

FAULT DETECTION OF UNCERTAIN MODELS USING POLYTOPE PROJECTION

Hicham JANATI IDRISSE, Olivier ADROT, José RAGOT and Frederic KRATZ*

*Centre de Recherche en Automatique de Nancy - CNRS UMR n°7039
Institut National Polytechnique de Lorraine
2, avenue de la Forêt de Haye - 54516 Vandœuvre-lès-Nancy Cedex, France
{hjanati, oadrot, jragot}@ensem.inpl-nancy.fr – Fax: +33 3 83 59 56 44*

**Laboratoire Vision et Robotique UPRES EA 2078
Université d'Orléans - IUT de Bourges
63 avenue de Lattre de Tassigny F-18020 Bourges Cedex, France*

Abstract: In this paper, a fault detection procedure for linear MIMO models with respect to the observations and the parameters is studied. The various undesirable phenomena being able to affect any physical system (parameter fluctuations, noise,...) are taken into account by describing them as uncertainties. Thus a fault detection procedure is established by providing a more robust decision with respect to the various imperfections. A more general case consists in using polytopes instead of the parallelotope, is treated in order to bring a better precision, by exploiting the Fourier-Motzkin elimination algorithm which is used to compute the projection of a polytope.
Copyright © 2002 IFAC

Keywords: uncertain model, intervals, fault detection, polytope.

1. INTRODUCTION

The purpose of a fault detection and isolation (F.D.I.) procedure is to specify in which operating condition is the studied physical system. More precisely, when a fault is present, one should be able to detect it, then to identify its nature i.e. to isolate the component that the fault has affected. These two stages help to reconfigure the considered system in order to make it operational by realizing the adequate action.

A problem met in the field of F.D.I. schemes lies in the fact that a model only defines an approximate behavior of a physical system. This is caused by modeling errors when a model is made linear or when some physical phenomena are not taken into account. However, a modeling error has not to be identified with a fault. In order to prevent a significant number of false alarms, the bounding approach consists in considering that parameter fluctuations and noise are represented by uncertain and bounded variables. Indeed, every uncertain parameter θ_i of the model is

considered bounded and time-variant variable $\theta_i \in [\theta_{\min i}; \theta_{\max i}]$, which only its bounds are known.

This paper is organized as the follows. After formulating the problem in section 2, an algorithm for polytope projection is explained in section 3 and is generalized in order to solve the Fault Detection (F.D.) problem when the parameter domain is a polytope. In the section 4, a fault detection method using bounding approach is proposed as a binary consistency test between measurements and their estimations using the model. Then, a relative volume providing a degree of consistency is defined in section 5, followed by an example in section 6.

2. PROBLEM FORMULATION

Fault detection procedure for a physical system using a model is viewed as a comparison between an observed behavior and an estimated one obtained by using a model. When model parameters are uncertain, the precision of the F.D. procedure is directly related

to the size of uncertainties: the more the size of the uncertainties is large, the less the F.D. procedure is precise. This is due to the fact that a fault may be confused with a possible parameter deviation which leads to a non-detection. Consequently, the more the size of the parameter domain is large, the taller the non-detected fault will be. Then, it is interesting to carry out the F.D. procedure by taking the smallest parameter domain containing all the real parameter values. In fact, the real parameter domain is generally unknown or not convex, that is why some parameter estimation techniques consist in finding an approximation of the real parameter domain as an ellipsoidal form or a parallelotope. This is not developed in this paper, the reader can refer to several works on parameter estimation field which have been collected in a collective volume that represents the main results (Milanese *et al.*, 1996). Thus, the precision of F.D. is related to the quality of the parameter estimation procedure and to the characteristics of the parameter domain.

In (Ploix *et al.*, 2000) and (Janati-Idrissi *et al.*, 2001), the parameter domain is considered as a parallelotope in order to have its generating expression (1), then a method based on interval analysis (Moore 1979, Moore *et al.*, 1988) is used to provide the estimated behavior. Indeed, if $\theta \in \mathbb{R}^p$ is the parameter vector belonging to a parallelotope Pr_θ , then its expression is:

$$\theta = \theta_c + T v \quad (1)$$

where $\theta_c \in \mathbb{R}^p$ is the center of Pr_θ and $T \in \mathbb{R}^{p \times q}$ is a matrix defining its form and its volume. The uncertain and the bounded nature of the parameter vector θ is traduced by the normalized vector $v \in \mathbb{R}^q$ such that $\|v\|_\infty \leq 1$. Finally, by injecting this expression in the model latter defined by (2) in the section 2.1, it becomes possible to provide the estimated behavior of the system and then to compare it to measurements. This technique can not be extended in case the parameter domain is considered as a polytope. But it is easy to see that a polytope is more precise in approximating the real parameter domain than a parallelotope (see figure 1).

