmatrix} S_S \\ S_O \\ X_{BH} \end{bmatrix} \qquad u = \begin{bmatrix} S_{S,in} \\ q_a \\ X_{BH,in} \end{bmatrix}$$

Constants parameters : $\theta = (\mu_H, b_H, f, Y_H, S_{O,sat}, K_S, K_{OH}, K)$

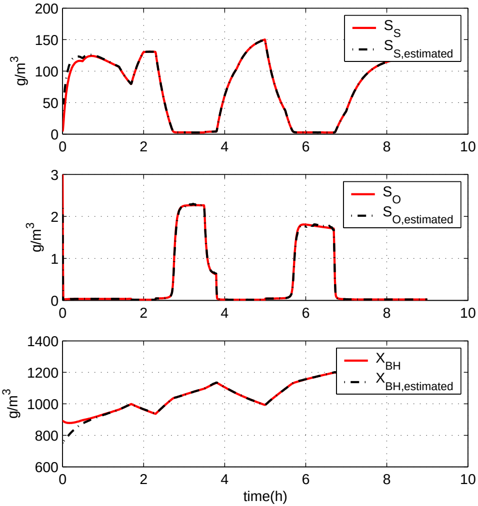
Slow and fast dynamics

CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ



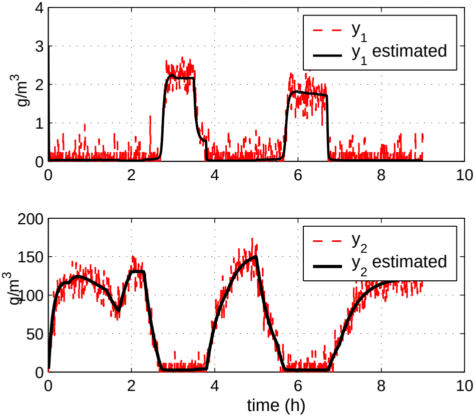
State estimation results



CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

Output estimation results



Conclusions and Future prospects

CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

Conclusions

1. Identification of slow and fast dynamics
2. Usage of MM with two time scales
3. State estimation using an unknown input observer and MM
4. Application to a part of the ASM1 model of a wastewater treatment plant

Future prospects

Using the Multiple Model form

1. System Diagnosis :

   - detect
   - isolate faults
   - identify

2. Apply to wastewater treatment plant model

CRAN

A. NAGY KISS,
G.MOUROT,
B. MARX,
J. RAGOT,
G. SCHUTZ

Thank you for your attention