State and unknown input estimation for discrete time multiple model

M. Chadli\textsuperscript{a,}\textsuperscript{*}, A. Akhenak\textsuperscript{b}, J. Ragot\textsuperscript{c}, D. Maquin\textsuperscript{c}

\textsuperscript{a}University of Picardie Jules Verne, Laboratoire de Modélisation, Information et Systèmes (E.A. 4290), 7, Rue du Moulin Neuf, 80000 Amiens, France
\textsuperscript{b}Institut de Recherche en Systèmes Electroniques Embarqués, Technopôle du Madrillet, Avenue Galilée, BP 10024, 76801 Saint-Etienne du Rouvray Cedex, France
\textsuperscript{c}Institut National Polytechnique de Lorraine, CRAN, CNRS-UHP-INPL (UMR 7039), 2, Avenue de la forêt de Haye, 54516 Vandoeuvre-lès-Nancy, France

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Abstract

This paper deals with the state estimation of nonlinear discrete systems described by a multiple model with unknown inputs. The main goal concerns the simultaneous estimation of the system’s state and the unknown inputs. This goal is achieved through the design of a multiple observer based on the elimination of the unknown inputs. It is shown that the observer gains are solutions of a set of linear matrix inequalities. After that, an unknown input estimation method is proposed. An academic example and an application dealing with message decoding illustrate the effectiveness of the proposed multiple observer.

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1. Introduction

This work focuses on the estimation of nonmeasured variables. The adopted approach belongs to the general framework of state reconstruction via observer. More precisely, our goal here is to estimate the input of the system. This can be also seen as an attempt to
inverse the system’s model. These problems will be studied within the multiple model approach framework [26]. The motivation of this approach is related to the fact that it is often difficult to design a model which takes into account all the complexity of the studied system. This approach, which includes the Takagi–Sugeno models [30] and polytopic linear differential inclusions (PLDI) [6], has been extensively considered (see among others [27,32,8,9] and references therein). However, all of these studies do not concern the multiple model with unknown input.

Over the past two decades, many researchers have paid attention to the problem of state estimation of dynamic linear systems subjected to both known and unknown inputs [34,28,14]. Unknown inputs can result either from model uncertainty or due to the presence of unknown external excitation (disturbances, unmodelled dynamics). For example in secure communication, we need to estimate not only the state for chaos synchronization but also the input confidential message, which can be considered as an unknown input [24,11,5]. In the field of fault detection and isolation, robust and fault-tolerant control unknown input observer is utilized for detecting and isolating both actuator and sensor faults [17,31,1]. Many approaches have developed full and/or reduced order unknown input observers to estimate the state of linear time-invariant dynamical system driven by both known and unknown inputs [3,13,16,18,19]. Recently, some researchers have further investigated this problem for singular systems [25,23,15]. However, up to date and to the best of our knowledge, the class of multiple model with unknown input has not yet been fully investigated and this will be the goal of this paper.

In this paper, for state and unknown input estimation, the suggested technique consists in associating to each local model a local unknown input observer [1]. The considered multiple observer is then a convex interpolation of these local observers. This interpolation is obtained throughout the same activation functions as the multiple model. Our contribution here lies in the design of the unknown input multiple observer by eliminating the unknown inputs from the dynamics of the state estimation error. The synthesis conditions of multiple observer are expressed in linear matrix inequalities (LMI) terms. The improvement of the multiple observer performances by pole assignment is also considered and leads to supplementary LMI constraints.

The rest of this paper is organized as follows. In Section 2, the general structure of the considered multiple model is presented. In Section 3, the proposed structure of unknown multiple model is described and the main result is presented. The derived conditions ensuring the global asymptotic convergence of the estimation error are given as a set of LMI with additional equality constraints. A method allowing to estimate the unknown input ends this section. The last section gives a numerical example to illustrate the effectiveness of the proposed results. The example concerns a particular, but important, application in the field of encoded or crypted communications. The observer, used as a decoder, is then designed in order to decode the crypted message.