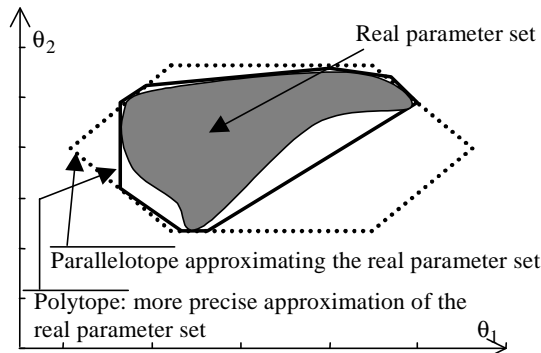


Fig.1 Parameter set approximation: difference between a polytope and a parallelotope.

Each one of these two domains defines a bounded subset in \mathbb{R}^p , limited by hyperplanes. In other words, a polytope (including the particular case of a

parallelotope) can be viewed as the intersection of different half spaces which can be described by their analytical expressions. In the parameter space, a half space is described by the following linear inequality :

$$a_i \theta \leq b_i$$

such that $b_i \in \mathbb{R}$ and a_i^T is a row vector in \mathbb{R}^p . The intersection of a set of r half spaces is mathematically represented by compacting all this single linear inequalities in a matrix form as:

$$A\theta \leq b$$

where $A \in \mathbb{R}^{r \times p}$ and $b \in \mathbb{R}^r$. Thus, such linear inequality describes a general form of a polytope.

2.1. System description

This paper focuses attention on a linear time-variant system which is described by the following model:

$$Y(k) = X(k)\theta(k) + E(k) \quad (2)$$

where $Y \in \mathbb{R}^n$, $X \in \mathbb{R}^{n \times p}$ are the measurable variables, their measurements are noticed respectively \tilde{Y} and \tilde{X} . Although no uncertainty is considered in the sensors, this notation of measurements is done only in order to highlight the difference between the estimation of variables (Y , X) and their measurements (\tilde{Y} , \tilde{X}). It is supposed that the matrix X is full row rank, if it is not the case, an elimination of some rows of X and the corresponding elements of Y which are linearly dependent is done to have a reduced model. $\theta(k) \in \mathbb{R}^p$ defines model parameters and k is the time index. Noting that such a model can describe also a dynamic system (AR, ARMA) since that can contain some components of Y shifted in the time.

Uncertainties affecting the model are classified in two categories. On the one hand, those acting directly on the outputs are additive uncertainties, and on the other hand, the multiplicative uncertainties (uncertainties multiplied by measurement) describe parameter variations:

♦ The multiplicative uncertainties represented by the uncertain parameter vector $\theta \in \mathbb{R}^p$ fluctuate inside an invariant polytope denoted P_θ , defined by some linear inequalities as:

$$A\theta(k) \leq b \text{ where } A \in \mathbb{R}^{r \times p} \text{ and } b \in \mathbb{R}^r$$

♦ The additive uncertainties represented by the uncertain vector $E(k) \in \mathbb{R}^n$, traduce the equation error of the model (1). The i^{th} component of $E(k)$, denoted $e_i(k)$ is considered as a bounded variable:

$$-\delta_i \leq e_i(k) \leq \delta_i, \text{ such that } \delta_i \in \mathbb{R} \text{ is known.}$$

Then the additive uncertainties can also be described by their analytical expressions as:

$$E(k) = Z(\delta)u(k) \quad \text{where} \quad Z(\delta) = \text{diag}(\delta_1, \dots, \delta_n)$$

($\delta = [\delta_1 \dots \delta_n]^T$) and $u(k) \in \mathbb{R}^n$ is a bounded and normalized vector: $\|u(k)\|_\infty \leq 1$.

The global description of model (2) becomes:

$$Y(k) = X(k)\theta(k) + Z(\delta)u(k),$$

under the following linear conditions:

$$A\theta(k) \leq b \text{ and } \|u(k)\|_\infty \leq 1.$$

Let us denote:

$\succ \alpha(k) = [\theta(k)^T \ u(k)^T]^T$: a new parameters model vector.

$\succ X(k) = [X(k) \ Z(\delta)]$ and $Y(k) = Y(k)$ with $X(k) \in \mathbb{R}^{n \times (p+n)}$, $Y(k) \in \mathbb{R}^n$.

