Design of Sliding Mode Unknown Input Observer for Uncertain Takagi-Sugeno Model

A. Akhenak*, M. Chadli†, J. Ragot‡, D. Maquin‡

*Ecole des Mines de Douai, Département Informatique et Automatique
941, rue Charles Bourseul, B.P. 838, 59508 Douai Cedex, France

†Centre de Robotique, d'Electrotechnique et d'Automatique
7, Rue du Moulin Neuf, 80000 Amiens, France

‡Centre de Recherche en Automatique de Nancy, UMR 7039 Nancy-University, CNRS
2, Avenue de la forêt de Haye, 54516 Vandoeuvre, France

***Abstract*— This paper addresses the analysis and design of a sliding mode observer on the basis of a Takagi-Sugeno (T-S) model subject both to unknown inputs and uncertainties. The main contribution of the paper is the development of a robust observer with respect to the uncertainties as well as the synthesis of sufficient stability conditions of this observer. The stabilization of the observer is performed by the search of suitable Lyapunov matrices. It is shown how to determine the gains of the local observers, these gains being solutions of a set of linear matrix inequalities (LMI). The validity of the proposed methodology is illustrated by an academic example.**

I. Introduction

State estimation of linear time-invariant dynamical system driven by both known and unknown inputs has been the subject of many research works [1], [2], [3]. Indeed, in practice, there are many situations where some of the inputs to the system are inaccessible. The recourse to the use of an unknown input observer is then necessary in order to be able to estimate the state of the considered system. This state estimate can be useful either for designing a control law and/or for supervision task. Indeed, in the context of instrument fault detection and isolation, most actuator failures can be generally modelled as unknown inputs to the system [2].

In parallel, sliding mode observers (SMO) have received large attention since it offers robustness properties with regard uncertainties [4], [5], [6]. Using an additive nonlinear discontinuous term, SMO constraints the trajectory of the estimation error to remain on a specific surface after finite time such that error is completely insensitive to the disturbances. This interesting property has been utilized either for state estimation [7], [8], [9] and fault detection and isolation [10].

All that relates to the linear models was largely developed in the literature. However, this assumption of linearity is checked only in a limited vicinity of a particular operating point. The T-S model approach can apprehend the nonlinear behaviour of a system, while keeping the simplicity of the linear models.

Indeed, the real physical systems are often nonlinear. As it is delicate to synthesize an observer for an unspecified nonlinear system, it is preferable to represent this system with a T-S model. The idea of the T-S model approach is to apprehend the global behaviour of a system by a set of local models (linear or affine), each local model characterizing the behaviour of the system in a particular zone of operation. The local models are then aggregated by means of an interpolation mechanism. This approach has been extensively considered (see among others [11], [12], [13], [14] and references therein).

Since the study of stability of the T-S models [15], [16], [17], [18], the researchers accentuated their work on the T-S observer design [13], [19], [20], [21]. Tanaka *et al.* [13], Ma *et al.* [19] and Yoneyama *et al.* [22] studied observer design for T-S fuzzy control systems, and they proved that a state feedback controller and an observer always yields a stabilizing output feedback controller provided that the stabilizing property of the control and asymptotic convergence of the observer are guaranteed by the Lyapunov method.

In recent papers, Park *et al.* [23] have presented the design of a robust adaptive fuzzy observer for uncertain nonlinear dynamic systems. The Lyapunov synthesis approach is used to guarantee a uniformly ultimately bounded property of the state observation error, as well as of all other signals in the closed-loop system. Tong *et al.* [24] have studied the robust fuzzy control problem for fuzzy model system in the presence of parametric uncertainties and unavailable state variables. A state observer was designed and sufficient conditions derived for robust stabilization in the sense of Lyapunov asymptotic stability. They are formulated using linear matrix inequalities (LMIs).

Recently, the notion of the sliding mode was introduced into the fuzzy observer synthesis. In [25] a sliding mode observer for Takagi-Sugeno models with matched and unmatched uncertainties is designed. Bergsten *et al.* [20] designed a sliding mode observer for a Takagi-Sugeno model in the difficult case when the weighting function depend on the estimated state. Chang *et al.* [26] present the design of a new type of fuzzy controller for complex single-input single-output systems. That paper presents a systematic design procedure of fuzzy model-based controllers with guaranteed stability and improved tracking performances.

