Paper:

Estimation of Bounded Model Uncertainties

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We identify parameters of a given input-output model so that estimated model output is consistent with the measured output of the system modeled. Parameter estimation based on a set-membership approach is a nonprobabilistic method for characterizing the uncertainty with which each model parameter is known. The model is consistent with data if the estimated output domain contains measured system output at each instant. Dynamic linear Multi-Input Multi-Output (MIMO) models are considered in this paper. Every equation error is bounded while model parameters fluctuate within a time-invariant domain represented by a zonotope. Our proposal helps find the characteristics of this domain, e.g., center, shape, size, by taking into account coupling between bounded variables of output equations to increase model accuracy.

Keywords: uncertain model, interval analysis, estimation theory

1. Introduction

This paper focuses on set-membership parameter estimation that calculates the bounds of uncertain variables in a model; so that this model explains a given dataset (consisting of measurements of system input and output). This model is affected by uncertainties in both additive equation error and multiplicative model parameters. Equation error is assumed to belong to an axis-aligned orthotope (a box) while model parameters fluctuate within a time-invariant bounded domain. The objective of this paper is to characterize this domain through the study of a class of structured Multi-Input Multi-Output (MIMO) models linear with uncertain parameters.

All coupling between model input-output equations due to bounded variables, called common variables, appearing simultaneously in several relations (Adrot, 2000 [1]; Adrot et al., 2000 [2]) are taken into account to improve model accuracy. This additional information enable us to work on the exact time-variant set containing model output (zonotope) instead of an overestimation (orthotope). Our proposal calculates different weights applied to coupling and adjusts the shape of the parameter value set, which is not fixed a priori.

Section 2 explains the context of this work and the principle of characterization. Section 3 discusses problem formulation for a model consisting of several input-output relations and details a solution based on the generation of strip constraints due to the elimination of common variables. Section 4 gives two examples of our proposal. Section 5 presents conclusions.

2. Background

2.1. Set-Membership Modeling

Model-based approaches developed in Automatic Control use mathematical representations consisting of analytical relations. Unfortunately, a model is only an approximation of a physical system. This inaccuracy is caused by modeling errors when a model is linearized, when physical phenomena are not taken into account, or infinitesimal precision cannot be achieved (noise, technological accuracy). To guarantee results obtained using a model, this model must describe all possible behavior of the physical system. This can be done by taking modeling errors into account.

Model inaccuracy can be modeled by parameter uncertainties. Conventionally, uncertainties are probabilistically described and represented as additive random noise with given probability density. In practice, the randomness assumption may be unrealistic, particularly for very simplified models, leading to important structural errors. If statistical hypotheses about parameters are made, the problem becomes very complex because of mathematical operations on probability densities, that do not conserve distribution laws, i.e., the product of two Gaussian laws is not Gaussian. Using parameters, whose support is infinite, may be problematic.

In this paper, we consider each uncertain parameter to be a bounded variable, i.e., an unknown parameter whose support is an interval. Because no probabilistic assumptions are made, a more important class of models can be used, where uncertainties have a multiplicative structure and affect any model parameter.

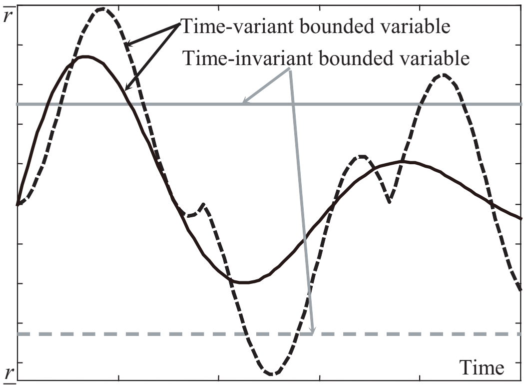

Fig. 1. Time-variant and time-invariant bounded variables.

2.2. Set-Membership Parameter Estimation

Set-membership parameter estimation started in the 80s and was originally designed to deal with a discrete-time model characterized by unknown but bounded equation error. This problem amounts to the determination of sets of parameter values called Feasible Parameter Sets (FPSs). For each point in this domain, the model is consistent with all available observations; i.e., each point is consistent with dataset, bounds of equation error (chosen a priori), and model structure.

Uncertain linear models lead to a theoretic FPS in the form of a convex polytope. The objective consists in circumscribing this domain by a simpler form as an ellipsoid (Fogel et al., 1982 [7]) or an axis-aligned orthotope (Milanese et al., 1982 [9]). Later, the objective consists in exactly determining the FPS by working on polytopes (Piet-Lahanier et al., 1990 [14]), (Mo et al., 1990 [11]). The next step treats bounded noise in model output (Clément et al., 1988 [6]) and on both sensor observations and equation error (Cerone, 1993 [5]). Set-membership parameter estimation can be viewed as a set-membership inversion problem too (Jaulin et al., 1993 [8]). Paving method is used to calculate overestimation of the FPS. Some of above works is combined in (Milanese et al., 1996 [10]).

