DESIGN OF OBSERVERS FOR TAKAGI-SUGENO SYSTEMS WITH IMMEASURABLE PREMISE VARIABLES : AN $\mathcal{L}_2$ APPROACH

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Abstract: A new observer design method is proposed for Takagi-Sugeno systems with immeasurable premise variables. Since the state estimation error can be written as a perturbed system, the proposed method is based on the $\mathcal{L}_2$ techniques to minimize the effect of the perturbations on the state estimation error. The convergence conditions of the observer are established by using the second method of Lyapunov and a quadratic function. These conditions are expressed in terms of Linear Matrix Inequalities (LMI). Finally, the performances of the proposed observer are augmented by a pole placement in LMI region.

Keywords: Multiple model approach; nonlinear observer; immeasurable premise variables; H_∞ optimization; pole placement; linear matrix inequality.

1. INTRODUCTION

The problem of nonlinear state estimation is a very vast field of research, having many applications, among them one can cite the use of the observers to estimate the immeasurable states of a system or to replace sensors which are expensive and difficult to maintain; these observers are also used for the state feedback control or for the diagnosis of the system.

The diagnosis methods for linear systems currently have a certain maturity, however assuming that the system to supervise can be correctly represented by a linear system is highly restrictive. Moreover, the direct extension of the methods developed in the linear case, to the nonlinear case is delicate. Nevertheless, interesting results have been obtained if the nonlinear systems are represented by a multiple model. This structure consists in a set of local linear models, each local model describing the behavior of the system in a particular region of the state-space.

In the context of the linear models, fault detection can be carried out by methods using state observers and residual generation (Maquin and Ragot, 2000). In general, fault isolation methods use banks of observers where each observer is driven by a subset of the inputs. The preceding technique cannot be immediately extended to the multiple model because of the couplings introduced into the structure. Generally, the design of an observer for a multiple model begins with the design of local observers, then a weighted interpolation is performed to obtain the estimated state. This design allows the extension of the analysis and synthesis tools developed for the linear systems, to the nonlinear systems.

(Tanaka *et al.*, 1998) proposed a study concerning the stability and the synthesis of regulators and observers for multiple models. In (chadli, 2002), (Tanaka *et al.*, 1998) and (Guerra *et al.*, 2006) tools directly inspired from the study of the linear systems are adapted for the stability study and stabilization of nonlinear systems. (Patton *et al.*, 1998) proposed a multiple observer based on the use of Luenberger observers, which was then used for the diagnosis. In (Akhenak, 2004) and (Akhenak *et al.*, 2006) observers with sliding mode developed for the linear systems, were transposed to the systems described by multiple model. The principal interest of this type of observers is the robustness with respect to the modeling uncertainties. Moreover, the unknown inputs observers designed for linear systems, were transposed, in the same way, into the case of nonlinear systems and application to fault diagnosis is envisaged in (Marx *et al.*, 2007).

However, in all these works, the authors supposed that the weighting functions depend on measurable premise variables. In the field of diagnosis, this assumption forces to design observers with weighting functions depending on the input $u(t)$, for the detection of the sensors faults, and on the output $y(t)$, for the detection of actuator faults. Indeed, if $\xi(t) = u(t)$ is used, for example in a bank of observers, even if the *ith* observer is not controlled by the input u_i, this input appears indirectly in the weighting function and it cannot be eliminated. For this reason, it is interesting to consider the case of weighting functions depending on immeasurable premise variables, like the state of the system. This assumption makes it possible to represent a large class of nonlinear systems. Only few works are based on this approach, nevertheless, one can cite (Bergsten and Palm, 2000), (Palm and Driankov, 1999), (Bergsten *et al.*, 2001) and (Bergsten *et al.*, 2002), in which a Luenberger observer is proposed, by using Lipschitz weighting functions. The stability conditions of the observer are formulated in the form of linear matrix inequalities (LMI) (Boyd *et al.*, 1994). Unfortunately, the Lipschitz constant appears in LMIs and reduces the applicability of the method if this constant has an important value. In (Palm and Driankov, 2000) and (Bergsten and Palm, 2000), the observer with sliding mode compensates the unknown terms of the system.