In this paper, the problem of the state estimation of an uncertain Takagi-Sugeno model subjected to the influence of the unknown inputs is addressed. The main contribution of this paper is the development of a robust sliding mode observer with the presence of unknown inputs and parametric uncertainties. New convergence conditions of the sliding mode observer are established while being based on the work presented in [27]. By using a quadratic Lyapunov function, the convergence conditions are expressed in the form of a set of linear matrix inequalities (LMI) [28].

The paper is organized as follows. In Section II, the general structure of the considered uncertain Takagi-Sugeno model is presented. In Section III, the proposed structure of a sliding mode observer is described and the main results are presented. The derived conditions ensuring the global asymptotic convergence of the estimation error are given as a set of LMI terms. The last section gives a numerical example to illustrate the effectiveness of the proposed approach.

Notation: Throughout the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ means that X is a symmetric positive definite matrix, $\mathbb{I}_M = \{1, 2, ..., M\}$ and $\|.\|$ represents the Euclidean norm for vectors and the spectral norm for matrices.

II. TAKAGI-SUGENO MODEL REPRESENTATION

Many physical systems are very complex in practice so that rigorous mathematical model can be very difficult to obtain, if not impossible. However, their nonlinear behaviour can always be captured in a limited vicinity of particular operating points by linear models. So the global model describing such systems in all of its functioning range can be expressed on the basis of such linear local models. It is however necessary to ensure the connexion of these models. This modelling technique is known as multiple model approach [29]. In this context, Takagi and Sugeno have proposed a model to describe complex systems [11]. Here, the following uncertain dynamic model representing a complex nonlinear system with unknown inputs is considered:

$$\begin{cases} \dot{x} = \sum_{i=1}^{M} \mu_i \Big((A_i + \Delta A_i) x + B_i u + R_i \bar{u} \Big) \\ y = \sum_{i=1}^{M} \mu_i C_i x \end{cases} \tag{1}$$

with $\sum_{i=1}^{M} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall i \in \mathbb{I}_M$, where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ the input vector, $\bar{u}(t) \in \mathbf{R}^q$, $q < n$, contains the unknown inputs and $y(t) \in \mathbf{R}^p$ the measured outputs. Matrices $A_i \in \mathbf{R}^{n \times n}$ and $B_i \in \mathbf{R}^{n \times m}$ denote the state matrix and the input matrix associated with the ith local model. Matrices $R_i \in \mathbf{R}^{n \times q}$ are the distribution matrices of unknown inputs. At last, $\xi(t)$ is the so-called decision vector which may depend on some subset of the known inputs and/or measured variables to define the operating regimes.

The matrices $\Delta A_i(t)$ are unknown time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the plant model. This kind of uncertainties is known as unmatched uncertainties. The unknown input $\bar{u}(t)$ is also assumed to be bounded.

$$\|\Delta A_i(t)\| \leq \delta_i \tag{2a}$$

$$\|\bar{u}(t)\| \leq \rho \tag{2b}$$

When $\mu_i(\xi(t)) = 1$, which implies $\mu_j(\xi(t)) = 0$, $\forall\, j \neq i$, model i is active. In fact, the value of the functions $\mu_i(\xi(t))$ are not Boolean and the state of the multiple model can be viewed as a weighted sum of the "local states". Notice however that, as this explanation helps to understand the structure of the considered model, it is not really exact as only one state vector exists: those of the multiple model, and local states don't really exist. When membership functions $\mu_i(\xi(t))$ are not Boolean ones, several local models are active at each time and the coefficients $\mu_i(\xi(t))\ i \in \mathbb{I}_M\}$ quantify the relative contribution of each local model to the global model. The choice of the number M of local models describing the multiple model may be intuitively done by taking into account a certain number of operating regimes. Matrices A_i, B_i, R_i and C_i can be obtained by using the direct linearization of an a priori nonlinear model around operating points, or alternatively by using an identification procedure [30], [31], [32]. From a practical point of view, matrices A_i, B_i, R_i and C_i describe the system's local behaviour around the ith regime.