**Notation**: Throughout the paper, the following useful notation is used: $X^T$ denotes the transpose of the matrix $X$, $X > 0$ means that $X$ is a symmetric positive definite matrix.

2. **General structure of a multiple model**

The formalization of the state estimation problem is made here starting from a multiple model representation. First, let us consider the class of nonlinear discrete systems subject to unknown inputs and represented by a multiple model. Such representation results from
the aggregation of $M$ local models as follows [1]:

$$x(t + 1) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i \bar{u}(t) + R_i \tilde{u}(t) + D_i)$$  \hspace{1cm} (1a)$$

$$y(t) = C x(t) + F \bar{u}(t)$$  \hspace{1cm} (1b)$$

with

$$\begin{align*}
\sum_{i=1}^{M} \mu_i(\xi(t)) &= 1 \\
0 \leq \mu_i(\xi(t)) &\leq 1 \quad \forall i \in \{1, \ldots, M\}
\end{align*}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the input vector, $\bar{u}(t) \in \mathbb{R}^q$, $q < n$, contains the unknown inputs and $y \in \mathbb{R}^p$ the measured outputs. Matrices $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ denote the state matrix and the input matrix associated to the $i$th local model. Matrices $R_i \in \mathbb{R}^{n \times q}$ and $F \in \mathbb{R}^{p \times q}$, with $\text{rank}(F) = q < p$ are the distribution matrices of unknown inputs. The matrices $D_i \in \mathbb{R}^n$ are introduced to take into account the operating point of the system and $C \in \mathbb{R}^{p \times n}$ is the output matrix. To finish with, $\xi(t)$ is the so-called decision vector which may depend on some subset of the known inputs and/or measured variables to define the operating regimes.

When $\mu_i(\xi(t)) = 1$, which implies $\mu_j(\xi(t)) = 0$, $\forall j \neq i$, model $i$ is active. In fact, the value of the functions $\mu_i(\xi(t))$ is not Boolean and the state of the multiple model can be viewed as a weighted sum of the “local models”. Notice, however, that, as this explanation helps to understand the structure of the considered model, it is not really exact as one state only exists: those of the multiple model, and local states do not really exist. When activation functions $\mu_i(\xi(t))$ are not Boolean ones, several local models are active and at each time, the coefficients $\mu_i(\xi(t)) i \in \{1, \ldots, M\}$ quantify the relative contribution of each local model to the global model. The choice of the number $M$ of local models for that multiple model may be intuitively chosen by taking into account a certain number of operating regimes. Matrices $A_i$, $B_i$, $R_i$ and $D_i$ can be obtained by using the direct linearization of an a priori nonlinear model around operating points, or alternatively by using an identification procedure [20,21,4]. From a practical point of view, matrices $A_i$, $B_i$, $R_i$ and $D_i$ describe the system’s local behavior around the $i$th regime.

The choice of variable $\xi(t)$ leads to different classes of models. It can depend on the measurable state variables, be a function of the measurable outputs of the system and possibly of the input. In this case, the multiple model describes a class of nonlinear systems or a Takagi–Sugeno model [30]. It can also be an unknown constant value, the multiple model then represents a PLDI [6].

In the following the considered problem concerns both the reconstruction of state variable $x(t)$ and unknown input $\bar{u}(t)$, using only the available information namely known input $u(t)$ and measured output $y(t)$.