In this paper, observer error dynamics are written as a perturbed system. So, with the use of $\mathcal{L}_2$ design (which is an extension of the H_∞ design), the influence of the immeasurable terms on the state estimation error can be minimized. According to this objective, we propose a new observer design for multiple model with immeasurable premise variables. The observer synthesis is carried out by using the second method of Lyapunov with a quadratic function and H_∞ optimization. The paper is organized as follows : section 2 introduces some works on the context of state estimation of multiple model with immeasurable premise variables. In section 3, the proposed observers are presented, convergence conditions of the proposed multiple observer are established. A design procedure to satisfy pole clustering constraints is also given. Simulation results are presented in section 4 and some conclusions and perspectives are given in section 5.

2. BACKGROUND RESULTS AND NOTATION

In this section we summarize some results on observer design for Takagi-Sugeno systems of the form:

$$\dot{x}(t) = \sum_{i=1}^{N} \mu_i(x(t)) \left(A_i x(t) + B_i u(t)\right) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input of the system, $y(t) \in \mathbb{R}^p$ is the output of yhe system. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are real known constant matrices. The weighting functions μ_i depend on immeasurable premise variables (state of a system), and verify:

$$\begin{cases} \sum_{i=1}^{N} \mu_i(x(t)) = 1 \\ 0 \leqslant \mu_i(x(t)) \leqslant 1 \ \forall i \in \{1, ..., N\} \end{cases} \quad (3)$$

Few works can be found concerning the class of nonlinear system with the assumption of immeasurable premise variables. The observer, proposed in (Bergsten and Palm, 2000), is in the form of Luenberger observer, namely:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{N} \mu_i(\hat{x}(t))(A_i \hat{x}(t) + B_i u(t) + G_i(y(t) - \hat{y}(t))) \quad (4)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5)$$

The observer error is given by:

$$e(t) = x(t) - \hat{x}(t) \quad (6)$$

then the observer error dynamics are given by:

$$\dot{e}(t) = \sum_{i=1}^{N} \mu_i(\hat{x}(t))(A_i - G_i C)e + \Delta(x, \hat{x}, u) \quad (7)$$

with:

$$\Delta(x, \hat{x}, u) = \sum_{i=1}^{N} (\mu_i(x) - \mu_i(\hat{x}(t))(A_i x + B_i u) \quad (8)$$

where (8) satisfies a Lipschitz condition in x, i.e,

$$\|\Delta(x, \hat{x}, u)\| \leqslant \alpha \|x - \hat{x}\| \quad (9)$$

Lemma 1. (Bergsten and Palm, 2000) The state estimation error between the multiple model (1) and the multiple observer (4) converges globally asymptotically toward zero, if there exists matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that the following conditions hold for $i = 1, \ldots, N$

$$A_i^T P + PA_i - C^T K_i^T - K_i C < -Q \quad (10)$$

$$\begin{bmatrix} -Q + \alpha^2 & P \\ P & -I \end{bmatrix} < 0 \quad (11)$$

The observer gains are given by $G_i = P^{-1} K_i$.

Lemma 1 recalls the design of the Thau-Luenberger observer introduced in (Bergsten and Palm, 2000). Unfortunately for an important value of the Lipschitz constant α, the set of LMI (10-11) may be unfeasible. Another method for state estimation of the system (1) is proposed in (Ichalal *et al.*, 2007). The contribution of (Ichalal *et al.*, 2007) is to obtain less restrictive existence condition for the observer. In this approach, the matrices A_i are decomposed into:

$$A_i = A_0 + \overline{A}_i \quad (12)$$

where A_0 is defined by:

$$A_0 = \frac{1}{N} \sum_{i=1}^{N} A_i \quad (13)$$

By substituting (12) in the equation of the multiple model (1) we obtain:

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{N} \mu_i(x(t)) (\overline{A}_i x(t) + B_i u(t)) \quad (14)$$

$$y(t) = Cx(t) \quad (15)$$

Based on this model, the following multiple observer is proposed:

$$\dot{\hat{x}}(t) = A_0 x(t) + \sum_{i=1}^{N} \mu_i(\hat{x}(t)) (\overline{A}_i \hat{x}(t) + B_i u(t) + G_i(y(t) - \hat{y}(t))) \quad (16)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (17)$$

