DATA RECONCILIATION USING INTERVAL ANALYSIS

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Abstract: A new method of data reconciliation for linear static systems which is able to take into account the knowledge about the uncertainties of the used model is proposed. The uncertainties are represented by bounded variables. The elementary operations relating to the intervals make it possible to propose a state estimation of the system taking into account the uncertainties without a priori knowledge on their statistic laws of distribution. The estimation result is also provided in an interval form.

Résumé: On propose une nouvelle méthode de réconciliation de données de systèmes statiques linéaires capable de prendre en compte les incertitudes paramétriques du modèle utilisé. Les incertitudes sont représentées par des variables bornées. Des opérations élémentaires portant sur les intervalles permettent de proposer une estimation de l'état du système prenant en compte ces incertitudes et sans connaissance a priori sur leurs lois de distribution statistique. Le résultat de l'estimation est également fourni sous une forme intervalle.

Key Words: Data reconciliation, uncertain models, bounded approach, interval analysis, fault detection and isolation, analytic redundancy.

Mots clés: Validation de données, modèles incertain, approche bornante, analyse par intervalles, détection et localisation de défauts, redondance analytique.

1 – Introduction

The problem of obtaining reliable estimates of process variables from measurements (data validation or reconciliation) and the related problem of detecting and isolating gross errors has been well studied. Historically speaking, likely due to measurement availability, static redundancy equations have been first utilised in the mineral processing and the chemical industries. The first studies (Ripps, 1962, Vaclavek, 1969, Smith et al., 1973) were concerned with data reconciliation using the now classical technique of equilibration of production's balances. In the following stages this data reconciliation principle has been generalised to processes which are described by algebraic equations either linear in the case of total flowrates (Crowe et al., 1983) or non linear in the case of chemical concentrations (Sood et al., 1979, Crowe, 1986).

At the same time, data reconciliation went into use for more general applications than establishing statistically coherent balances. It was then applied to more fundamental problems such as: detection, localisation and estimation of gross errors (Narasimhan et al., 1989, Ragot et al., 1990), diagnosis and observability of systems (Kretsovalis et al., 1988), (Crowe, 1989), (Ragot et al., 1998), optimization of sensor locations (Maquin et al., 1987, 1996) and study of the reliability of a measurement system (Turbatte et al., 1991).

Data reconciliation methods are mainly based on the knowledge of a model which must describe, as well as possible, the behaviour of the concerned system. They exploit the analytic redundancy embedded in the mathematical description of the model by verifying the coherency between the actual measurements and some estimations issued from the model. This redundancy generally leads to a discrepancy between the equations and the data which have to be reconciled; so it provides a check on the reliability of a given set of measurements. At last, it seems relevant to validate and adjust the measurements taking into account the degree of precision of each measurement and key physical laws. Most of the methods used techniques based upon statistical considerations, where the noise affecting the records is often characterised by the mean and the covariance of an amplitude probability function. Maximising the likelihood function issued from this probability function allows to express the estimation of the true data to be expressed. A survey of the methods used in data reconciliation can be found in (Crowe, 1996).

As mentioned above, most previous investigators have used statistical criteria such as least squares as the criteria for calculating the best estimates of process variables. However, the results provided by such estimators are valid under the following restrictive conditions:

the nature of the measurement noise has to be known
the model of the process has to be perfectly known.

It can be a difficult task to estimate the validity of such assumptions and, in a certain number of applications, it is obvious that both conditions are not fully respected (Ploix, 1998), (Adrot 2000). Therefore, it becomes very hazardous and mathematically not correct to reconcile operation data with regard to an uncertain model without taking this fact into account. Some attempts in this direction have been already published (Mandel et al., 1998), (Maquin et al., 2000). An alternative method is proposed here which do not use any hypothesis about the noise distribution nor perfection of the used model. The only information needed about the noise or the model parameters is the value of the bounds of their interval variations. On an historical point of view, the paper of Himmelblau (1985) is probably one of the first which gives the formulation of such data rectification principles. This idea has been confirmed in (Harikumar et al., 1993) and in (Kyriakopoulos et al., 1996).