III. SLIDING MODE OBSERVER

This section is dedicated to the state estimation of the model (1). The proposed sliding mode unknown input fuzzy observer (SMUIFO) is based on a nonlinear combination of local unknown input observers involving sliding terms allowing to compensate the uncertainties and the unknown inputs. The proposed sliding mode observer of the Takagi-Sugeno model has the following form:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{M} \mu_i \Big(A_i \hat{x} + B_i u + G_i \left(y - C \hat{x} \right) + \nu_i + \alpha_i \Big) \\ \hat{y} = \sum_{i=1}^{M} \mu_i C_i \hat{x} \end{cases} \tag{3}$$

The aim of the design is to determine gain matrices G_i and variables $\nu_i(t) \in \mathbf{R}^n$ and $\alpha_i(t) \in \mathbf{R}^n$, that guarantee the asymptotic convergence of $\hat{x}(t)$ towards $x(t)$. Let us note that $\nu_i(t)$ and $\alpha_i(t)$ can be considered as variables which compensate respectively the errors due to the unknown inputs and the model uncertainties. Their specific structures will be described further.

A. Stability conditions

In order to establish the conditions for the asymptotic convergence of the observer (3), let us define the state and output estimation errors:

$$e = x - \hat{x} \tag{4}$$

$$r = y - \hat{y} = \sum_{i=1}^{M} \mu_i C_i e \tag{5}$$

The dynamic of the state estimation error can be evaluated using the equations (1) and (3):

$$\dot{e} = \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \Big(\bar{A}_{ij} e + \Delta A_i x + R_i \bar{u} - \nu_i - \alpha_i \Big) \tag{6}$$

with:

$$\bar{A}_{ij} = A_i - G_i C_j \tag{7}$$

Theorem 1: the state of the observer (3) converges globally asymptotically to the state of the Takagi-Sugeno model (1), if there exists a symmetric positive definite matrix $P \in \mathbf{R}^{n \times n}$, matrices $W_i \in \mathbf{R}^{n \times p}$ and positive scalars β_1, β_2 and β_3 checking the following conditions for all $i, j \in \mathbb{I}_M$:

$$\begin{bmatrix} A_i^T P + P A_i - C_j^T W_i^T - W_i C_j + \gamma I & P \\ P & -\beta_1 I \end{bmatrix} < 0 \tag{8}$$

with $\gamma = \beta_2 \delta_i^2 + \beta_3$.

The gains G_i and the terms $\nu_i(t)$ and $\alpha_i(t)$ of the observer (3) are given by the following equations:

$$\begin{array}{l} \text{If } r \neq 0 \left\{ \begin{array}{l} \nu_i = \rho^2 \beta_3^{-1} \dfrac{\|PR_i\|^2}{2\, r^T r} P^{-1} \displaystyle\sum_{j=1}^{M} \mu_j C_j^T r \\ \alpha_i = \beta_1 \left(1 + \beta_4\right) \delta_i^2 \dfrac{\hat{x}^T \hat{x}}{2\, r^T r} P^{-1} \displaystyle\sum_{j=1}^{M} \mu_j C_j^T r \end{array} \right. \\ \text{If } r = 0 \left\{ \begin{array}{l} \nu_i = 0 \\ \alpha_i = 0 \end{array} \right. \end{array} \tag{9}$$

with: $\beta_4 = \frac{\beta_1}{\beta_2 - \beta_1}$

$$G_i = P^{-1} W_i. \tag{10}$$

The proof of the asymptotic convergence of this observer (3) rests on the following lemma 1.