3. Multiple observer design

In this section, we explain how to design a multiple observer. The structure of that observer results from the interpolation of local unknown input observers throughout the same activation functions as those used in the unknown input multiple model (1). The
considered structure of the multiple observer is the following [10,1]:

\[
\begin{align*}
\dot{z}(t) &= \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} + L_i y(t)) \\
\dot{x}(t) &= z(t) - E \gamma(t)
\end{align*}
\]

where \( N_i \in \mathbb{R}^{n \times n} \), \( G_{i1} \in \mathbb{R}^{n \times m} \), \( G_{i2} \in \mathbb{R}^n \), \( E \in \mathbb{R}^{n \times p} \), \( L_i \in \mathbb{R}^{n \times p} \) are the observer gains to be determined. Indeed, the observer only uses known variables \( u(t) \) and \( y(t) \), \( \bar{u}(t) \) being nonmeasured. This set of matrices has to be properly defined to ensure the convergence of estimated state \( \hat{x}(t) \) towards true state \( x(t) \). For that purpose, let us define the state estimation error:

\[
e(t) = x(t) - \hat{x}(t)
\]

From that definition and using Eqs. (2b) and (1b), the estimation error can be written as

\[
e(t) = (I + EC)x(t) - z(t) + EF \bar{u}(t)
\]

Thus, at time \( t + 1 \), the estimation error is

\[
e(t + 1) = \sum_{i=1}^{M} \mu_i(\xi(t))(P(A_i x(t) + B_i u(t) + R_i \bar{u}(t) + D_i)

- N_i z(t) - G_{i1} u(t) - G_{i2} - L_i y(t)) + EF \bar{u}(t + 1)
\]

with

\[
P = I + EC
\]

Replacing \( y(t) \) and \( z(t) \) by their respective expressions given by Eqs. (1b) and (2b), the estimation error takes the form

\[
e(t + 1) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i e(t) + (PB_i - G_{i1}) u(t) + (PA_i - N_i - K_i C)x(t)

+ (PD_i - G_{i2}) + (PR_i - K_i F) \bar{u}(t)) + EF \bar{u}(t + 1)
\]

with

\[
K_i = N_i E + L_i
\]

If the following conditions are fulfilled:

\[
N_i = PA_i - K_i C \quad (9a)
\]

\[
G_{i1} = PB_i \quad (9b)
\]

\[
G_{i2} = PD_i \quad (9c)
\]

\[
PR_i = K_i F \quad (9d)
\]

\[
EF = 0 \quad (9e)
\]

where \( P \) and \( K_i \) are defined in Eqs. (6) and (8), respectively, Eq. (7) is reduced to

\[
e(t + 1) = \sum_{i=1}^{M} \mu_i(\xi(t))N_i e(t)
\]

The decay of the state estimation error depends on matrix \( N = \sum_{i=1}^{M} \mu_i(\xi)N_i \). It is important to note that the stability of matrices \( N_i \), \( \forall i \in \{1, \ldots, M\} \) does not guarantee the
stability of $N$. One way to ensure the stability of matrix $N = \sum_{i=1}^{M} \eta_i(x_i)N_i$ is to verify that a matrix $X > 0$ exists such that

$$N_i^T X N_i - X < 0 \tag{11}$$

Thus, constraints (9) and (11) allow the complete synthesis of observer (2) for the multiple model with unknown inputs (1).

### 3.1. Global convergence of the multiple observer

In this part, sufficient conditions for the global asymptotic convergence of the state estimation error (10) are established in LMI terms with additional structural constraints.

**Theorem 1.** The state estimation error between multiple model (1) and unknown input multiple observer (2) converges globally asymptotically towards zero if there exists matrices $X > 0$, $S$ and $W_i$ such that the following conditions hold $\forall i \in \{1, \ldots, M\}$:

$$
\begin{align*}
X & \succ 0 \tag{12a} \\
(XA_i + SCA_i - W_i C) & \succ W_i F \tag{12b} \\
SF & = 0 \tag{12c}
\end{align*}
$$

Multiple observer (2) is then completely defined by

$$
\begin{align*}
E & = X^{-1} S \\
G_{i1} & = (I + X^{-1}SC)B_i \\
G_{i2} & = (I + X^{-1}SC)D_i \\
N_i & = (I + X^{-1}SC)A_i - X^{-1}W_i C \\
L_i & = X^{-1}W_i - N_i E 
\end{align*} \tag{13a-e}
$$