In this paper, a quite different approach is proposed. The uncertainties are represented by bounded variables and the all the treatments are based on interval analysis (Moore, 1979), (Neumaier, 1990). In order to simplify, the presentation essentially deals with static linear models of the following form:

$$\begin{aligned} &Ax = 0 \\ &x \in R^n, \quad A \in R^{m.n} \end{aligned} \tag{1}$$

The entries $A_{j,i}$ of the process constraint matrix A are interval variables $A_{j,i} = \left[\underline{A}_{j,i}, \quad \overline{A}_{j,i}\right]$ where $j = 1..m$, $i = 1..n$, $\underline{A}_{j,i}$ and $\overline{A}_{j,i}$ denote respectively the lower and upper bounds of the interval.

The measurement system is represented by the uncertain following model:

$$y_i = h_i(1 + a_i h_i)x_i + g_i(1 + b_i n_i) \qquad i = 1..n \tag{2}$$

The h_i and n_i variables are bounded and normalised, i.e. $|h_i| \le 1$ and $|n_i| \le 1$, a_i, b_i, h_i and g_i are positive constants. The term $h_i a_i h_i x_i$ represents the multiplicative error while $g_i(1 + b_i n_i)$ stands for additive error.

Based on the knowledge of the process model (1) and measurement system model (2) as well as the measurements y_i, $i = 1..n$, the main objective is to reconcile the measurements using a state estimation method based on interval algebra. In order to simplify the presentation, all the state variables of the process are assumed to be measured. If this is not the case, a preliminary observability analysis allows the

2 – Fault detection

Coherency study is based on residual analysis. These residuals are obtained by substituting to the true (but unknown !) state variables, in model equation (1), estimated values obtained from measurement equation (2). Indeed, from (2), one obtains:

$$\hat{x}_i = \frac{y_i - g_i(1+b_i n_i)}{h_i(1+a_i h_i)} \tag{3}$$

Remembering that n_i and h_i are normalised bounded variables, the *i*th estimation is such that:

$$\hat{x}_i \in \left[\frac{y_i - g_i(1+b_i)}{h_i(1+a_i)}, \quad \frac{y_i - g_i(1-b_i)}{h_i(1-a_i)} \right] = \left[\underline{\hat{x}}_i, \quad \overline{\hat{x}_i} \right] \qquad i = 1 \ldots n \tag{4}$$

The $\underline{\hat{x}}_i$ and $\overline{\hat{x}_i}$ variables are respectively upper and lower bounds of this *i*th interval estimation $\hat{x}_i$. Taking into account the model equation (1), residuals may now be defined as:

$$R_j = A_j \hat{x} \qquad \text{with } \hat{x} = \left[\underline{\hat{x}}, \quad \overline{\hat{x}} \right] \text{ and } j=1 \ldots m \tag{5}$$

In this last expression, A_j denotes the *j*th row of the *A* matrix. As the entries of *A* are also interval variables, a residual is defined by the scalar product of two interval variable vectors. The scalar product between an interval variable vector $\hat{x}$ and an ordinary vector (real value vector) $\underline{A}_j$ is defined as follows:

$$\underline{A}_j \hat{x} = \frac{1}{2} \underline{A}_j \left(\underline{\hat{x}} + \overline{\hat{x}} \right) + \frac{1}{2} \left| \underline{A}_j \right| \left[\underline{\hat{x}} - \overline{\hat{x}}, \quad \overline{\hat{x}} - \underline{\hat{x}} \right] \tag{6a}$$