Lemma 1: for any matrices X and Y with appropriate dimensions, the following property holds for any positive scalar β:

$$X^T Y + Y^T X \leq \beta X^T X + \beta^{-1} Y^T Y$$

Proof: in order to demonstrate the asymptotic convergence of the observer (3), let us consider the following Lyapunov quadratic function:

$$V = e^T P e \tag{11}$$

Using equations (4) and (6), the derivative $\dot{V}(e(t))$ along the trajectory of the system is given by:

$$\dot{V} = \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j \Big(e^T (\bar{A}_{ij}^T P + P \bar{A}_{ij}) e + x^T \Delta A_i^T P e + e^T P \Delta A_i x - 2\alpha_i^T P e + 2 e^T P R_i \bar{u} - 2 e^T P \nu_i \Big) \tag{12}$$

Lemma 1 allows to write:

$$\dot{V} \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j \Big(e^T (\bar{A}_{ij}^T P + P \bar{A}_{ij}) e + \beta_1^{-1} e^T P^2 e + \beta_1 x^T \Delta A_i^T \Delta A_i x - 2\alpha_i^T P e + 2 e^T P R_i \bar{u} - 2 e^T P \nu_i \Big) \tag{13}$$

Using the expression of the state estimation error (4), the inequality (13) becomes:

$$\dot{V} \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j \Big(e^T (\bar{A}_{ij}^T P + P \bar{A}_{ij} + \beta_1^{-1} P^2) e + 2 e^T P R_i \bar{u} + \beta_1 \delta_i^2 (\hat{x} + e)^T (\hat{x} + e) - 2\alpha_i^T P e - 2 e^T P \nu_i \Big)$$

$$\dot{V} \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j \Big(e^T (\bar{A}_{ij}^T P + P \bar{A}_{ij} + \beta_1^{-1} P^2) e + 2 e^T P R_i \bar{u} + \beta_1 \delta_i^2 \big(\hat{x}^T \hat{x} + e^T e\big) + \beta_1 \delta_i^2 \big(\hat{x}^T e + e^T \hat{x}\big) - 2\alpha_i^T P e - 2 e^T P \nu_i \Big)$$

Using again lemma (1), this last expression can be rewritten as follows:

$$\dot{V} \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j \Big(e^T \big(\bar{A}_{ij}^T P + P \bar{A}_{ij} + \beta_1^{-1} P^2 + \beta_2 \delta_i^2 I\big) e + \beta_1 (1 + \beta_4) \delta_i^2 \hat{x}^T \hat{x} - 2\alpha_i^T P e + 2 e^T P R_i \bar{u} - 2 e^T P \nu_i \Big)$$

with $\beta_2 = \beta_1 (1 + \beta_4^{-1})$.

Two cases can therefore be distinguished according to the value of the output residual:

Case 1: $r \neq 0$.

In this case, it is easy to notice from relation (9) that:

$$\begin{aligned} 2\alpha_i^T P e &= \beta_1 (1 + \beta_4) \delta_i^2 \frac{\hat{x}^T \hat{x}}{r^T r} r^T \sum_{j=1}^{M} \mu_j C_j P^{-1} P e \\ 2\alpha_i^T P e &= \beta_1 (1 + \beta_4) \delta_i^2 \frac{\hat{x}^T \hat{x}}{r^T r} r^T r \\ 2\alpha_i^T P e &= \beta_1 (1 + \beta_4) \delta_i^2 \hat{x}^T \hat{x} \end{aligned} \tag{14}$$

$$\begin{aligned} 2 e^T P R_i \bar{u} &= e^T P R_i \bar{u} + \bar{u}^T R_i^T P e \\ 2 e^T P R_i \bar{u} &\leq \beta_3 e^T e + \beta_3^{-1} \|P R_i \bar{u}\|^2 \\ 2 e^T P R_i \bar{u} &\leq \beta_3 e^T e + \rho^2 \beta_3^{-1} \|P R_i\|^2 \end{aligned} \tag{15}$$

$$\begin{aligned} 2 e^T P \nu_i &= \rho^2 \beta_3^{-1} \frac{\|P R_i\|^2}{r^T r} e^T P P^{-1} \sum_{j=1}^{M} \mu_j C_j^T r \\ 2 e^T P \nu_i &= \rho^2 \beta_3^{-1} \|P R_i\|^2 \end{aligned} \tag{16}$$

From (14), (15) and (16) one can easily deduce:

$$\dot{V} \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i \mu_j e^T U_{ij} e \tag{17}$$

with: $U_{ij} = \bar{A}_{ij}^T P + P \bar{A}_{ij} + \beta_1^{-1} P^2 + \beta_2 \delta_i^2 I + \beta_3 I$.

