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Robust fault and state estimation for discrete time-varying uncertain systems

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Abstract

In this paper, we consider the robust Kalman filtering for uncertain discrete time-varying systems, to solve the problem of simultaneously state and fault estimation. The system under consideration is subjected to time-varying norm-bounded parameter uncertainty in both the state and measurement matrices. The approach suggested rests on the use of the Augmented State Robust Kalman Filter (ASRKF) based on the optimization of an upper bound on the variance error of the state estimation. A necessary and sufficient condition for the existence of the filter is established in terms of a pair of Riccati equations. The proposed filter is tested by an illustrative example.

Index terms_ *Robust Kalman filtering, uncertain discrete time-varying systems, robust state estimation, robust fault estimation.*

1. Introduction:

This paper is concerned with the problem of joint fault and state estimation of linear discrete-time stochastic system. In spite of the presence of the parameter uncertainty the robust estimate of the state and the fault enables us to implement a fault tolerant control (FTC). A simple idea consists to use an architecture FTC resting on the compensation of the effect of the fault, see e. g. [1], [2].

Initially, we refer to the robust Kalman filtering problem largely treated in the literature by different approach. There are essentially two approaches to the robust estimation problem. The first is the H_∞ filtering, which minimizes the worst case energy gain from the noise inputs to the estimation error. In [3] a robust H_∞ filtering technique was proposed to satisfy state estimation error variance constraint as well as prescribed H_∞ performance for all admissible perturbations. This method is based on the solution of two discrete Riccati difference equations (DRE). Also, a new technique that ensures a more efficient evaluation of robust H_2 *and* H_∞ performance was developed by [4].This method is based on the use of a matrix inequality conditions which contain additional free parameters as compared to existing characterizations. These parameters provide extra degree of freedom and lead to a less conservative design. Duan et al. [5] extends the existing result on the robust H_2 *and* H_∞ filtering problem and introduces a new structure of slack variables to provide extra free dimension in the solution space for the H_2 *and* H_∞ optimization. The other approach is the so-called guaranteed cost filtering or robust Kalman filtering, the idea is to design a filter to minimize the estimation error covariance, see [6]-[8]. In many applications, an extra model for the system may not be available. In this situation, the error variance for all allowed parametric uncertainties is desirable. In [8]-[12] both the finite and the infinite-horizon filtering problem were addressed. Necessary and sufficient conditions for the existence of robust filter with an optimized upper bound for the error variance are given in terms of a pair of parameterized Riccati equations.

In the present work, we consider the problem of joint state and fault estimation for linear discrete time-varying systems with norm-bounded parameter uncertainty. The state and the measurement noises are assumed white signal with known statistics. The problem is addressed to design a robust filter that can solve the latter problem. In this case, when dynamical evolution of the fault is available, we may use the augmented state robust Kalman filter (ASRKF).

This paper is organized as follows, in section 2, we give the problem formulation. In section 3, the problem of state and fault estimation over finite-horizon is developed. A numerical example is illustrated in section 4.

2. Problem formulation

The problem consists of designing a filter that gives a robust state and fault estimation for discrete time-varying uncertain system. This problem is described by the bloc diagram of Fig. 1.

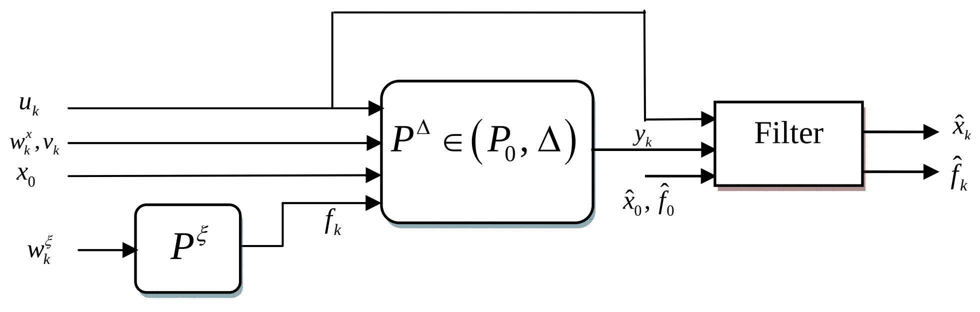

Fig.1. State and fault estimator filter

The plant P^{Δ} represents the uncertain discrete time-varying systems with additive fault and is described by