Then the parameter domain is defined by:

$$M\alpha(k) \leq N$$

where: $M = \begin{bmatrix} A & 0 \\ 0 & I_n \\ 0 & -I_n \end{bmatrix}$, $N = \begin{bmatrix} b \\ \mathbf{1}_n \\ \mathbf{1}_n \end{bmatrix}$,

$$M \in \mathbb{R}^{(r+2n) \times (p+n)} \text{ and } N \in \mathbb{R}^{(r+2n)}.$$

such that $\mathbf{1}_n$ is a vector in \mathbb{R}^n of which all the elements are equal to 1 and I_n is the identity matrix in $\mathbb{R}^{n \times n}$. Finally, the model (2) becomes:

$$Y(k) = X(k)\alpha(k) \quad (3)$$

under the linear conditions:

$$M\alpha(k) \leq N$$

The measurements of variables $X(k)$ and $Y(k)$ are:

$$\tilde{X}(k) = [\tilde{X}(k) \ Z(\delta)] \text{ and } \tilde{Y}(k) = \tilde{Y}(k)$$

According to parameter values at a time k , the model produces one behavior among several possible ones. Consequently, in the F.D. operation, the consistency test must take into account the uncertain nature of the model. In the next section a method for treating the collected data and for making a decision in the presence of uncertainties is presented.

3. CONSISTENCY TEST

When the physical system described in the section 2.1 operates normally, the measurements ($\tilde{X}(k)$, $\tilde{Y}(k)$) must be consistent with the model (3) i.e. at a time k , it exists a parameter vector value α_0 such that:

$$\tilde{Y}(k) = \tilde{X}(k)\alpha_0 \text{ with } M\alpha_0 \leq N \quad (4)$$

Then, finding the unknown variable α_0 is a linear programming (LP) problem widely treated in the literature.(Nash, 1996). In general, the value of α_0 is not unique, consequently, a given solution at a time k does not necessary correspond to exact parameter values. Then, it can not reflect the real operating state of the system. This is due on the one hand to the fact that the model (3) is not injective (in general, even if the parameters would have been invariant and certain, there would be an infinity of solutions which would be consistent with the model for a given data) and on the other hand it is caused by the lack of the information (uncertain model).

In the following, two methods for solving (4) are

proposed. The first one, called the parametric approach, reasons in the parameter space. The second method consists in considering the measurement space using a set-membership approach.

3.1. Parametric approach

The system described by (3) operates normally if the following condition holds:

$$\exists \alpha \in \mathbb{R}^{p+n} \text{ such that } \tilde{Y}(k) = \tilde{X}(k)\alpha \text{ with } M\alpha \leq N \quad (5)$$

In order to simplify notations, the time index k is omitted in the following. It means that all the procedure is performed at each time k . Since the matrix \tilde{X} is full row rank, then it exists at least one matrix $H \in \mathbb{R}^{(n+p) \times p}$ and a vector $g \in \mathbb{R}^{n+p}$ such that:

$$\alpha = H\beta + g, \text{ where } \beta \in \mathbb{R}^p$$

By injecting this expression of α in the inequality $M\alpha \leq N$; the final form of the condition (5) is:

$$\exists \beta \in \mathbb{R}^p / \Psi\beta \leq \Gamma \quad (6)$$

$$\text{where } \Psi = MH \text{ and } \Gamma = N - Mg$$

Thus, the consistency test is easier to establish because the number of variables occurring in the inequality system to be treated is smaller: $(p+n \rightarrow p)$.

The F.D. procedure can be summarized by the following set of rules:

\succ If the linear inequality $\Psi\beta \leq \Gamma$ is feasible then the system described by (3) operates normally.

\succ If $\Psi\beta \leq \Gamma$ is not feasible then the system described by (3) is in an abnormal operating state.

Several techniques can be used for solving the feasibility of a set of linear inequalities. The obvious one, consists in using the algorithms solving linear matrix inequality (LMI) problems (Boyd, *et al.* 1994) by rewriting (6) as $r+2n$ scalar LMIs:

$$\Psi_{i,1}\beta_1 + \dots + \Psi_{i,j}\beta_j + \dots + \Psi_{i,p}\beta_p \leq \Gamma_i, \ i = 1, \dots, (r+2n)$$

such that the elements of the vector β (β_j , $j = 1, \dots, p$) are unknown matrices (scalar matrices).