Case 2: $r(t) = 0$.

In the general case, the error $e(t)$ is not orthogonal to the term $\sum_{i=1}^{M} \mu_i(\xi(t)) C_i$, therefore the nullity of $r(t)$ implies that of $e(t)$. In this case, from the equations (11) and (12), one can easily notice that the Lyapunov function and its derivative are null. In the particular case where, for some t, the error $e(t)$ is orthogonal to the term $\sum_{i=1}^{M} \mu_i(\xi(t)) C_i$, we cannot conclude about the negativity of the derivative of the Lyapunov function. However, clearly, this situation is necessary "instantaneous" and cannot last a long time as $e(t)$ evolves. Therefore, that case has no impact on the proposed analysis.

The analysis of these two cases has shown that the derivative of the considered Lyapunov function is systematically negative if the following inequalities hold:

$$(A_i - G_iC_j)^T P + P(A_i - G_iC_j) + \beta_1^{-1} P^2 + \beta_2 \delta_i^2 I + \beta_3 I < 0$$

Notice that these latter are nonlinear in P, G_i, β_1. To linearize them and to obtain the constraints (8), one can proceed to the following change of variable:

$$W_i = PG_i \tag{18}$$

After that, the use of the Schur complement [28] leads to the obtention of linear matrix inequalities in P, W_i, β_1, β_2 and β_3 that can be easily solved by the means of LMI methods.

If equation (8) holds, the RHS of inequality (17) is clearly negative and the asymptotic convergence of the observer (3) is guaranteed. In conclusion, the state estimation error converges asymptotically towards zero, if the conditions (9) and the inequalities (8) are checked.

B. Relaxed stability conditions

In order to reduce the conservatism of the inequalities (8), the result proposed in [18] is exploited.

Theorem 2: the state estimation error between the model (1) and the sliding mode observer (3) converges globally asymptotically towards zero, if there exists symmetric positive definite matrices P and Q_{ii}, matrices Q_{ij} and positive scalars β_1, β_2 and β_3 satisfying the following inequalities for all $i, j \in \mathbb{I}_M$:

$$\begin{cases} \begin{bmatrix} A_i^T P + PA_i - C_i^T W_i^T - W_iC_i + Q_{ii} + \gamma I & P \\ P & -\beta_1 I \end{bmatrix} < 0 \\ \begin{bmatrix} T + \gamma I & P \\ P & -\beta_1 I \end{bmatrix} < 0 \\ \begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1M} \\ Q_{12}^T & Q_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1M}^T & \cdots & \cdots & Q_{MM} \end{pmatrix} > 0 \end{cases} \tag{19}$$

with:

$$\begin{aligned} T &= T_1 - T_2 + T_3 \\ T_1 &= \tfrac{(A_i + A_j)^T}{2} P + P \tfrac{(A_i + A_j)}{2} \\ T_2 &= C_j^T W_i^T + W_iC_j + C_i^T W_j^T + W_jC_i \\ T_3 &= Q_{ij} + Q_{ij}^T \\ \gamma &= \beta_2 \delta_i^2 + \beta_3 \end{aligned}$$

The gains G_i and the terms $\nu_i(t)$ and $\alpha_i(t)$ of the observer (3) are already given by the equations (9) and (10).

Proof: the proof of this theorem is performed following the same steps as in theorem 1 and exploiting the result developed in [18].

It should be noted that the introduction of matrices Q_{ij} leads to relaxed stability conditions. These matrices are not necessary positive definite. It is then possible to relax the constraints using the cross terms ($i \neq j$). Let us note that these matrices are not also necessary symmetrical and this fact constitutes additional degrees of freedom, in comparison with what was done in [33].