$$\left(P_0,\Delta\right):\begin{cases} x_{k+1} = (A_k + \Delta A_k)x_k + G_k u_k + F_k^x f_k + B_k^x w_k^x \\ y_k = (C_k + \Delta C_k)x_k + F_k^y f_k + v_k \end{cases} \tag{1}$$

where $u_k \in \Re^n$ is the known input, $x_k \in \Re^n$ is the system state, $f_k \in \Re^p$ is the addition fault vector, $y_k \in \Re^m$ is the measurement vector, $w_k \in \Re^n$ and $v_k \in \Re^m$ white noise sequences of zero-mean. The matrices $A_k, B_k, C_k, G_k, F_k^x$ and F_k^y are known and have appropriate dimensions. Whereas, ΔA_k and ΔC_k represent the time-varying parameters uncertainties in the matrices A_k and C_k respectively. These uncertainties have the following form:

$$\begin{bmatrix} \Delta A_k \\ \Delta C_k \end{bmatrix} = \begin{bmatrix} H_{1,k} \\ H_{2,k} \end{bmatrix} F_k E_k$$

where $H_{1,k}$, $H_{2,k}$ and E_k are known real matrix of appropriate dimensions, and F_k is an unknown perturbation matrix that satisfies the constraint $F_k^T F_k \le I$, $\forall\ k \in \left[0, N\right]$.

We treat the additive fault f_k as a stochastic process generated by

$$P^{\xi}\ :\ \begin{cases} \xi_{k+1} = A_k^{\xi}\xi_k + B_k^{\xi} w_k^{\xi} \\ f_k = C_k^{\xi}\xi_k \end{cases} \tag{2}$$

where A_k^{ξ}, B_k^{ξ} *and* C_k^{ξ} are known real matrices with appropriate dimensions.

Assumptions

***A1* :** the noises w_k^x *and* v_k are uncorrelated white noise sequences with the following covariance:

$$\mathcal{E}\left[w_k^x w_l^{xT}\right] = W_k^x \delta_{kl}\ ;\ \ \mathcal{E}\left[v_k v_l^T\right] = V_k \delta_{kl}\ ;\ \mathcal{E}\left[w_k^x v_l^T\right] = 0$$

where, $\mathcal{E}[\bullet]$ denotes the expectation operator and δ is the Kronecker delta function.

***A2* :** the noise w_k^{ξ} is zero-mean white noise sequence with the following covariance:

$$\mathcal{E}\left[w_k^{\xi} w_k^{\xi T}\right] = W_k^{\xi} \delta_{kl}\ ;\ \ \mathcal{E}\left[w_k^x w_l^{\xi T}\right] = \mathcal{E}\left[v_k w_l^{\xi T}\right] = 0$$

***A3* :** the initial state is a Gaussian random variable and is uncorrelated with the white noise process w_k^x, w_k^{ξ} and v_k .

$\boldsymbol{A_4}$ **:** conditions on matrices ranks

$\operatorname{rank}\left(F_k^x\right) = p$; $\operatorname{rank}\left(F_k^y\right) = p$; $\operatorname{rank}\left(C_k\right) = m\left(m \geq p\right)$

First, we will establish the state space model of the augmented system following from system (1) and filter (2). Define a new state vector as $\breve{x}_k = \begin{bmatrix} x_k \\ \xi_k \end{bmatrix}$, and then the state dynamic equation of the failed model (1) can be augmented with the failure model of equation (2) to give:

$$\begin{aligned} \breve{x}_{k+1} &= (\breve{A}_k + \breve{H}_{1,k} F_k \breve{E}_k)\breve{x}_k + \breve{G}_k u_k + \breve{B}_k \breve{w}_k \\ y_k &= (\breve{C}_k + \breve{H}_{2,k} F_k \breve{E}_k)\breve{x}_k + \breve{D}_k v_k \\ f_k &= \tilde{C}_k \breve{x}_k \end{aligned} \tag{3}$$

where, $\breve{x}_k = \begin{bmatrix} x_k \\ \xi_k \end{bmatrix}$; $\breve{A}_k = \begin{bmatrix} A_k & F_k^x C_k^\xi \\ 0 & A_k^\xi \end{bmatrix}$; $\breve{B}_k = \begin{bmatrix} B_k^x & 0 \\ 0 & B_k^\xi \end{bmatrix}$; $\breve{G}_k = \begin{bmatrix} G_k \\ 0 \end{bmatrix}$