3.2. Set membership approach

Another alternative to test the consistency of the measurements, is the set membership approach. It consists in estimating the system output Y and to compare it with its measurement \tilde{Y} . Since the model (3) is uncertain due to the fact that the parameter vector α has several possible values, then the output estimation is not a punctual value. If the system described by (3) operates normally and the parameter vector belongs to the domain D_α defined by:

$$D_\alpha = \{\alpha \in \mathbb{R}^{n+p} / M\alpha \leq N\}$$

then the model output Y take several possible values in the domain D_Y described as:

$$D_Y = \{\tilde{X}\alpha / \alpha \in D_\alpha\}$$

Thus, the F.D. procedure can be based on a consistency test of measurements which is completely equivalent to that used in the previous subsection and which is presented in the following way:

➤ If $\tilde{\mathbf{Y}} \in D_Y$ then the system described by (3) is in a normal operating state.

➤ If $\tilde{\mathbf{Y}} \notin D_Y$ then the system described by (3) is in an abnormal operating state.

Therefore, this consistency test requires the determination of the domains D_Y . Several works are elaborated in this context (Ploix *et al.*, 2000); however, they treat only the case where the parameter domain is a parallelotope. The methods used in these works are based on interval analysis which can not be applied to give an exact description of the domain D_Y when D_α is a polytope. In this paper, the proposed technique is based on another point of view. It consists in computing the projection of the polytope D_α on the measurement space in order to deduce the exact description of D_Y .

4. PROJECTION OF A POLYTOPE

In this section, an algorithm for polytope projection using Fourier-Motzkin elimination appearing in the book of (Ziegler, 1995) is presented. It starts from a projection in a single direction and then is generalized in this paper to compute the projection of a domain on an affine space.

4.1. Projection in a direction

Considering a given polytope \mathbf{P} defined by some linear inequalities:

$$\mathbf{x} \in \mathbf{P} \Leftrightarrow \mathbf{Q}\mathbf{x} \leq \mathbf{c} \quad / \quad \mathbf{Q} = (q_{i,j})_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n_x}}, \quad \mathbf{c} \in \mathbb{R}^m \quad \text{and}$$

$\mathbf{x} \in \mathbb{R}^{n_x}$ x_i and c_i are respectively the i^{th} elements of vectors \mathbf{x} and \mathbf{c} . The canonical basis in \mathbb{R}^{n_x} is (d_1, \dots, d_{n_x}) . The projection of \mathbf{P} in the direction of d_j , is defined as:

$$\text{proj}_j(\mathbf{P}) = \left\{ \mathbf{x} \in \mathbb{R}^{n_x} / x_j = 0, \exists t \in \mathbb{R} : \mathbf{x} + td_j \in \mathbf{P} \right\}$$

Analytically, it consists in eliminating the component x_j from the inequality: $\mathbf{Q}\mathbf{x} \leq \mathbf{c}$ and is the classical perception of the projection. For more clearness, considering an example of a polytope \mathbf{P} defined by the following linear inequalities:

$$\begin{array}{ll} -x_1 - 4x_2 \leq -9 & x_1 \leq 4 \\ -2x_1 - x_2 \leq -4 & 2x_1 + x_2 \leq 11 \\ x_1 - 2x_2 \leq 0 & -2x_1 + 6x_2 \leq 17 \end{array}$$

The computation of the projection of \mathbf{P} on the subspace $\{x \in \mathbb{R}^2 / x_2 = 0\}$ requires the elimination of x_2 from the previous linear inequalities by currying out all possible linear combinations of two constraints, such that in one of them x_2 is multiplied by a positive scalar and in the second one, x_2 is multiplied by a

negative scalar. By applying this procedure, the projection of \mathbf{P} will be defined as:

$$\text{proj}_2(\mathbf{P}) = \left\{ x \in \mathbb{R}^2 / x_2 = 0, 0.5 \leq x_1 \leq 4 \right\}$$

Figure 2 shows the projection of \mathbf{P} in the direction of d_2 , which is an orthogonal projection on the linear space $\{x \in \mathbb{R}^2 / x_2 = 0\}$.