IV. Simulation example

Consider the model (20), made up of two local models and involving two outputs and three states. The output vector of the model $y(t)$ is a nonlinear combination of the states.

$$\begin{cases} \dot{x} = \sum_{i=1}^{2} \mu_i \Big((A_i + \Delta A_i)x + B_iu + R_i\bar{u} \Big) \\ y = \sum_{i=1}^{2} \mu_i C_i x \end{cases} \tag{20}$$

Here, the membership functions depend on the input of the system. The numerical values of the matrices A_i, B_i, C_i and R_i are as follows:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -6 \end{bmatrix} \quad A_2 = \begin{bmatrix} -3 & 2 & 2 \\ 5 & -8 & 0 \\ 0.5 & 0.5 & -4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 0.25 \end{bmatrix} \quad R_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1 \\ 0.5 \\ 2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The model uncertainties are such that:

$$\Delta A_{i,(j,k)}(t) = \theta A_{i,(j,k)} \eta(t) \quad j, k \in \{1, 3\} \text{ and } i \in \{1, 2\}$$

where $A_{i,(j,k)}$ denotes the (j,k)th element of A_i and $\theta = 0.2$. The function $\eta(t)$ is a piece-wise constant function which magnitude is uniformly distributed on the interval $[0 \;\; 1]$. Its time evolution is depicted on Fig. 1.

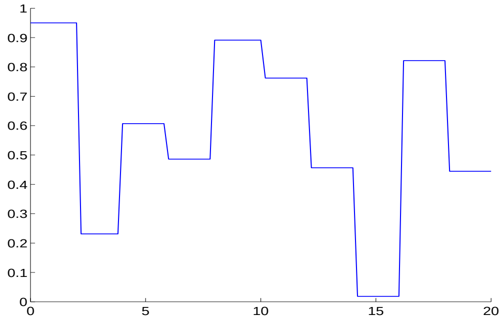

Fig. 1. The piece-wise constant function $\eta(t)$

The considered sliding mode observer that estimates the state vector of the uncertain model is described by:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{2} \mu_i \Big(A_i \hat{x}(t) + B_i u(t) + G_i \left(y - \hat{y} \right) + \nu_i + \alpha_i \Big) \\ \hat{y} = \sum_{i=1}^{2} \mu_i C_i \hat{x} \end{cases} \tag{21}$$

It is important to note that the implementation of this sliding mode observer induces a practical problem: when the estimation error $r(t)$ tends towards zero, the magnitude of $\nu_i(t)$ and $\alpha_i(t)$ may increase without bound. This problem is overcame as follows:

$$\text{If } \|r\| \geq \varepsilon \begin{cases} \nu_i = \rho^2 \beta_3^{-1} \dfrac{\|PR_i\|^2}{2r^T r} P^{-1} \sum_{j=1}^{2} \mu_j C_j^T r \\ \alpha_i = \beta_1 \left(1 + \beta_4\right) \delta_i^2 \dfrac{\hat{x}^T \hat{x}}{2\ r^T r} P^{-1} \sum_{j=1}^{2} \mu_j C_j^T r \end{cases}$$

$$\text{If } \|r\| < \varepsilon \begin{cases} \nu_i = 0 \\ \alpha_i = 0 \end{cases}$$

The terms $\nu_i(t)$ and $\alpha_i(t)$ are fixed to zero when the output estimation error is such that $\|r(t)\| \leq \varepsilon$, where ε is a threshold chosen by the user. In this case, the estimation error cannot converge to zero asymptotically but to a small neighborhood of zero depending on the choice of ε. For this example, ε is chosen equal to 10^{-3}.

The resolution of inequalities (19), using classical LMI solver, leads to the following matrices G_i, P, Q_{11}, Q_{12} and Q_{22}:

$$G_1 = \begin{bmatrix} 0.55 & 2.18 \\ 1.58 & -0.67 \\ 0.18 & -0.93 \end{bmatrix} \quad G_2 = \begin{bmatrix} 2.62 & 1.04 \\ -1.34 & 1.29 \\ 2.22 & -2.19 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Q_{11} = \begin{bmatrix} 1.66 & -0.13 & -0.44 \\ -0.13 & 1.44 & -0.12 \\ -0.44 & -0.12 & 2.16 \end{bmatrix}$$

$$Q_{22} = \begin{bmatrix} 1.87 & -0.27 & 0.18 \\ -0.27 & 1.66 & 0.12 \\ 0.18 & 0.12 & 1.39 \end{bmatrix}$$

$$Q_{12} = \begin{bmatrix} 1.16 & 0.14 & 0.22 \\ 0.34 & 0.17 & 0.66 \\ -0.33 & 0.87 & -0.69 \end{bmatrix}$$

Remark: Let us note that for $\theta = 0.2$, the conditions (8) of theorem 1 (and the developed conditions in [27]) fail to synthesize a sliding mode observer for the model (20). This fact illustrates the real contribution of the relaxed stability conditions.