The scalar product between $\hat{x}$ and $\overline{A}_j$ is defined as well :

$$\overline{A}_j \hat{x} = \frac{1}{2} \overline{A}_j \left(\underline{\hat{x}} + \overline{\hat{x}} \right) + \frac{1}{2} \left| \overline{A}_j \right| \left[\underline{\hat{x}} - \overline{\hat{x}}, \quad \overline{\hat{x}} - \underline{\hat{x}} \right] \tag{6b}$$

where $\underline{A}_j \in R$ is the row vector which entries are the lower bounds of the *j*th row and $\overline{A}_j \in R$ the upper bounds row vector. Generalising these expressions, the scalar product of the two interval variable vectors A_j and $\hat{x}$ that defines the R_j residual is then expressed as:

$$\begin{aligned} & R_j = \left[\underline{R}_j, \quad \overline{R}_j \right] \\ & \underline{R}_j = \min\left(\min(\underline{A}_j \hat{x}), \min(\overline{A}_j \hat{x}) \right), \ \overline{R}_j = \max\left(\max(\underline{A}_j \hat{x}), \max(\overline{A}_j \hat{x}) \right) \end{aligned} \tag{7}$$

In this expression $\underline{R}_j$ represents the lower bound of the R_j residual and $\overline{R}_j$ it's upper bound. Incoherences between the different measurements may now be highlighted through an analysis of these residuals. Indeed, an interval residual will be considered as « normal » if it contains the zero value. In that case, all the measurements intervening in the corresponding residual are assumed to be fault-free and coherent with the model. Otherwise, the residual is abnormal, and the measurements from which it is calculated are not coherent with the model.

3 – Fault isolation

Isolation of faults is based on the analysis of a classical fault signature matrix. Indeed, each redundancy equation (or a subset of them) represents a partial model of the system, which has been obtained either directly, if all the state variables are measured or by rewriting the original constraints so as to be expressed using only known variables. All these partial models are obviously "true" (then the corresponding residual is said "normal") when the system operates normally (since they described in fact

rows as residuals and as many columns as state variables. The number of residuals is not a priori known. Indeed, one can separate the so-called primary residuals corresponding to a set of independent equations (directly those describing the process constraints if all the state variables are measured) and linear combinations of these primary residuals which aim to eliminate one or several particular variables. The entries of this matrix are evaluated as follows. If a residual is "normal" (i.e. contains the zero value), respectively "abnormal", all the elements corresponding to the state variables intervening in it's expression are set to "1", respectively "0", the other elements are "undetermined".

Isolation of faulty measurements is therefore very simple: they are associated to the columns of the signature matrix that don't contain "1" elements. Indeed, the absence of "1", in a column corresponding to a particular state variable, means that this latter belongs systematically to abnormal residuals It is therefore natural to think that it is this particular variable that renders abnormal the residuals in which it intervenes. The example of section 5 will illustrate the proposed analysis.

4 – State estimation

Before precisely describing the state estimation process, let us formulate a remark and introduce some notations. Based on the analytic redundancy of the process model, several "local" estimations of a state variable may be proposed. In fact, a different interval estimation of a particular state variable may be evaluated from any process constraint in which it intervenes. To precise this idea, let us consider any constraint, say the *j*th, in which the *k*th variable intervenes. Let us also define the $B_j^{(k)}$ vector built from the *j*th row of process constraint matrix A_j such that it's entries are defined by:

$$\begin{aligned} B_{j,i}^{(k)} &= -\frac{A_{j,i}}{\left[\underline{A}_{j,k},\quad \overline{A}_{j,k}\right]} \quad \forall (i,j,k),\ i=1..n,\ i\neq k,\ A_{j,k}\neq 0 \\ B_{j,k}^{(k)} &= \left[\underline{B}_{j,k}^{(k)},\quad \overline{B}_{j,k}^{(k)}\right] = [0,\quad 0] \end{aligned} \tag{8}$$

Therefore, the different estimations $\hat{x}_{k,j}$ of the state variable x_k may be computed as follows:

$$\hat{x}_{k,j} = B_j^{(k)}\hat{x} \tag{9}$$

In the notation $\hat{x}_{k,j}$, the first subscript indicates the number of the estimated variable while the second one corresponds to the number of the constraint used for that estimation. On a practical point of view, these estimations are defined by a scalar product between two interval variable vectors, so the calculus principle previously described may be applied.