In the general case, let us consider that \mathbf{P} is a polytope defined in \mathbb{R}^{n_x} by: $\mathbf{A}\mathbf{x} \leq \mathbf{b}$, with

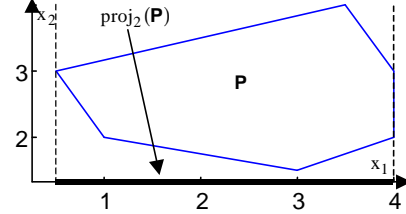


Fig. 2 Projection of \mathbf{P} in the direction of x_2

$\mathbf{A} \in \mathbb{R}^{m \times n_x}$ and $\mathbf{b} \in \mathbb{R}^m$, and choose $s \leq n_x$. The projection of \mathbf{P} in the direction of d_s is a polytope defined by given linear inequalities: $\mathbf{A}^{/s}\mathbf{x} \leq \mathbf{b}^{/s}$ such that the matrix $\mathbf{A}^{/s}$ and the vector $\mathbf{b}^{/s}$ are defined by the following algorithm:

Fourier-Motzkin elimination:

The rows of the matrix $\mathbf{A}^{/s}$ are

- ♦ the rows a_i of \mathbf{A} , for all i with $a_{i,s} = 0$, and
- ♦ the sums $a_{i,s}a_j + (-a_{j,s})a_i$ for all i, j with $a_{i,s} > 0$ and $a_{j,s} < 0$.

The elements of $\mathbf{b}^{/s}$ are

- ♦ $x \in \mathbf{P}$, for all i with $a_{i,s} = 0$, and
- ♦ $a_{i,s}b_j + (-a_{j,s})b_i$ for all i, j with $a_{i,s} > 0$ and $a_{j,s} < 0$.

Finally the projection of \mathbf{P} in the direction of d_s is described by: $\text{proj}_s(\mathbf{P}) = \left\{ \mathbf{x} \in \mathbb{R}^{n_x} / \mathbf{A}^{/s}\mathbf{x} \leq \mathbf{b}^{/s} \right\}$

4.2. Projection on an affine space

In this section, the Fourier-Motzkin elimination algorithm is generalized to the case of a non orthogonal projection of a polytope on an affine space, in order to be able to construct the domain D_Y . Consider again the model (3):

$$\mathbf{Y}(k) = \mathbf{X}(k)\alpha(k) / M\alpha(k) \leq N.$$

The domain D_Y is the projection of D_α on the affine subspace Λ_k spanned by the rows of the matrix $\mathbf{X}(k)$:

$$\Lambda_k = \left\{ \alpha \in \mathbb{R}^{n+p} / \mathbf{Y}(k) = \mathbf{X}(k)\alpha \right\}$$

the method proposed in this section is applied for all k , then the temporal index is omitted in the following.

Since this projection is not orthogonal, it is necessary to carry out a change of variable. Firstly, let us consider the permutation matrix $\mathbf{P} \in \mathbb{R}^{(n+p) \times (n+p)}$ such that:

$$\tilde{\mathbf{X}}\mathbf{P}^T = \begin{pmatrix} \tilde{\mathbf{X}}_1 & \tilde{\mathbf{X}}_2 \end{pmatrix}$$

where $\tilde{\mathbf{X}}_1 \in \mathbb{R}^{n \times n}$ is invertible and $\tilde{\mathbf{X}}_2 \in \mathbb{R}^{n \times p}$, and let \mathbf{T} be a matrix defined as: $\mathbf{T} = \mathbf{P} \begin{pmatrix} \tilde{\mathbf{X}}_1^{-1} & -\tilde{\mathbf{X}}_1^{-1}\tilde{\mathbf{X}}_2 \\ 0 & \mathbf{I}_p \end{pmatrix}$.

By noting $\bar{\alpha} = \mathbf{T}^{-1}\alpha$, $\bar{\mathbf{M}} = \mathbf{MT}$, $\bar{\alpha}(k) = [\bar{\alpha}_1^T \ \bar{\alpha}_2^T]^T$ and $\bar{\mathbf{M}} = [\bar{\mathbf{M}}_1 \ \bar{\mathbf{M}}_2]$ such that: $\bar{\mathbf{M}}_1 \in \mathbb{R}^{(r+2n) \times n}$, $\bar{\mathbf{M}}_2 \in \mathbb{R}^{(r+2n) \times p}$, $\bar{\alpha}_1 \in \mathbb{R}^n$ and $\bar{\alpha}_2 \in \mathbb{R}^p$, model (3) becomes:

$$\mathbf{Y}(k) = (\mathbf{I}_n \ 0) \bar{\alpha} \text{ where } \bar{\mathbf{M}} \bar{\alpha} \leq \mathbf{N} \\ \Leftrightarrow \mathbf{Y} = \bar{\alpha}_1 \text{ where } \bar{\mathbf{M}}_1 \bar{\alpha}_1 + \bar{\mathbf{M}}_2 \bar{\alpha}_2 \leq \mathbf{N}$$

It is easy to show that, after the change of variable carried out by the matrix \mathbf{T} , the two subvectors $\bar{\alpha}_1$ and $\bar{\alpha}_2$ evolve in two different orthogonal subspaces.