$\breve{C}_k = \begin{bmatrix} C_k & F_k^y C_k^\xi \end{bmatrix}$; $\tilde{C}_k = \begin{bmatrix} 0 & C_k^\xi \end{bmatrix}$; $\breve{D}_k = D_k$;

$\breve{H}_{1,k} = \begin{bmatrix} H_{1,k} & 0 \\ 0 & 0 \end{bmatrix}$; $\breve{H}_{2,k} = \begin{bmatrix} H_{2,k} & 0 \end{bmatrix}$; $\breve{E}_k = \begin{bmatrix} E_k & 0 \\ 0 & 0 \end{bmatrix}$; $\breve{w}_k = \begin{bmatrix} w_k^x \\ w_k^\xi \end{bmatrix}$

The objective of this paper is to design a robust Kalman filter to joint fault and state estimations of a discrete time-varying system in the presence of norm-bounded parameter uncertainty in both the state and output matrices, in the finite case. We can consider that the filter has the following form:

$$\hat{\breve{x}}_{k+1} = A_k^f \hat{\breve{x}}_k + \breve{G}_k u_k + K_k^f y_k \quad ; \hat{\breve{x}}_k = \begin{bmatrix} \hat{x}_k & \hat{\xi}_k \end{bmatrix}^T \tag{4}$$

where A_k^f and K_k^f are time-varying matrices to be determinate in order that the variance of the estimation error is guaranteed to be smaller than a certain bound for all uncertainty matrices. The estimation error dynamics satisfies:

$$\mathcal{E}\left[(\breve{x}_k - \hat{\breve{x}}_k)(\breve{x}_k - \hat{\breve{x}}_k)^T\right] \leq S_k$$

With S_k being an optimized upper bound of filtering error covariance over the class of robust quadratic filters.

3. Filter design

In this section, a solution to the robust state and fault estimation problem over finite horizon [0, N] will be given, by the application of the robust Kalman filters presented in ([1], [8] and [9]). This problem of simultaneously state and fault estimation is solved by the use of the augmented state robust Kalman filter (ASRKF) approach. A sufficient condition for the existence of such a filter is proposed in term of Riccati equations.

First we assume that the initial condition $\breve{x}(0)$ is a zero mean Gaussian random variable with an unknown covariance matrix that satisfies the following assumption:

***A*5 [11-12]:**

- $\mathcal{E}\left[\breve{x}_0\breve{x}_0^T\right] \leq \bar{S}_0$, where $\bar{S}_0 = \bar{S}_0^T > 0$ is a known matrix.
- $rank\left[\breve{A}_k \ \breve{H}_{1,k} \ \breve{B}_k\breve{W}_k^{1/2}\right] = n + p$

where $\breve{W}_k = \mathcal{E}\left[\breve{w}_k\breve{w}_k^T\right] = \begin{bmatrix} W_k^x & 0 \\ 0 & W_k^{\xi} \end{bmatrix}$

Define the state estimation error $\tilde{x}_k$ by

$$\tilde{x}_k = \breve{x}_k - \hat{\breve{x}}_k$$

A state-space model describing the augmented system formed by combining (3) and (4) can be expressed as follow:

$$\begin{cases} \zeta_{k+1} = (A_{c1,k} + H_{c1,k}F_kE_{c1,k})\zeta_k + G_{c1,k}u_k + M_{c1,k}\eta_k \\ \tilde{x}_k = L\zeta_k \end{cases} \tag{5}$$

where

$$\zeta_k = \begin{bmatrix} \tilde{x}_k \\ \hat{\breve{x}}_k \end{bmatrix};\ \zeta_0 = \begin{bmatrix} x_0 \\ f_0 \\ 0 \\ 0 \end{bmatrix};\ \eta_k = \begin{bmatrix} \breve{w}_k \\ v_k \\ 0 \end{bmatrix};\ A_{c1,k} = \begin{bmatrix} \breve{A}_k - K_k^f\breve{C}_k & \breve{A}_k - A_k^f - K_k^f\breve{C}_k \\ K_k^f\breve{C}_k & A_k^f + K_k^f\breve{C}_k \end{bmatrix};\ L = \begin{bmatrix} I & 0 \end{bmatrix};$$

$$M_{c1,k} = \begin{bmatrix} \breve{B}_k & -K_k^f \\ 0 & K_k^f \end{bmatrix};\quad E_{c1,k} = \begin{bmatrix} \breve{E}_k & \breve{E}_k \end{bmatrix};\quad H_{c1,k} = \begin{bmatrix} \breve{H}_{1,k} - K_k^f\breve{H}_{2,k} \\ K_k^f\breve{H}_{2,k} \end{bmatrix};\quad G_{c1,k} = \begin{bmatrix} 0 \\ \breve{G}_k \end{bmatrix}$$