Lemma 2. (Ichalal *et al.*, 2007) The state estimation error between the multiple model (1) and the multiple observer (16) converges globally asymptotically toward zero, if there exists matrices $P = P^T > 0$, $Q = Q^T > 0$ and positive scalars λ_1, λ_2 and γ such that the following conditions hold for $i = 1, \ldots, N$

$$A_0^T P + PA_0 - K_i^T P - PK_i < -Q \quad (18)$$

$$\begin{bmatrix} -Q + \lambda_1 M_i^2 I & P\overline{A}_i & PB_i & N_i \gamma I \\ \overline{A}_i^T P & -\lambda_1 I & 0 & 0 \\ B_i^T P & 0 & -\lambda_2 & 0 \\ N_i \gamma I & 0 & 0 & -\lambda_2 I \end{bmatrix} < 0 \quad (19)$$

$$\gamma - \beta_1 \lambda_2 > 0 \quad (20)$$

where β_1 is the bound on the input $u(t)$. The gains of the observer are computed by $G_i = P^{-1} K_i$.

The LMI (18-20) in lemma 2 may have solutions, even for great values of the Lipschitz constant and of the bound on the input β_1. The drawback of this method is that, if the bound β_1 increases, then the band-width of the observer increases and thus the observer estimates the measurement noise.

The contribution of this paper is to obtain a minimal influence of the unknown premise variables on the estimation quality, and moreover to satisfy pole clustering in prescribed regions of the complex plane. In order to quantify the influence of an input signal on the output of a system, the $\mathcal{L}_2$-norm of a system, based on the L_2-norm of a signal, is introduced.

Definition (L_2-norm) The L_2-norm of a signal $z(t)$, denoted $\|z(t)\|_2$ is defined by

$$\|z(t)\|_2^2 = \int_0^\infty z^T(t) z(t) dt \quad (21)$$

It is supposed that all the signals studied in this paper are measurable functions (or square integrable) that is to say: of finite energy. The space of measurable functions is denoted $\mathbb{L}_2$.

Definition ($\mathcal{L}_2$-norm) Consider a system of input $u(t) \in \mathbb{L}_2$ and of output $y(t) \in \mathbb{L}_2$. The $\mathcal{L}_2$-norm of the system is defined by

$$\gamma = \sup_{u(t) \in \mathbb{L}_2} \frac{\|y(t)\|_2}{\|u(t)\|_2} \quad (22)$$

It is well known that the $\mathcal{L}_2$-norm is a extension to the nonlinear systems, of the H_∞-norm of the linear systems (for a linear system $G(s)$, the $\mathcal{L}_2$-norm and the H_∞-norm defined by $\|G(s)\|_\infty = \sup_{\omega \in \mathbb{R}} \sigma_{max}(G(j\omega))$, where σ_{max} denotes the maximal singular value, are equal).

In the remaining of the paper, the following assumptions are made:

Hypothesis 3. The weighting functions μ_i are Lipschitz

$$\|\mu_i(x) - \mu_i(\hat{x})\| \leqslant N_i \|x - \hat{x}\| \quad (23)$$

$$\|\mu_i(x)x - \mu_i(\hat{x})\hat{x}\| \leqslant M_i \|x - \hat{x}\| \quad (24)$$

3. MAIN RESULT

In this section the observer design is detailed. The chosen structure of the observer used to estimate the state variables of the multiple model presented in (14) (or equivalently (1)), is a Luenberger observer as follows :

$$\dot{\hat{x}}(t) = A_0 x(t) + \sum_{i=1}^{N} \mu_i(\hat{x}(t))(\overline{A}_i \hat{x}(t) + B_i u(t) + G_i(y(t) - \hat{y}(t))) \quad (25)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (26)$$

The observer error dynamics is given as :

$$\dot{e}(t) = \sum_{i=1}^{N} (\mu_i(\hat{x}(t))\Phi_i e + A_i\delta_i + \Delta_i B_i u) \quad (27)$$

where :

$$\begin{cases} \delta_i(t) = \mu_i(x)x - \mu_i(\hat{x})\hat{x} \\ \Delta_i(t) = \mu_i(x) - \mu_i(\hat{x}) \\ \Phi_i = A_0 - G_i C \end{cases} \quad (28)$$

The objective is to determine the gains of the observer that minimize the $\mathcal{L}_2$-norm from the unknown terms on the state estimation error and such that the estimation error dynamics satisfy pole clustering constraints.