**Proof.** Using Eq. (11), the hypothesis $X > 0$ and using the Schur complement, one can easily deduce that

$$
\begin{pmatrix}
X & * \\
X N_i & X
\end{pmatrix} > 0 
$$

Using Eqs. (6) and (9a), the preceding inequality becomes

$$
\begin{pmatrix}
X & * \\
XA_i + XECA_i - XK_i C & X
\end{pmatrix} > 0 
$$

However, this last inequality (14) is nonlinear in synthesis variables $X$, $E$ and $K_i$. In order to convert these conditions into an LMI formulation, we consider the following changes of variables:

$$
\begin{align*}
W_i & = XK_i \tag{15a} \\
S & = XE \tag{15b}
\end{align*}
$$
Using the new variables, inequality (14) becomes
\[
\begin{pmatrix} X \\ XA_i + SCA_i - W_iC X \end{pmatrix} > 0
\] (16)

The two equality constraints (12b) and (12c) are obtained by pre-multiplying the last two constraints (9d) and (9e) by \( X > 0 \) with the change of variable (15):
\[
\begin{align*}
XPR_i &= XK_iF \\
XEF &= 0 \\
(Y + SC)R_i &= W_iF \\
SF &= 0
\end{align*}
\]

Therefore classical numerical tools may be used to solve LMI problem (12a) subject to linear equality constraints (12b) and (12c). After having solved this problem, the different matrices defining the proposed observer \( N_i, G_{i1}, G_{i2}, L_i \) and \( E \) can be deduced from the knowledge of \( X, S \) and \( W_i \) as given in Eqs. (13). This completes the proof. \( \square \)

Remark 1. The resolution of Eq. (12) is conditioned by research of a common Lyapunov matrix checking several inequality constraints (12a) under equalities constraints (12b) and (12c). To relax these conditions, one way consists in using nonquadratic Lyapunov functions (see for example [22,7,35]). However, using nonquadratic Lyapunov functions in our case lead to nonlinear constraints (i.e. conditions (12) become nonlinear).

3.2. Pole assignment

In this part, we examine how to improve the performances of the multiple observer in particular with regard to the rate of convergence towards zero of the state estimation error. For better estimating the state variables of the multiple model, the dynamics of the multiple observer is selected in a manner which is appreciably faster than that of the multiple model. As the multiple observer is a nonlinear observer, exact pole assignment cannot be achieved. It is only possible to assign the poles to a specific sub-region in the complex plane [12]. If the prescribed region is a circle centered at the origin

![Fig. 1. Poles assignment.](image-url)
with a radius $\alpha<1$ (Fig. 1). The LMI formulation of the previous problem is expressed by the following corollary:

**Corollary.** If there exist matrices $X$, $S$ and $W_i$ such that the following conditions hold $\forall i \in \{1, \ldots, M\}$:

\[
\begin{align}
\alpha X & \star \\
XA_i + SCA_i - W_iC & \alpha X \\
(X + SC) R_i & = W_i F \\
SF & = 0
\end{align}
\]

(17a)

(17b)

(17c)

where $1 > \alpha > 0$, then the multiple observer (2) is globally asymptotically convergent with the performance defined by the complex region $S(0, \alpha)$. The observer parameters are as defined by Eq. (13).

**Remark 2.** By using expression (9a), inequality (17a) can be rewritten as follows:

\[
N_i^T X N_i - \alpha^2 X < 0
\]

(18)

**Remark 3.** It is possible to choose other kind of sub-region included in the unit circle. For example, the specification of the eigenvalues inside the circle with center at $(\sigma, 0)$ and radius $r$ as shown in Fig. 2 can be taken into account by simply replacing the matrices $N_i$ by $(N_i - \sigma I)/r$ in the stability conditions (11).

### 3.3. Unknown input estimation

In some applications (for example in diagnosis), the estimation of the unknown input $\bar{u}(t)$ has to be performed. Thus, several works were developed for the unknown input estimation within the framework of linear dynamic systems [23,29]. In [17,31], the authors presented an unknown input estimation method based on the sliding mode observer and applied to fault detection and isolation.