Definition [11], [12]: The filter (4) is said to be quadratic filter if and only if for some $\alpha_k > 0$, there exists a bounded $\Sigma_k = \Sigma_k^T \geq 0$ that satisfies the following Riccati difference equation (RDE):

$$\Sigma_{k+1} = A_{c1,k}\Sigma_k A_{c1,k}^T + \alpha_k^{-1} H_{c1,k} H_{c1,k}^T + M_{c1,k}\overline{W} M_{c1,k}^T + A_{c1,k}\Sigma_k E_{c1,k}^T (\alpha_k^{-1} I - E_{c1,k}\Sigma_k E_{c1,k}^T)^{-1} E_{c1,k}\Sigma_k A_{c1,k}^T \quad (6)$$

where $I - \alpha_k E_{c1,k}\Sigma_k E_{c1,k}^T > 0$, $\Sigma_0 = diag\{S_0, 0\}$ and $\overline{W} = diag\{W^x, W^\xi, V, 0\}$.

The following result can be obtained by the application of the lemma1 presented in reference [6].

Lemma [12]: Consider the uncertain system (3) satisfying assumption ***A₁***, and let (4) be a given quadratic filter associated with a guaranteed cost matrix $\Sigma_k = \Sigma_k^T \geq 0$. Then the covariance matrix of ζ_k of the error system (5) satisfies the bound $\mathcal{E}\left[\zeta_k \zeta_k^T\right] \leq \Sigma_k \ \forall k \in \left[0, N\right]$, for all admissible uncertainties. Furthermore, $\mathcal{E}\left[\tilde{x}_k \tilde{x}_k^T\right] \leq L\Sigma_k L^T$, $\forall k \in \left[0, N\right]$ and $\tilde{x}_k$ is the estimation error.

Remark 1: The necessary condition on the filter for optimality of the upper bound on the error variance, $tr\left(L\Sigma_k L^T\right)$, $\forall k \in \left[0, N\right]$ is that the optimal solution Σ_k of (6) should be of the following partitioned form:

$$\Sigma_k = \begin{bmatrix} \Sigma_{11,k} & \Sigma_{12,k} \\ \Sigma_{21,k} & \Sigma_{22,k} \end{bmatrix}$$

Next we can introduce the following form:

$$\Sigma_k = \begin{bmatrix} \Sigma_{11,k} & 0 \\ 0 & P_k - \Sigma_{11,k} \end{bmatrix}$$

where $P_k = \mathcal{E}\left[\breve{x}_k \breve{x}_k^T\right]$ and $\Sigma_{12,k} = \Sigma_{21,k} = 0$, $\forall\ k \in \left[0, N\right]$ which is argued similar to the continuous -time case as in [13].

In the case, where the system is without uncertainty this simply implies the orthogonality of estimation error $\tilde{x}_k$ to the estimate $\hat{\breve{x}}_k$ of $\breve{x}_k$, which is necessary for $\hat{\breve{x}}_k$ to be optimal. Furthermore, $\Sigma_{12,k} = P_k - \Sigma_{11,k}$ follows from the fact:

$$P_k = \Sigma_{11,k} + \Sigma_{12,k} + \Sigma_{21,k} + \Sigma_{22,k}$$

As derived in ([6], [11] and [12]), the filter parameters (the state and the gain matrices) are optimized to give a minimal upper bound on the state error covariance estimation for all admissible uncertainties. By the multiplying both sides of (6) by L and L^T , respectively, it follows that:

$$
\begin{aligned}
\Sigma_{11,k+1} &= L\Sigma_{k+1}L^T \\
&= (\breve{A}_k - K_k^f \breve{C}_k)\Sigma_{11,k}(\breve{A}_k - K_k^f \breve{C}_k)^T + (\breve{A}_k - A_k^f - K_k^f \breve{C}_k)(P_k - \Sigma_{11,k})\times \\
&\quad (\breve{A}_k - A_k^f - K_k^f \breve{C}_k)^T + [(\breve{A}_k - A_k^f - K_k^f \breve{C}_k)P_k + A_k^f \Sigma_{11,k}]\breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k \times \\
&\quad [(\breve{A}_k - A_k^f - K_k^f \breve{C}_k)P_k + A_k^f \Sigma_{11,k}]^T + \alpha_k^{-1}(\breve{H}_{1,k} - K_k^f \breve{H}_{2,k})(\breve{H}_{1,k} - K_k^f \breve{H}_{2,k})^T \\
&\quad + \breve{B}_k \breve{W} \breve{B}_k^T + K_k^f V (K_k^f)^T
\end{aligned} \tag{7}
$$

where, $\tilde{M}_k = \alpha_k^{-1} I - \breve{E}_k P_k \breve{E}_k^T > 0$

Considering $\Sigma_k \geq 0$, we know that $P_k - \Sigma_{11,k} \geq 0$. Using this fact and noting $\tilde{M}_k \geq 0$, it is obvious that $I + \breve{E}_k^T \tilde{M}_k^{-1} \breve{E}_k \left(P_k - \Sigma_{11,k}\right)$ is invertible and so is $I + \left(P_k - \Sigma_{11,k}\right)\breve{E}_k^T \tilde{M}_k^{-1} \breve{E}_k$. Note that $\left[I + \left(P_k - \Sigma_{11,k}\right)\breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right]^{-1}\left(P_k - \Sigma_{11,k}\right) = \left(P_k - \Sigma_{11,k}\right)\left[I + \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\left(P_k - \Sigma_{11,k}\right)\right]^{-1}$. Then it follow from (7) that

$$
\begin{aligned}
\Sigma_{11,k+1} &= \Delta K_k^f + \left\{A_k^f \left[I + \left(P_k - \Sigma_{11,k}\right)\breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right] - \left(\breve{A}_k - K_k^f \breve{C}_k\right)\left(I + P_k \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right)\right\} \\
&\quad \times \left[I + \left(P_k - \Sigma_{11,k}\right)\breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right]^{-1}\left(P_k - \Sigma_{11,k}\right) \\
&\quad \times \left\{\left[I + \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\left(P_k - \Sigma_{11,k}\right)\right]A_f^T - \left(I + P_k \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right)\left(\breve{A}_k - K_k^f \breve{C}_k\right)^T\right\}
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
\Delta K_k^f &= \left(\breve{A}_k - K_k^f \breve{C}_k\right)\Sigma_{11,k}\left(\breve{A}_k - K_k^f \breve{C}_k\right)^T + \breve{B}_k \breve{W}_k \breve{B}_k^T + K_k^f V_k K_k^{fT} + \frac{1}{\alpha_k}\left(\breve{H}_{1,k} - K_k^f \breve{H}_{2,k}\right)\left(\breve{H}_{1,k} - K_k^f \breve{H}_{2,k}\right)^T \\
&\quad + \left(\breve{A}_k - K_k^f \breve{C}_k\right)\left(P_k - \Sigma_{11,k}\right)\left(\breve{A}_k - K_k^f \breve{C}_k\right)^T + \left(\breve{A}_k - K_k^f \breve{C}_k\right)P_k \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k P_k^T\left(\breve{A}_k - K_k^f \breve{C}_k\right)^T \\
&\quad - \left(\breve{A}_k - K_k^f \breve{C}_k\right)\left(I + P_k \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right)\times\left[I + \left(P_k - \Sigma_{11,k}\right)\breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right]^{-1}\left(P_k - \Sigma_{11,k}\right) \\
&\quad \times\left(I + P_k \breve{E}_k^T \tilde{M}_k^{-1}\breve{E}_k\right)\left(\breve{A}_k - K_k^f \breve{C}_k\right)^T
\end{aligned}
$$

Note that $\Sigma_{11,k+1} = S_{k+1}$

Since $\partial^2 tr\left(S_{k+1}\right)/\partial\left(A_k^f\right)^2 > 0$ and $\partial^2 tr\left(S_{k+1}\right)/\partial\left(K_k^f\right)^2 > 0$, $tr\left(S_{k+1}\right)$ is minimal if $\partial tr\left(S_{k+1}\right)/\partial A_k^f = 0$ and $\partial tr\left(S_{k+1}\right)/\partial K_k^f = 0$. Moreover, from $\partial tr\left(S_{k+1}\right)/\partial A_k^f = 0$, we have

$$
A_k^f = [\breve{A}_k + \alpha_k \breve{A}_k S_k \breve{E}_k^T (I - \alpha_k \breve{E}_k S_k \breve{E}_k^T)^{-1}\breve{E}_k] - K_k^f [\breve{C}_k + \alpha_k \breve{C}_k S_k \breve{E}_k^T (I - \alpha_k \breve{E}_k S_k \breve{E}_k^T)^{-1}\breve{E}_k] \tag{9}
$$