3.1 Observer design

Theorem 4. The state estimation error between the multiple model (14) and the multiple observer (16) converges globally asymptotically toward zero satisfying the H_∞ constraint, if there exists matrices $P = P^T > 0$, $Q = Q^T > 0$ and positive scalar λ_1, λ_2 and γ such that the following conditions hold

$$\begin{aligned} & A_0^T P + PA_0 - K_i^T P - PK_i < -Q \\ & \begin{bmatrix} -Q + \lambda_1 M_i^2 I & P\overline{A}_i & PB_i \\ \overline{A}_i^T P & -\lambda_1 I & 0 \\ B_i^T P & 0 & -\lambda_2 \end{bmatrix} < 0 \\ & \lambda_2 - \gamma^2 > 0 \end{aligned} \quad (29)$$

where $K_i = PG_i$.

Proof.

To show the convergence of the estimation error toward zero, let us consider the following quadratic function of Lyapunov :

$$V(e(t)) = e(t)^T P e(t), P = P^T > 0 \quad (30)$$

The convergence of the observer and the satisfaction of the H_∞ (**??**) constraint are given if

$$\dot{V}(e(t)) + e(t)^T e(t) - \gamma^2 u(t)^T u(t) < 0 \quad (31)$$

Then, by using (27):

$$\dot{V}(e) = \sum_{i=1}^{n} (\delta_i^T \overline{A}_i^T Pe + e^T P\overline{A}_i\delta_i + u^T\Delta_i^T B_i^T Pe + e^T PB_i\Delta_i u + \mu_i(\hat{x})(e^T\Phi_i^T Pe + e^T P\Phi_i e)) \quad (32)$$

Taking into account the definition (**??**), definition () and assumptions 2, one has then :

$$\begin{cases} \delta_i^T\delta_i \leqslant M_i^2 e^T e \\ \Delta_i^T\Delta_i \leqslant 1 \end{cases} \quad (33)$$

Lemma 5. for all matrices X and Y of adapted size, λ being a nonnull constant, the following property holds :

$$X^T Y + Y^T X \leqslant \lambda X^T X + \lambda^{-1} Y^T Y \quad \lambda > 0 \quad (34)$$

By applying this lemma and (**??**), we have:

$$\begin{aligned} \delta_i^T\overline{A}_i^T Pe + e^T P\overline{A}_i\delta_i &\leqslant \lambda_1\delta_i^T\delta_i + \lambda_1^{-1} e^T P\overline{A}_i\overline{A}_i^T Pe \\ &\leqslant \lambda_1 M_i^2 e^T e + \lambda_1^{-1} e^T P\overline{A}_i\overline{A}_i^T Pe \end{aligned} \quad (35)$$

and :

$$\begin{aligned} u^T\Delta_i^T B_i^T Pe + e^T PB_i\Delta_i u &\leqslant \lambda_2 u^T\Delta_i^T\Delta_i u + \lambda_2^{-1} e^T PB_iB_i^T Pe \\ &\leqslant \lambda_2 u^T u + \lambda_2^{-1} e^T PB_iB_i^T Pe \end{aligned} \quad (36)$$

The derivative of Lyapunov function (**??**) can then be written in the following way:

$$\dot{V} \leqslant \sum_{i=1}^{n} e^T(\mu_i(\hat{x})(\Phi_i^T P + P\Phi_i) + (\lambda_1 M_i^2 + 1)I + \lambda_1^{-1} P\overline{A}_i\overline{A}_i^T P + \lambda_2^{-1} PB_iB_i^T P)e + u^T(\lambda_2 N_i^2 - \gamma^2)u \quad (37)$$

The negativity of (**??**) is assured if:

$$e^T(\mu_i(\hat{x})(\Phi_i^T P + P\Phi_i) + (\lambda_1 M_i^2 + 1)I + \lambda_1^{-1} P\overline{A}_i\overline{A}_i^T P + \lambda_2^{-1} PB_iB_i^T P)e + u^T(\lambda_2 N_i^2 - \gamma^2)u < 0 \quad (38)$$

what leads to the following conditions:

$$\Phi_i^T P + P\Phi_i < Q \quad (39)$$

$$Q + M_i^2(\lambda_1\alpha^2 + 1)I + \lambda_1^{-1} P\overline{A}_i\overline{A}_i^T P + \lambda_2^{-1} PB_iB_i^T P < 0 \quad (40)$$

$$\lambda_2 N_i^2 - \gamma^2 < 0 \quad (41)$$

With the change of variables $K_i = PG_i$ and by applying the Schur complement, one obtains the following linear matrices inequalities :