The state estimation process is then divided into two consecutive steps. The first one consists to "correct" the primary estimations (3) issued from the use of the measurement model if the corresponding measurement has been detected faulty. Let us consider that the *k*th measurement is faulty, the new estimation of this variable is obtained as the intersection of all the possible estimations $\hat{x}_{k,j}$:

$$\hat{x}_k = \bigcap_j \hat{x}_{k,j} \tag{10}$$

At this step, all the estimations $\hat{x}_i$, $i=1..n$, are coherent with the process model ; all the residuals evaluated with these primary or corrected estimations contain the zero value. However, it is still possible to enhance them by using more completely the analytical redundancy of the model. Indeed, the multiple estimation process described previously for state variables corresponding to faulty measurements may now be also applied to the other state variables.

The final estimation $\hat{\hat{x}}_i$ of the *i*th variable exploits all these "local" estimations as it is obtained by their intersection:

$$\hat{\hat{x}}_i = \left(\bigcap_j \hat{x}_{i,j} \right) \cap \hat{x}_i \tag{11}$$

5 – Example

5.1 – Description

The chosen academic example describes a system characterised by ten variables and five constraint equations. Some parameters of the constraints are perfectly known and the others are described by an interval variable.

$$A = \begin{pmatrix} [0.98,\ 1.02] & -[0.99,\ 1.01] & 0 & -1 & 0 & [0.98,\ 1.02] & 0 & 0 & 0 & 0 \\ 0 & [0.99,\ 1.01] & 0 & 0 & 0 & 0 & 1 & -[0.95,\ 1.05] & 0 & 0 \\ 0 & 0 & 0 & 1 & -[0.99,\ 1.01] & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & [0.99,\ 1.01] & -[0.98,\ 1.02] & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -[0.9,\ 1.1] & -1 \end{pmatrix} \tag{12}$$

The uncertain model of the measurement system is described by:

$$y_i = (1 + a_i \eta_i) x_i, \qquad |\eta_i| \le 1, \qquad i = 1..10 \tag{13}$$

In order to simplify the presentation, only the multiplicative errors have been taken into account in this example. The measurement vector and it's corresponding vector of a_i coefficients are the following:

$$y = (21.17 \quad 5.17 \quad 5.91 \quad 11.71 \quad 11.55 \quad 5.60 \quad 10.20 \quad 15.40 \quad 0.97 \quad 4.90)^T \tag{14a}$$

$$a = (0.1 \quad 0.1 \quad 0.15 \quad 0.1 \quad 0.15 \quad 0.1 \quad 0.25 \quad 0.1 \quad 0.1 \quad 0.1)^T \tag{14b}$$

Remembering that h_i is a normalised bounded variable ($|h_i| \le 1$), one deduces from (14) that:

$$\frac{y_i}{1 + a_i} \le \hat{x}_i \le \frac{y_i}{1 - a_i} \qquad i = 1..10 \tag{15}$$

Substituting the measurements in this inequality allows the estimations $\hat{x}_i$ (i =1…10) to be obtained:

$$\begin{aligned} &\hat{x}_1 = [19.25,\ \ 23.52],\ \hat{x}_2 = [4.70,\ \ 5.74],\ \hat{x}_3 = [5.14,\ \ 6.95],\ \hat{x}_4 = [10.65,\ \ 13.01] \\ &\hat{x}_5 = [10.04,\ \ 13.59],\ \hat{x}_6 = [5.09,\ \ 6.22],\ \hat{x}_7 = [8.16,\ \ 13.60],\ \hat{x}_8 = [14.00,\ \ 17.11] \\ &\hat{x}_9 = [0.88,\ \ 1.08],\ \hat{x}_{10} = [4.45,\ \ 5.44] \end{aligned} \tag{16}$$