Finally, one can deduce the linear inequalities relating the vector of outputs \mathbf{Y} and the vector $\bar{\alpha}_2$:

$$\bar{\mathbf{M}}_1 \mathbf{Y} + \bar{\mathbf{M}}_2 \bar{\alpha}_2 \leq \mathbf{N} \quad (7)$$

Since \mathbf{Y} and $\bar{\alpha}_2$ evolve in two orthogonal subspaces, the domain D_Y is the orthogonal projection of the polytope $\bar{\mathbf{P}}$ ($\bar{\mathbf{P}} = \{\bar{\alpha} \in \mathbb{R}^{p+n} / \bar{\mathbf{M}} \bar{\alpha} \leq \mathbf{N}\}$) defined in \mathbb{R}^{n+p} by inequalities (7), on the linear subspace defined by: $\left\{ \mathbf{z} = [\mathbf{Y}^T \ \bar{\alpha}_2^T]^T \in \mathbb{R}^{n+p} / \bar{\alpha}_2 = 0 \right\}$.

Consequently the domain D_Y is well defined as:

$$D_Y = \left\{ \mathbf{z} = [\mathbf{Y}^T \ \bar{\alpha}_2^T]^T \in \mathbb{R}^{n+p} / \bar{\alpha}_2 = 0, \exists t_1, \dots, t_p \in \mathbb{R}, \right. \\ \left. (\mathbf{z} - t_1 \bar{\mathbf{d}}_{n+1} - \dots - t_p \bar{\mathbf{d}}_{n+p}) \in \bar{\mathbf{P}} \right\}$$

such that $(\bar{\mathbf{d}}_1, \dots, \bar{\mathbf{d}}_{n+p})$ is the canonical basis of \mathbb{R}^{n+p} . By considering the previous notation, the domain D_Y is described by:

$$D_Y = \text{proj}_{n+p} \left(\text{proj}_{n+p-1} \left(\dots \left(\text{proj}_{n+1}(\bar{\mathbf{P}}) \right) \dots \right) \right)$$

This consists in applying the projection procedure explained in the section 4.1 in order to eliminate all the components of the vector $\bar{\alpha}_2$. Finally, this technique leads to an analytical expression of the measurement domain containing the estimations of all possible values of model outputs according to the parameter domain and data, by providing a matrix $\mathbf{A} \in \mathbb{R}^{\bar{r} \times n}$ and a vector $\mathbf{b} \in \mathbb{R}^{\bar{r}}$ such that:

$$D_Y = \left\{ \mathbf{Y} \in \mathbb{R}^n / \mathbf{A} \mathbf{Y} \leq \mathbf{b} \right\}, \text{ where } \bar{r} \text{ is the number of the linear constraints describing } D_Y.$$

The consistency test becomes:

- If $\mathbf{A} \tilde{\mathbf{Y}}(k) \leq \mathbf{b}$ then the system operates normally.
- If not then the system is affected by a fault.

5. NOTION OF RELATIVE VOLUME

The set-membership test of a point $\tilde{\mathbf{Y}}$ to a parallelotope D_Y by checking or not some linear inequalities has a binary result. It allows to verify if

$\tilde{\mathbf{Y}} \in D_Y$ or $\tilde{\mathbf{Y}} \notin D_Y$. But, it can not give more information about the degree of consistency (i.e. the position of $\tilde{\mathbf{Y}}$ and the size of D_Y). The figure 3 shows how it is possible to extract more information from the measurements and the model.

The polytope D_1 in the figure 3 is described by the following linear inequalities:

$$\begin{aligned} -y_1 - 2y_2 &\leq -6 & -y_1 + 3y_2 &\leq 4 \\ y_1 - y_2 &\leq 3 & y_1 &\leq 5 \end{aligned}$$

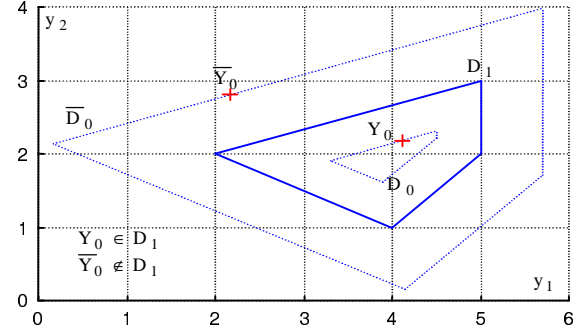


Fig 3 Relative volume

The relative volume of the polytope D_1 in \mathbb{R}^2 with respect to a point Y_0 is:

$$\text{Vr}(D_1 / Y_0) = \frac{\text{Vol}(D_0)}{\text{Vol}(D_1)} \quad (\text{see figure 3})$$

where $\text{Vol}(D_0)$ is the volume of the polytope D_0 . For the point $Y_0 \in D_1$ (resp. $\bar{Y}_0 \notin D_1$) in the figure 3, the relative volume is smaller (resp. bigger) than 1:

$$\text{Vr}(D_1 / Y_0) \leq 1; \text{Vr}(D_1 / \bar{Y}_0) \geq 1.$$

In the general case, the relative volume of a polytope D in \mathbb{R}^q with respect to a point Y_0 such that:

$$D = \left\{ \mathbf{x} \in \mathbb{R}^q / \mathbf{A} \mathbf{x} \leq \mathbf{b} \right\} / \mathbf{A} \in \mathbb{R}^{\bar{q} \times q} \text{ and } \mathbf{b} \in \mathbb{R}^{\bar{q}} \text{ are known, is defined by making the following steps:}$$

- Normalization of the linear inequality $\mathbf{A} \mathbf{x} \leq \mathbf{b}$:
if \mathbf{a}_i is the i^{th} row of \mathbf{A} and \mathbf{b}_i is the i^{th} element of \mathbf{b} , then the normalized form of the inequality $\mathbf{A} \mathbf{x} \leq \mathbf{b}$ is $\mathbf{A}_N \mathbf{x} \leq \mathbf{b}_N$ such that if \mathbf{a}_{Ni} is the i^{th} row of \mathbf{A}_N and \mathbf{b}_{Ni} is the i^{th} element of \mathbf{b}_N then:

$$\mathbf{a}_{Ni} = \frac{\mathbf{a}_i}{\|\mathbf{a}_i\|_2} \text{ and } \mathbf{b}_{Ni} = \frac{\mathbf{b}_i}{\|\mathbf{a}_i\|_2}.$$

- By taking σ as the maximum of the components of the vector $\mathbf{A}_N Y_0 - \mathbf{b}_N$ and the polytope D_0 defined by:

$$D_0 = \left\{ \mathbf{x} \in \mathbb{R}^q / \mathbf{A}_N \mathbf{x} \leq \mathbf{b}_N + \sigma \mathbf{1}_{\bar{q}} \right\}$$

$$\text{then the relative volume is: } \text{Vr}(D / Y_0) = \frac{\text{Vol}(D_0)}{\text{Vol}(D)}.$$

The volume of a polytope can be computed by using the algorithm given by (Lasserre 1983).

The calculation of the relative volume of the domain D_Y (containing the estimation of all possible values of the output model \mathbf{Y}) with respect to the

measurement $\tilde{\mathbf{Y}}$ makes it possible to carry out the consistency test as the following way:

If $\text{Vr}(\mathbf{D}_Z / \tilde{\mathbf{Y}}) \leq 1$ then the system described by operates normally, if not, it is in an abnormal operating state. Moreover, if $\text{Vr}(\mathbf{D}_Z / \tilde{\mathbf{Y}})$ tends to 0, the measurement $\tilde{\mathbf{Y}}$ is strongly consistent with the model and the more the $\text{Vr}(\mathbf{D}_Z / \tilde{\mathbf{Y}})$ tends towards 1, the closer the point $\tilde{\mathbf{Y}}$ gets to a border of \mathbf{D}_Y , in other words, the more the limit between consistency and inconsistency is slight.

6. EXAMPLE

In order to illustrate the method used herein for the F.D. procedure, an example is tacked in this section. Considering the following static model:

$y(k) = x(k)\theta(k)$ where $y(k) \in \mathbb{R}^2$, $x(k) \in \mathbb{R}^{2 \times 3}$ and $\theta = (\theta_1, \theta_2, \theta_3)^T$. The parameter domain is defined by the following linear inequalities:
 $-\theta_1 - 2\theta_2 + \theta_3 \leq -6$; $\theta_1 - \theta_2 - \theta_3 \leq 4$; $-\theta_3 \leq 0$
 $-\theta_1 - 2\theta_3 \leq 5$; $\theta_3 \leq 1$; $-\theta_1 + 3\theta_2 + \theta_3 \leq 3$

A variable fault is applied on the first output (10% of the output value) in the interval [5,10], as illustrated on the figure 4, this fault is detected along this window (except at $k=10$), it is traduced by the fact that the relative volume is big than 1 (figure 6). Another variable fault affects the second output (20% of the output value) in the interval [20,25]. This second fault is not detected at $k=20$ and $k=21$. Note that a non detection of a fault is due to the uncertainties affecting the model, in this case a small fault can be confused with a possible parameter fluctuation.