Next substituting (9) into (8) we have $S_{k+1} = \Delta K_k^f$. After some algebraic manipulation,

$$S_{k+1} = \breve{A}_k Q_k \breve{A}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{1,k}^T + \breve{B}_k \breve{W}_k \breve{B}_k^T + \left[K_k^f \left(\breve{R}_{\varepsilon k} + \breve{C}_k Q_k \breve{C}_k^T \right) - \left(\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T \right) \right]$$
$$\times \left(\breve{R}_{\varepsilon k} + \breve{C}_k Q_k \breve{C}_k^T \right)^{-1} \left[K_k^f \left(\breve{R}_{\varepsilon k} + \breve{C}_k Q_k \breve{C}_k^T \right) - \left(\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T \right) \right]^T$$
$$- \left(\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T \right) \times \left(\breve{R}_{\varepsilon k} + \breve{C}_k Q_k \breve{C}_k^T \right)^{-1} \left(\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T \right)^T \tag{10}$$

where, $Q_k^{-1} = S_k^{-1} - \alpha_k \breve{E}_k^T \breve{E}_k$ and $\breve{R}_{\varepsilon k} = V + \frac{1}{\alpha_k} \breve{H}_{2,k} \breve{H}_{2,k}^T$.

And from $\partial tr\left(S_{k+1}\right) / \partial K_k^f = 0$, we have:

$$K_k^f = (\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T)(\breve{R}_{ek} + \breve{C}_k Q_k \breve{C}_k^T)^{-1} \tag{11}$$

Now, substituting (11) into (10), we have

$$S_{k+1} = \breve{A}_k Q_k \breve{A}_k^T - (\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T)(\breve{R}_{ek} + \breve{C}_k Q_k \breve{C}_k^T)^{-1}$$
$$\times (\breve{A}_k Q_k \breve{C}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{2,k}^T)^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{1,k}^T + \breve{B}_k \breve{W} \breve{B}_k^T \tag{12}$$

Furthermore, the covariance matrix of the augmented state satisfies

$$P_{k+1} = \varepsilon\left[\breve{x}_k \breve{x}_k^T \right]$$
$$= \left(\breve{A}_k + \breve{H}_{1,k} F_k \breve{E}_k \right) P_k \left(\breve{A}_k + \breve{H}_{1,k} F_k \breve{E}_k \right)^T + \breve{B}_k \breve{W}_k \breve{B}_k^T$$

By the use of the lemma1 presented in reference [6], an upper bound of P_{k+1} is given by

$$P_{k+1} \le \breve{A}_k P_k \breve{A}_k^T + \breve{A}_k P_k \breve{E}_k^T (\frac{I}{\alpha_k} - \breve{E}_k P_k \breve{E}_k^T)^{-1} \breve{E}_k P_k \breve{A}_k^T + \frac{1}{\alpha_k} \breve{H}_{1,k} \breve{H}_{1,k}^T + \breve{B}_k \breve{W} \breve{B}_k^T \tag{13}$$

A robust quadratic filter for the uncertain system (3) that minimizes the bound on the error variance can exist if and only if, for some $\alpha_k > 0$, exist a solution $P_k = P_k^T > 0$ over $\left[0, N\right]$ to the RDE (13) and such that $\left(P_k\right)^{-1} - \alpha_k \breve{E}_k^T \breve{E}_k > 0$. Under these conditions are verified, the robust quadratic filter with an optimized upper bound for error covariance is given by:

Remark 2: It is clear that when parameter uncertainty in system (3) disappears, the robust filter reduces to the standard finite horizon Kalman filter.

Remark 3: The scaling parameter α_k could be exploited to optimize the filtering performance. Furthermore, if $\bar{\alpha}_k$ is the supremum of α_k such that $\bar{\alpha}_k \breve{E}_k P_k \breve{E}_k^T < I$ the RDEs (12) and (13) admit a positive definite solution for any $\alpha_k \in (0, \bar{\alpha}_k)$ and $trace(P_k)$, $trace(S_k)$ are both convex function of α_k ([11], [12]).