$$A_0^T P + PA_0 - C^T K_i^T - K_i C < -Q \quad (42)$$

$$\begin{bmatrix} -Q + \lambda_1 M_i^2 I & P\overline{A}_i & PB_i \\ \overline{A}_i^T P & -\lambda_1 I & 0 \\ B_i^T P & 0 & -\lambda_2 I \end{bmatrix} < 0 \quad (43)$$

$$\lambda_2 - \gamma^2 < 0 \quad (44)$$

3.2 Pole placement

One notes in simulation that, if the value of (H_∞ Attenuation γ) decreases, the poles of the matrices $(A0 - GiC)$ increase in absolute value. For that, a pole placement makes it possible to solve this problem. we propose then the following theorem which makes it possible to place the eigenvalues of the multiple observer in particular regions. In this section we propose an extension of the previous method of synthesis by placing the eigenvalues of the observer in LMI region S (Fig. 1).

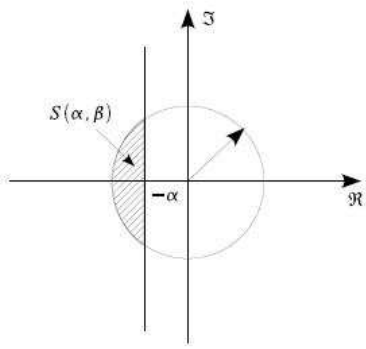

Fig. 1. LMI region

Theorem 6. The state estimation error between the multiple model (14) and the multiple observer (16) converges globally asymptotically toward zero satisfying the H_∞ constraint and the poles of the matrix $(A_0 - G_i C)$ are in the LMI region (Fig.1), if there exists matrices $P = P^T > 0$, $Q = Q^T > 0$, gain matrices K_i and positive scalar λ_1, λ_2 and γ such that the following conditions hold

$$\begin{bmatrix} \beta P & P(A_0 - G_i C) \\ (A_0 - G_i C)^T P & \beta P \end{bmatrix} > 0 \quad (45)$$

$$A_0^T P + PA_0 - C^T K_i^T - K_i C + 2\alpha P < -Q \quad (46)$$

$$\begin{bmatrix} -Q + \lambda_1 M_i^2 I & P\overline{A}_i & PB_i \\ \overline{A}_i^T P & -\lambda_1 I & 0 \\ B_i^T P & 0 & -\lambda_2 I \end{bmatrix} < 0 \quad (47)$$

$$\lambda_2 - \gamma^2 < 0 \quad (48)$$

where $K_i = PG_i$.

Proof. Using the concept of $\mathcal{D}$-stability presented in (Chilali, 1996) and (Bong-Jae and Sangchul, 2006), the constraint to place the poles of the matrix $(A_0 - G_i C)$ in the shaded region of Fig. 1 can be expressed in terms of LMIs as :

$$\begin{bmatrix} RP & P(A_0 - G_i C) \\ (A_0 - G_i C)^T P & RP \end{bmatrix} > 0 \quad (49)$$

$$(A_0 - G_i C)^T P + P(A_0 - G_i C) + 2\alpha P < 0 \quad (50)$$

4. SIMULATION RESULTS

We consider the following example to show the advantages of the using proposed H_∞ observer. This is an example of the form (1) with

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -6 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The weight functions are

$$\begin{cases} \mu_1(x) = \dfrac{1 - \tanh(x_1)}{2} \\ \mu_2(x) = 1 - \mu_1(x) = \dfrac{1 + \tanh(x_1)}{2} \end{cases} \quad (51)$$

The Lipschitz constants in hypothesis 1 are $M_1 = M_2 = 1.1$. We choose an input signal in the form (Fig.2)

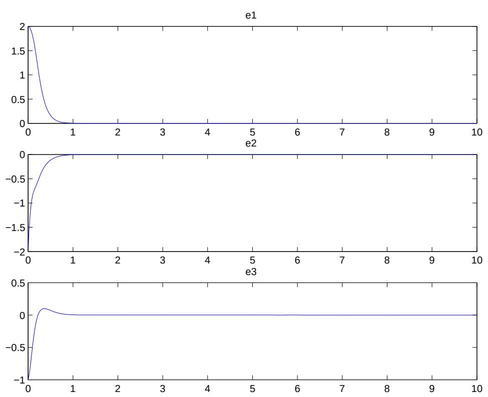

Fig. 2. Estimation error

5. CONCLUSION

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