As is explained later, the problem considered in this paper is different. Constant model parameters do not generally lead to the most accurate model in the sense of the proposed criterion. An uncertain parameter can be considered to be time-variant in some cases. Consider, for example, an electrical resistance r depending on temperature. The value of r is not always the same and r may be viewed as a time-variant parameter described by bounded variable $r \in [\underline{r}, \overline{r}]$, where $\underline{r}$ and $\overline{r}$ define its lower and upper bounds (**Fig.1**).

Another objective may be to represent a nonlinear phenomenon by a linear set-membership relation to simplify model equations. Function $y = f(x)$, for example, may be represented by the grey domain in **Fig.2** defined by relation $y = (1+\rho\theta)x$, where ρ is a chosen weight and θ a normalized bounded variable: $\theta \in [-1, 1]$. The unknown value of uncertain parameter θ should vary in its constant support to describe y.

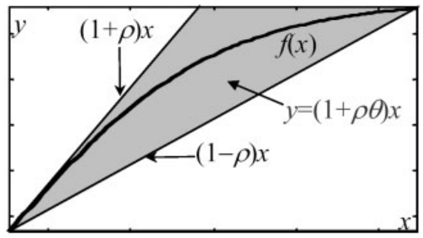

Fig. 2. Set-membership linearization.

Another problem consists in modeling uncertainties under initial conditions intervening on a state representation used for trajectory calculation. These uncertainties may come from sensor imprecision or nonobservability of some of physical variables of the monitored system and can be considered to be time-variant variables, especially with noise.

Uncertain parameters may be time-variant, so they are defined by random variables with bounded realizations, the objective being to characterize the time-invariant domain in which they fluctuate. We propose a nonprobabilistic technique for determining inaccuracy with which each model parameter is known. If $\boldsymbol{\theta}$ is a bounded vector, then $\mathrm{S}(\boldsymbol{\theta})$ designs the value set of $\boldsymbol{\theta}$ corresponding to the set of all admissible values of $\boldsymbol{\theta}$.

Only structured models linear with uncertain parameters and measurements (topped with a tilde) are considered. The term "structured" indicates that uncertain parameters are localized in mathematical model equations. At time k, they are represented by generic bounded vector $\boldsymbol{\theta}_k$. This notation requires some explanation because the studied model is dynamic, so parameter v^i may appear with time delays in model equations. Vector $\boldsymbol{\theta}_k$ may contain the same uncertain model parameter v^i expressed at different times $(k, k-1, \ldots)$:

$$\begin{aligned}\boldsymbol{\theta}_k &= [\theta_{k,1} \cdots \theta_{k,p}]^T \\ &= [\cdots\ v_k^i \cdots v_{k-1}^i \cdots v_{k-s_i}^i\ \cdots]^T,\ s_i \in \mathbb{N} \quad . \quad . \quad (1)\end{aligned}$$

where $\theta_{k,i}$ is the i^{th} component of $\boldsymbol{\theta}_k$ and v_k^i denotes parameter v^i at time k.

The following class of dynamic input-output models is considered:

$$\boldsymbol{y}_k = \tilde{\boldsymbol{X}}_k \boldsymbol{\theta}_k + \boldsymbol{\varepsilon}_k$$

$$\begin{bmatrix} \vdots \\ y_{k,i} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \tilde{\boldsymbol{x}}_{k,i}^T \\ \vdots \end{bmatrix} \boldsymbol{\theta}_k + \begin{bmatrix} \vdots \\ \varepsilon_{k,i} \\ \vdots \end{bmatrix} \quad . \quad . \quad . \quad . \quad (2)$$

$$\begin{aligned}&\boldsymbol{y}_k \in \mathbb{R}^m,\ y_{k,i} \in \mathbb{R} \\ &\tilde{\boldsymbol{X}}_k \in \mathbb{R}^{m \times p},\ \tilde{\boldsymbol{x}}_{k,i} \in \mathbb{R}^p \\ &\boldsymbol{\theta}_k \in \mathbb{R}^p, \\ &\boldsymbol{\varepsilon}_k \in \mathbb{R}^m,\ \varepsilon_{k,i} \in \mathbb{R}\,.\end{aligned}$$

Vector $\boldsymbol{y}_k$ denotes model output obtained at time k, $\forall k \in \{1, \ldots, h\}$. Scalar $\varepsilon_{k,i}$ defines i^{th} independent equation error and corresponds to a bounded variable comprised between $-\delta_{\varepsilon,i}$ and $\delta_{\varepsilon,i}$. Bound $\delta_{\varepsilon,i} \in \mathbb{R}^+$ is assumed to be a