In this section, we show how to reconstruct unknown inputs $\bar{u}(t)$ from known matrices $A_i$, $B_i$, $D_i$, $F$, $C$ and measured variables $u(t)$ and $y(t)$. We have previously shown that if the
conditions of Theorem 1 hold, the state estimation error tends towards zero. Then substituting true state \( x(t) \) by its estimate \( \hat{x}(t) \) in Eq. (1), we get

\[
\begin{align*}
\dot{x}(t + 1) &= \sum_{i=1}^{M} \mu_i(\zeta(t))(A_i\hat{x}(t) + B_iu(t) + R_i\hat{u}(t) + D_i) \\
y(t) &= C\hat{x}(t) + F\hat{u}(t)
\end{align*}
\]  
(19)

where \( \hat{u}(t) \) denotes an estimation of the unknown input vector. Assuming that matrix

\[
W = \begin{pmatrix}
\sum_{i=1}^{M} \mu_i(\zeta(t))R_i \\
F
\end{pmatrix}
\]  
(20)

is of full column rank, unknown input \( \bar{u}(t) \) is then estimated by using

\[
\hat{u}(t) = (W^TW)^{-1}W^T \left( \dot{x}(t + 1) - \sum_{i=1}^{M} \mu_i(\zeta(t))(A_i\hat{x}(t) + B_iu(t) + D_i) \right) \\
y(t) - C\hat{x}(t)
\]  
(21)

Summarizing the estimation procedure, the design of multiple observer and the estimation of unknown inputs can be implemented as follows:

1. Solve the linear constraints (12) with LMITOOL software [33].
2. Deduce the observer parameters \( N_i, G_{i1}, G_{i2}, L_i \) and \( E \) of the multiple observer (2) using Eqs. (13).
3. Estimate unknown input estimation using Eq. (21) with the matrix \( W \) (Eq. (20)) is of full column rank.

4. Simulation examples

In this section, two examples are presented in order to illustrate the performances of the proposed multiple observer. The first is an academic example and the second describes a secure communication system.

4.1. An academic example

Consider the following multiple model involving two outputs and three states:

\[
\begin{align*}
x(t + 1) &= \sum_{i=1}^{2} \mu_i(\hat{\zeta}(t))(A_i\hat{x}(t) + B_iu(t) + R_i\hat{u}(t)) \\
y(t) &= C\hat{x}(t) + F\hat{u}(t) + \nu(t)
\end{align*}
\]  
(22)

In this example, \( \hat{\zeta}(t) \) represents known input \( u(t) \) and \( \nu(t) \) is a vector of centered random noise with a variance equal to 0.01. The activation functions are as follows:

\[
\begin{align*}
\hat{\zeta}(t) &= u(t) \\
\mu_1(\hat{\zeta}(t)) &= \frac{1}{2}(1 - \tanh(\hat{\zeta}(t))) \\
\mu_2(\hat{\zeta}(t)) &= 1 - \mu_1(\hat{\zeta}(t))
\end{align*}
\]  
(23)
The numerical values of matrices $A_i$, $B_i$, $R_i$, $C$ and $F$ are as follows:

$$
A_1 = \begin{bmatrix}
-0.2 & 0.1 & 0.1 \\
0.1 & -0.3 & 0.1 \\
0.2 & 0.1 & 0.3
\end{bmatrix},
A_2 = \begin{bmatrix}
-0.3 & 0.2 & 0.2 \\
0.1 & -0.3 & 0 \\
0.5 & 0.05 & -0.3
\end{bmatrix}
$$

$$
B_1 = \begin{bmatrix}
0.5 \\
-0.25 \\
-0.25
\end{bmatrix},
B_2 = \begin{bmatrix}
-0.25 \\
0.5 \\
-0.125
\end{bmatrix},
R_1 = \begin{bmatrix}
0.5 \\
-0.5 \\
0.5
\end{bmatrix},
R_2 = \begin{bmatrix}
0.5 \\
0.25 \\
-1
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
1 & 1 & 1 \\
1 & 0 & 1
\end{bmatrix},
F = \begin{bmatrix}
1
\end{bmatrix}
$$