5.2 – Residual analysis

From the estimations (16), the process model (12) and equations (6) and (7), the following residuals are then evaluated:

$$R_1 = [0.98,\ \ 1.02]\hat{x}_1 - [0.99,\ \ 1.01]\hat{x}_2 - \hat{x}_4 + [0.98,\ \ 1.02]\hat{x}_6 = [5.04,\ \ 13.01] \tag{17a}$$

$$R_2 = [0.99,\ \ 1.01]\hat{x}_2 - [0.95,\ \ 1.05]\hat{x}_8 + \hat{x}_7 = [-5.15,\ \ 6.10] \tag{17b}$$

$$R_3 = \hat{x}_4 - [0.99,\ \ 1.01]\hat{x}_5 - \hat{x}_7 = [-11.24,\ \ -10.53] \tag{17c}$$

$$R_4 = [0.99,\ \ 1.01]\hat{x}_5 - \hat{x}_3 - [0.98,\ \ 1.02]\hat{x}_6 = [-3.36,\ \ 3.60] \tag{17d}$$

$$R_5 = \hat{x}_8 - [0.9,\ \ 1.1]\hat{x}_9 - \hat{x}_{10} = [-1.49,\ \ 1.70] \tag{17d}$$

The zero element doesn't belong to the first and third residuals (17a) and (17c), so, they are declared "abnormal". Therefore, there exist incoherences between the measurements and the process model. At least, one of the measurements intervening in the expressions of these residuals, i.e. y_1, y_2, y_4, y_5, y_6 and y_7 is suspected to be faulty.

5.3 – Fault isolation by signature analysis

For this particular small dimension example, the generic procedure proposed in section 3 may be reduced. As only two residuals are suspected, the more suspect measurement is that which is common to R_1 and R_3, i.e. y_4. Combining the expressions of these two residuals, it is possible to eliminate this variable leading to the new residual R_{31}.

$$R_{31} = [0.98,\ \ 1.02]\hat{x}_1 - [0.99,\ \ 1.01]\hat{x}_2 + [0.98,\ \ 1.02]\hat{x}_6 - [0.99,\ \ 1.01]\hat{x}_5 - \hat{x}_7 \qquad (18)$$

The value of this residual is then numerically evaluated taking into account estimations (16): $R_{31} = [-9.28,\ \ 7.58]$. Clearly this residual is not suspect as the interval contains the zero value. The following signature table exhibits this analysis.

Measurement / Residual	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	Normal
R_1	0	0		0		0					No
R_2		1					1	1			Yes
R_3				0	0		0				No
R_4			1		1	1					Yes
R_5			1						1	1	Yes
R_{31}	1	1			1	1	1				Yes

Table 1: Signature table

5.4 – State estimation

The measurement y_4 is faulty ; it is then necessary to estimate the corresponding state variable from the other fault-free measurements. As it intervenes into two process constraints (the first and the third row of the *A* matrix, it is possible to propose the two following estimations:

$$\hat{x}_{4,3} = [0.99,\ \ 1.01]\hat{x}_5 + \hat{x}_7 = [18.10,\ \ 27.32] \qquad (19a)$$

$$\hat{x}_{4,1} = [0.98,\ \ 1.02]\hat{x}_1 - [0.99,\ \ 1.01]\hat{x}_2 + [0.98,\ \ 1.02]\hat{x}_6 = [18.05,\ \ 25.69] \qquad (19b)$$