Step 1 : Initialisation for $k = 0$ $\breve{x}_0$; $\hat{\breve{x}}_0$ $S_0 = P_0$ ***Step 2*** : Implementation of ASRKF   For $k = 1 : N$ ***Step 2.1:*** Calculate of optimal values α_k^* and S_k^* $\alpha_k^* = \arg \min_{\alpha_k \in [0, \bar{\alpha}_k]} trace(S_k(\alpha_k))$ *Such that equations* (12) *and* (13) $S_k^* = S_k(\alpha_k^*)$ $P_k^* = P_k(\alpha_k^*)$ ***Step 2.2***: Estimation of augmented state $\hat{\breve{x}}_{k+1} = (\breve{A}_k + \Delta\breve{A}_{ek})\hat{\breve{x}}_k + \breve{G}_k u_k + K_k^f [y_k - (\breve{C}_k + \Delta\breve{C}_{ek})\hat{\breve{x}}_k]$ $\Delta\breve{A}_{ek} = \alpha_k^* \breve{A}_k S_k^* \breve{E}_k^T (I - \alpha_k^* \breve{E}_k S_k^* \breve{E}_k^T)^{-1} \breve{E}_k$ $\Delta\breve{C}_{ek} = \alpha_k^* \breve{C}_k S_k^* \breve{E}_k^T (I - \alpha_k^* \breve{E}_k S_k^* \breve{E}_k^T)^{-1} \breve{E}_k$ $K_k^f = (\breve{A}_k Q_k \breve{C}_k^T - \alpha_k^{-1} \breve{H}_{1,k} \breve{H}_{2,k}^T)(\breve{R}_{ek} + \breve{C}_k Q_k \breve{C}_k^T)^{-1}$

Table 1: Recursive algorithm of the augmented state robust Kalman filter (ASRKF)

4. Illustrative example

This section is focused to the application of the proposed filter ASRKF. The system under consideration is given by

$$\begin{cases} x_{k+1} = \begin{bmatrix} a_1(k) & -0.5 \\ 0.3 & 0.4+\delta(k) \end{bmatrix} x_k + \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} u_k + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} f_k + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} w_k^x \\ y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} f_k + v_k \end{cases}$$

$$\begin{cases} \xi_{k+1} = \begin{bmatrix} 0.2 & 0.7 \\ 0.1 & 0.3 \end{bmatrix} \xi_k + \begin{bmatrix} 3 \\ -1.5 \end{bmatrix} w_k^{\xi} \\ f_k = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \xi_k \end{cases}$$

where, $a_1(k) = 0.4 + 0.1 * \sin(0.6 * k)$

The varying noise $w_k^{\xi} \sim \begin{cases} N(-0.6,\ W^{\xi}) & \text{if } k \le 30 \\ N(\ 0.8,\ W^{\xi}\ ) & \text{if } 30 < k \le 70 \\ N(-0.6,\ W^{\xi}) & \text{if } k > 70 \end{cases}$

The above system is of the form of system (1)-(2) with:

$$H_{1,k} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ H_{2,k} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\ E_k = \begin{bmatrix} 0 & 0.3 \end{bmatrix}$$

$$W_k^x = 0.1;\ V_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix};\ W_k^{\xi} = 0.1$$

where $\delta(k)$ is the uncertain parameter satisfying $\delta(k) = 0.3 * \sin(0.5 * k)$

The initial conditions are given by

$$x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \hat{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad f_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \hat{f}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The optimal value of the state covariance matrix P_k^* and the optimized upper bound of error covariance matrix S_k^* are shown in Figure2 respectively.