The resolution of the conditions of Theorem 1 leads to the following results:

$$
X = \begin{bmatrix}
2.8073 & 0.0057 & 0.9173 \\
0.0057 & 2.3897 & 0.0062 \\
0.9173 & 0.0062 & 1.9580
\end{bmatrix},
W_1 = \begin{bmatrix}
-0.1010 & 1.1738 \\
-0.0055 & -0.0040 \\
-0.4607 & 0.6552
\end{bmatrix}
$$

$$
W_2 = \begin{bmatrix}
0.1336 & 0.7475 \\
-0.0228 & 0.0271 \\
-0.4701 & -0.4076
\end{bmatrix}
$$

$$
E = \begin{bmatrix}
0.1742 & -0.1742 \\
-0.9906 & 0.9906 \\
1.1883 & -1.1883
\end{bmatrix},
L_1 = \begin{bmatrix}
0.5026 & -0.0897 \\
-0.0105 & 0.0058 \\
-0.9379 & 0.8438
\end{bmatrix},
L_2 = \begin{bmatrix}
0.6999 & -0.1563 \\
0.0002 & 0.0022 \\
-0.9371 & 0.2342
\end{bmatrix}
$$

$$
N_1 = \begin{bmatrix}
0.0057 & -0.0011 & 0.0057 \\
0.4130 & 0.0015 & 0.5130
\end{bmatrix},
N_2 = \begin{bmatrix}
0.0014 & 0.0063 & -0.0024 \\
0.8718 & 0.0033 & 0.4029
\end{bmatrix}
$$

$$
G_1 = \begin{bmatrix}
0.4565 \\
-0.0024
\end{bmatrix},
G_2 = \begin{bmatrix}
0.0047 \\
-0.5471
\end{bmatrix}
$$

Fig. 3 depicts the time evolution of known input $u(t)$ while Fig. 4 shows the unknown input $\bar{u}(t)$. Fig. 5 shows, on the same graph, the time evolution of the unknown input and its estimate obtained from Eq. (21). Figs. 6–8 show the three state estimation errors obtained with initial conditions $x_0 = (100)^T$, and $\bar{x}_0 = (21 - 1)^T$.

### 4.2. Second example: application to secure communication

The approaches developed in Sections 3.1 and 3.3 can be applied to synthesize a secure communication system. The problem we are faced with consists in transmitting some coded message with a signal broadcasted by a communication channel. On the receiver side, the hidden signal is recovered by a decoding system. The increasing need of secure
communications leads to the development of many techniques which make difficult the detecting of transmitted message [24,11,5]. In this section, the proposed multiple observer is used to design a secure communication scheme. We consider a discrete SISO multiple model that results from the aggregation of two local models:

\[
\begin{align*}
    x(t + 1) &= \sum_{i=1}^{2} \mu_i(\zeta(t))(A_i x(t) + R_i \bar{u}(t)) \\
    y(t) &= C x(t) + F \bar{u}(t)
\end{align*}
\]  

(24)
System (24) has the particularity to be controlled by an unique unknown input $\bar{u}(t)$. The activation functions are expressed with exponential functions that only depend on the multiple model output ($\bar{x}(t) = y(t)$):

$$
\begin{align*}
\bar{x}(t) &= y(t) \\
\mu_1(\bar{x}(t)) &= \frac{1}{2}(1 - \tanh(\bar{x}(t))) \\
\mu_2(\bar{x}(t)) &= 1 - \mu_1(\bar{x}(t))
\end{align*}
$$

(25)
The numerical values of matrices are as follows:

\[
\begin{align*}
A_1 &= \begin{bmatrix} -1.1 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0.8 & -0.1 \\ 1 & 1.1 \end{bmatrix}, & C &= [0.5 & 0.5], & F &= 5
\end{align*}
\]

The structure of the considered system is described in Fig. 9. The message to be encoded constitutes the so-called unknown input of the multiple model which plays the role of the encoder. The output of this model is transmitted using a public channel. On the receiving side, an unknown input multiple observer serves as a decoder in order to re-build the
Fig. 9. Structure of the secure communication system.