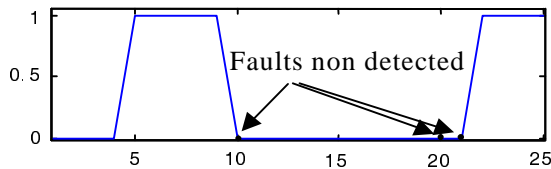


Fig. 4 Alarm signal

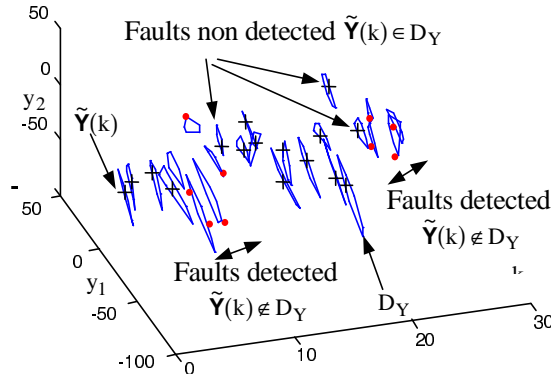


Fig. 5 Measurements and its estimations

The figure 5 shows the domains \mathbf{D}_Y and the measurements $\tilde{\mathbf{Y}}(k)$ for $k=1..25$. At a given time k the measurement $\tilde{\mathbf{Y}}(k)$ is represented by the symbol

'+' when $\text{Vr}(\mathbf{D}_Z / \tilde{\mathbf{Y}}(k)) \leq 1$ (or $\tilde{\mathbf{Y}}(k) \in \mathbf{D}_Y$) and by a point '.' if the measurement is not consistent with the model $\text{Vr}(\mathbf{D}_Z / \tilde{\mathbf{Y}}(k)) \geq 1$ (or $\tilde{\mathbf{Y}}(k) \notin \mathbf{D}_Y$). The alarm signal in the figure 4, is equal to 1 if a fault is detected and 0 if not.

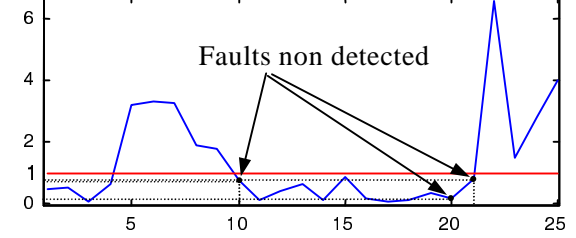


Fig. 6 Evolution of the relative volume

7. CONCLUSION

Fault detection of uncertain system when the parameter domain is a polytope is studied in this paper in order to give more precision. Two methods to test the consistency of the measurements with the model are proposed: the parametric approach gives a binary result as opposed to the set-membership approach which provides an additional information about the degree of consistency via a the relative volume defined while being based on polytope theory..

The result described only concerns one class of uncertain systems, it would be also interesting to extend these techniques to the case of state space representation with uncertain parameters in which the set-membership approach is more delicate to be applied because of the uncertainty propagation.

REFERENCES

- Boyd S. et al., Linear Matrix Inequalities in systems and control theory, Philadelphia, PA: SIAM, 1994
- Janati-Idrissi H., Adrot O., Ragot J., Multi-Fault detection of systems with bounded uncertainties, *40th IEEE Conference on Decision and Control, CDC'01*, Orlando (USA), December 7-10, 2001.
- Lasserre J.B., 1983, An analytical expression and an algorithm for the volume of a convex polyhedron in \mathbb{R}^n , *Journal of Optimization Theory and Applications*, **Vol 39**, N° 3, pp 363-377, 1983
- Milanese M.; J. Norton, H. Piet-Lahanier and E. Walter (ed.) (1996). Bounding approach to system identification, *Plenum Press*, New-York and London.
- Moore R.E. (1979). Methods and application of interval analysis, *SIAM Philadelphia, Pennsylvania*.
- Mo S. H., J.P. Norton (1988). Recursive parameter bounding algorithms which compute polytope bounds, *12th IMACS World Congress*, **2**, 477-480.
- Nash, S., and Sofer, A., Linear and Nonlinear Programming, *McGraw-Hill*, 1996.
- Ploix S., O. Adrot, J. Ragot (2000) Bounding approach to the diagnosis of a class of uncertain static systems, *Safeprocess2000*
- Ziegler G. M., *Lectures on Polytopes* Graduate texts in mathematics; Vol 152 Springer-Verlag, New York 1995.