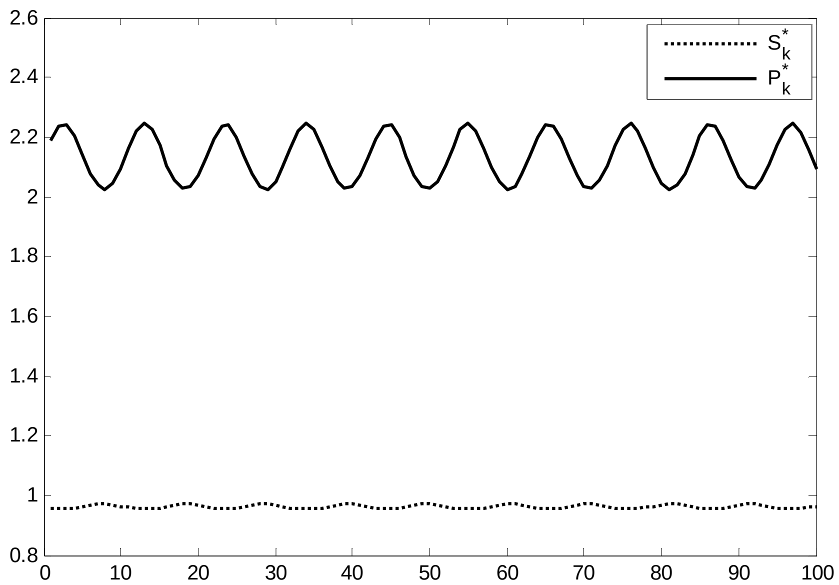

Fig.2. The optimal value of $trace(P_k^*)$ and $trace(S_k^*)$

In figure 2, we observe that the optimal value of the derived upper bound of the covariance matrix S_k is below the state covariance matrix P_k^* for any admissible uncertainty.

Figures 3 and 4, present the state, the fault and their estimations respectively.

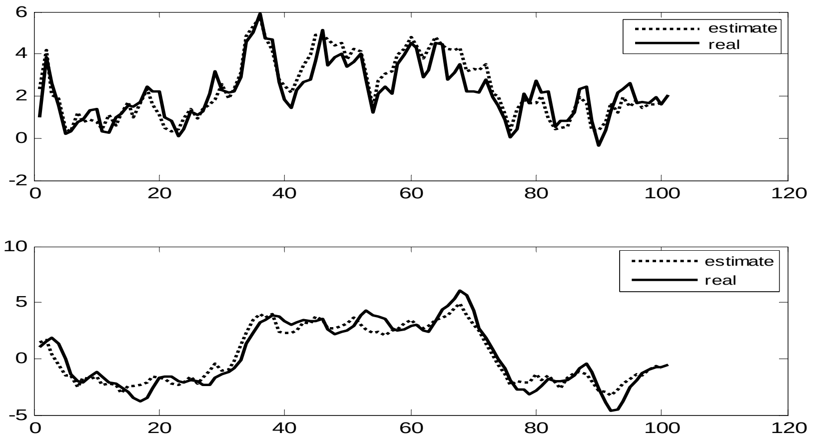

Fig.3. State estimation

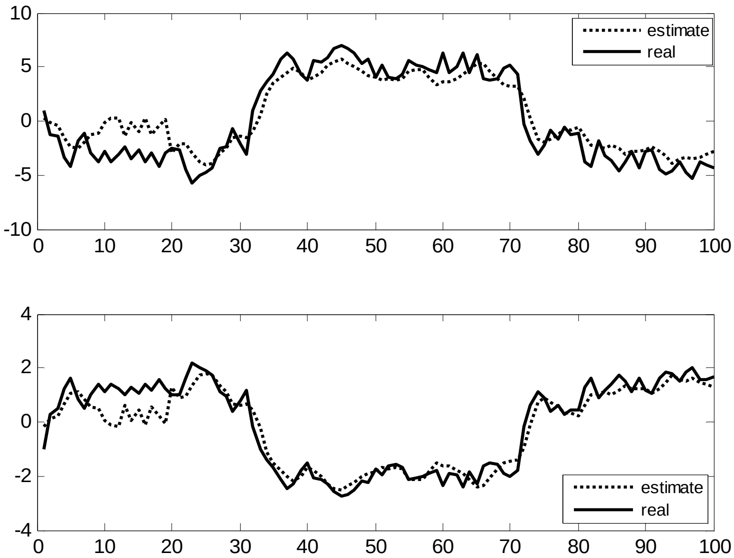

Fig.4. Fault estimation

The performances of the resultant filter are given in Figures 3 and 4 which verifies that the robust filter is optimal in spite of the presence of time-varying parameter uncertainties.

5. Conclusion

In this paper, the problem of joint robust fault and state estimation for linear discrete time-varying uncertain systems has been established. A recursive solution using an augmented state robust Kalman filter (ARSKF) is proposed. The filter has been tested by an illustrative example that has given a robust estimation of the state and the fault in spit the presence of norm-bounded parameter uncertainties in both the state and the measurement matrices.

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