Due to measurement errors it's natural that these two estimations don't matched perfectly. The primary estimation $\hat{x}_4 = [10.65,\ \ 13.01]$ is then replaced by $\hat{x}_4 = (\hat{x}_{4,1} \cap \hat{x}_{4,3}) = [18.10,\ \ 25.69]$. All the other multiple estimations are then computed on the basis of $\hat{x}_i$, $i = 1 \ldots n$. One obtains:

$$\hat{x}_{1,1} = [16.41,\ \ 26.50] \quad \hat{x}_{2,1} = [-1.84,\ \ 12.24] \quad \hat{x}_{2,2} = [-0.30,\ \ 9.81] \quad \hat{x}_{3,4} = [3.60,\ \ 8.74]$$

$$\hat{x}_{3,5} = [5.25,\ \ 6.63] \quad \hat{x}_{4,1} = [18.05,\ \ 25.69] \quad \hat{x}_{4,3} = [18.10,\ \ 27.32] \quad \hat{x}_{5,3} = [4.50,\ \ 17.53]$$

$$\hat{x}_{5,4} = [10.13,\ \ 13.30] \quad \hat{x}_{6,1} = [-1.24,\ \ 12.63] \quad \hat{x}_{6,4} = [2.99,\ \ 8.59] \quad \hat{x}_{7,2} = [7.50,\ \ 13.31]$$

$$\hat{x}_{7,3} = [4.38\ \ 15.74] \quad \hat{x}_{8,2} = [12.81,\ \ 19.40] \quad \hat{x}_{9,5} = [-0.31,\ \ 2.50] \quad \hat{x}_{10,5} = [3.95,\ \ 6.16]$$

The final estimations are then evaluated on the basis of the previous one:

$$\hat{\hat{x}}_1 = \hat{x}_{1,1} \cap \hat{x}_1 = [19.25,\ 23.52] \qquad \hat{\hat{x}}_2 = \hat{x}_{2,1} \cap \hat{x}_{2,2} \cap \hat{x}_2 = [4.70,\ 5.74]$$

$$\hat{\hat{x}}_3 = \hat{x}_{3,4} \cap \hat{x}_{3,5} \cap \hat{x}_3 = [5.25,\ 6.63] \qquad \hat{\hat{x}}_4 = \hat{x}_{4,1} \cap \hat{x}_{4,3} = [18.10,\ 25.69]$$

$$\hat{\hat{x}}_5 = \hat{x}_{5,3} \cap \hat{x}_{5,4} \cap \hat{x}_5 = [10.13,\ 13.30] \qquad \hat{\hat{x}}_6 = \hat{x}_{6,1} \cap \hat{x}_{6,4} \cap \hat{x}_6 = [5.09,\ 6.22]$$

$$\hat{\hat{x}}_7 = \hat{x}_{7,2} \cap \hat{x}_{7,3} \cap \hat{x}_7 = [8.16,\ 13.31] \qquad \hat{\hat{x}}_8 = \hat{x}_{8,2} \cap \hat{x}_8 = [14.00,\ 17.11]$$

$$\hat{\hat{x}}_9 = \hat{x}_{9,5} \cap \hat{x}_9 = [0.88,\ 1.08] \qquad \hat{\hat{x}}_{10} = \hat{x}_{10,5} \cap \hat{x}_{10} = [4.45,\ 5.44]$$

The analysis of these results shows that all the information embedded both in the process and measurement models has been exploited. The measurement bias affecting y_4 has been detected, isolated and corrected. Some final estimations are directly issued from measurement model ($\hat{\hat{x}}_1, \hat{\hat{x}}_2$ or $\hat{\hat{x}}_6$ for example), the use of the process model cannot enhanced the first estimations (16). On the contrary, when measurements are less accurate, the interval estimations are restricted by using of the process model ; it is the case of $\hat{\hat{x}}_3, \hat{\hat{x}}_5$ or $\hat{\hat{x}}_7$ for example.

6 – Conclusion

The exposed technique represents an interesting alternative to the classical technique of data reconciliation using the principle of the likelihood function maximisation based on the distribution of the measurement errors. It requires very few theoretic hypotheses for its implementation and is essentially based on a semi-empirical knowledge relative to the variable and parameter plausible confidence domains. Future works will be dedicated to the extension of the proposed method to non linear system, particularly to system described by both linear and bilinear equations

7 – References

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