The system (20) was simulated using the known and unknown inputs depicted in Fig. 2 and 3. Fig. 4 shows the form of the membership functions $\mu_1(u(t))$ and $\mu_2(u(t))$.

Fig. 5 presents the comparison between one of the state of the model and its estimate from the sliding mode observer. The two layouts are superimposed except in the vicinity of the origin (due to the choice of the initial conditions of the multiple observer).

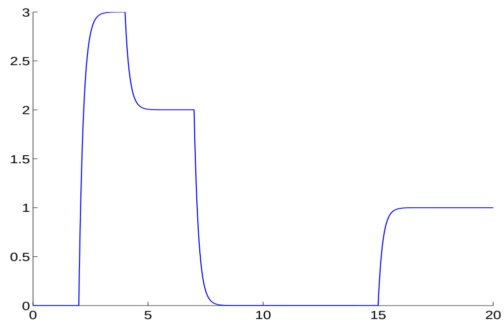

Fig. 2. Known input $u(t)$

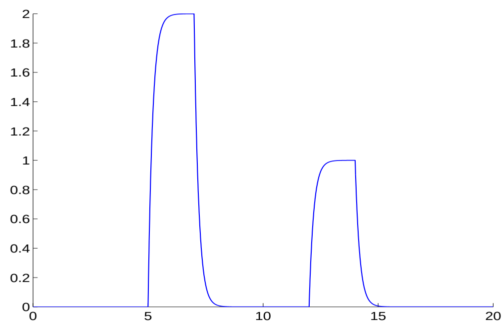

Fig. 3. Unnown input $\bar{u}(t)$

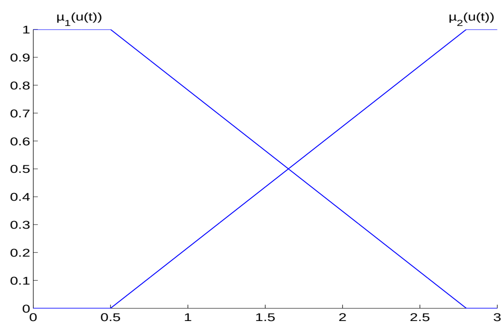

Fig. 4. Membership functions

V. Conclusion

In this paper, based on a Takagi-Sugeno uncertain model representation, the design of a sliding mode observer using the principle of interpolation of several local observers has been proposed in the case where some inputs of the system are unknown. The stability of the

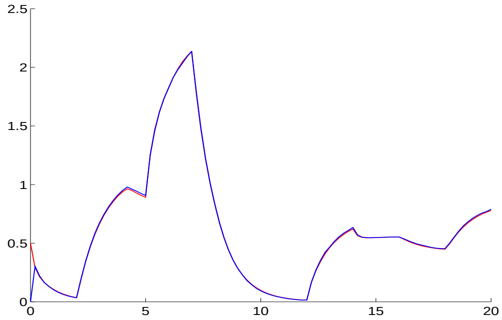

Fig. 5. $x_3(t)$ and its estimate $\hat{x}_3(t)$

observer requires the consideration of coupling constraints between these local observers; these constraints lead to the resolution of a LMI problem. The calculation of the gains of the observer is then returned to a simultaneous calculation of the gains of the local observers. Assuming the existence of suitable matrices, we showed that the reconstruction of the state vector of the uncertain Takagi-Sugeno model is possible. The stability conditions (convergence) are relaxed by using the results obtained in [18]. An academic example illustrates the effectiveness of the derived conditions.

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