Fig. 10. Message \( \bar{u}(t) \).

Fig. 11. Output \( y(t) \).
message. Clearly, the choice of the “local” models \((A_i \text{ and } B_i)\) matrices, their number, as well as the nature of the activation function \((\mu_i(\xi(t)))\) are key elements for an external person to be able to decode, from the sole knowledge of \(y(t)\), the embodied crypted message. The goal of the proposed example is only to show the feasibility of the proposed secure communication system.

From structure (24) of the model, we can deduce immediately the following values:

\[
E = 0, \quad G_{i1} = 0, \quad G_{i2} = 0
\]

For this very particular example, the encoding system (initial model) is conceived at the same time as the decoding system (observer); the choice of matrices \(R_i\) is thus free. This is why matrix inequality (12a) are solved without taking into account constraints (12b) and (12c). The conditions of Theorem 1 allow the determination of matrices \(X\), \(W_1\) and \(W_2\):

\[
X = \begin{bmatrix} 1.6718 & -2.0563 \\ -2.0563 & 7.7169 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -3.9158 \\ 9.0362 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -2.3610 \\ 13.5810 \end{bmatrix}
\]

Matrices \(N_i\) and \(L_i\) defining the observer are then, respectively, deduced using Eqs. (13d) and (13e):

\[
L_1 = \begin{bmatrix} -1.3418 \\ 0.8134 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1.1192 \\ 2.0581 \end{bmatrix}, \\
N_1 = \begin{bmatrix} -0.4291 & 1.1709 \\ -0.1067 & 0.2933 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.2404 & -0.6596 \\ -0.0291 & 0.0709 \end{bmatrix}
\]

Knowing \(W_i\) and \(X\), it is easy to compute the values of \(R_1\) and \(R_2\). This is done by the means of Eq. (12b):

\[
R_1 = \begin{bmatrix} -6.7090 \\ 4.0671 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 5.5959 \\ 10.2906 \end{bmatrix}
\]

Fig. 12. \(x_1(t)\) true and its estimate.
Fig. 10 shows the message to be transmitted while Fig. 11 depicts the output of the multiple model, i.e. the encoded message. Figs. 12 and 13, respectively, show the true and estimated states which are perfectly superposed. Finally, Fig. 14 presents the estimated message (unknown input estimate). Except around the time origin, the estimated message perfectly matches the true one. Concerning the transmission of a crypted message on a public channel of communication, one can wonder about the possibility of detecting and retrieving the message from the transmitted signal. In [2], some answers are given and one of them is satisfied here with a simple “visual appreciation”. Fig. 15, plotted in the phase plane of the system, does not show any particular behavior of periodic type or with
commutation. These two remarks cannot constitute evidence of inviolability of the signal transmitted compared to the message to transmit, but simply a first indication on the masking of the message to be transmitted.

5. Conclusion

In this paper, based on multiple model representation, the design of an unknown input multiple observer using the principle of interpolation of local observers has been proposed. Moreover, the case where some inputs of the system are unknown has been considered. The stability of the multiple observer requires, however, the consideration of coupling constraints between these local observers; these constraints lead to the resolution of an LMI problem under structural constraints. The calculation of the gains of the multiple observer is then returned to a simultaneous calculation of the gains of the local observers. Assuming the existence of suited matrices, we showed that the reconstruction of the state and unknown inputs vectors of the multiple model is possible. An academic example is given to show the effectiveness of the derived conditions. A particular application of the proposed method deals with communication decryption; the objective is to recover a message imbedded in a signal generated by a dynamical nonlinear system.

References


