State estimation for affine LPV systems

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Abstract

In this paper, the design of a scheduled observer which allows to estimate the state of an affine LPV[1] system is investigated. The state and the gain matrices of the observer are scheduled by using an interpolation method which is linear according to each parameter but which is nonlinear according to the parameter vector. The stability of the estimation error is based on the existence of an affine parameter-dependent Lyapunov function. The problem of the observer design and the existence of a such Lyapunov function is interpreted as an LMI feasibility problem with a rank constraint. An example is presented.

Keywords: scheduled observer, linear interpolation, BMI stability conditions.

1 Introduction

To justify the synthesis of observers by an interpolation-based approach, we point out the interest of the methods implementing the technique of gain scheduling. These methods make it possible to construct a nonlinear control (estimation) law, by combining the features of a family of linear time-invarying controllers (observers). The gain scheduling is an attractive approach to increase performance and robustness for systems with nonlinearity, parameter variation, uncertainties. Also, for important classes of LPV systems, the gain-scheduling techniques offer a good approach to get a control (observer) structure. For a review of the main theoretical results and design procedures relating to gain-scheduling see [13].

In this paper, we study the design of observers for LPV systems by the gain scheduling technique. In [11], Hyde *et al.* proposed a state observer whose gain is obtained by linear interpolation, by aiming to obtain a gain scheduled robust observer-based controler. The interpolation algorithm requires a slow variation of the system matrices A, B, C compared to the operating point, which ensures a slow variation of the observer gain. Stilwell [14] developed a linear interpolation technique for arbitrary state feedback gains and dynamic controllers with observers state-feedback structure. In the case of a scalar scheduling variable, a stability preserving interpolation in terms of frozen values of the scheduling variable has been proposed. When the scheduling variable is time-varying, the stability is established by imposing a bound on its rate of variation.

This paper is organised as follows. In Section 2, we state the problem under consideration. In Section 3, we present the interpolation algorithm and the structure of the observer according to this interpolation. In Section 4, the design of the observer is formulated as an LMI feasibility condition with a rank constraint or as a BMI feasibility condition.

[1] linear parameters varying

2 Problem formulation

We consider the class of linear parameter-varying (LPV) systems of which state matrix depends affinely on the parameter vector. This class of systems can be described by:

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $A(\rho) = A_0 + \sum_{i=1}^{i=K} \rho_i(t)A_i$, $B(\rho) = B_0 + \sum_{i=1}^{i=K} \rho_i(t)B_i$ and A_0, A_1, ..., A_K, B_0, B_1, ..., B_K are known constant matrices, ρ_1, ρ_2, ..., ρ_K are time-varying parameters, $x(t) \in I\!R^n$ is the state vector, $u(t) \in I\!R^m$ is the input vector and $y(t) \in I\!R^p$ is the output vector.

We assume that:

1. each parameter ρ_i ranges between known extremal values $\rho_i(t) \in [\underline{\rho_i}, \overline{\rho_i}]$ and $\underline{\rho_i} < 0$, $\overline{\rho_i} > 0$
2. the variation rate of each parameter $\dot{\rho}_i(t)$ is limited by known upper and lower bounds $\dot{\rho}_i(t) \in [\underline{\dot{\rho}_i}, \overline{\dot{\rho}_i}]$.

Note that the assumptions 1. and 2. are not restrictive for the class of models considered.

The parameter vector $\rho(t)$ remains in an hyper-rectangle called the parameter box of which 2^K vertices are defined by:

$$\mathcal{V} = \{(\omega_1, \omega_2, \ldots, \omega_K) |\ \omega_i \in \{\underline{\rho_i}, \overline{\rho_i}\}\}.$$

Similarly, the rate of variation of the parameters belongs to the hyper-rectangle defined by the following set of vertices:

$$\mathcal{S} = \{(\tau_1, \tau_2, \ldots, \tau_K) |\ \tau_i \in \{\underline{\dot{\rho}_i}, \overline{\dot{\rho}_i}\}\}.$$

Our aim is to design a parameter varying observer for the LPV system. To this end, linear observers are built for extremal values of ρ and the gain scheduled observer is built by interpolation of these observers, in real time, according to the parameter $\rho(t)$.

We look for a full order observer for the system (1)-(2) with the following structure:

$$\dot{z}(t) = H(\rho)z(t) + L(\rho)y(t) + J(\rho)u(t) \quad (3)$$
$$\hat{x}(t) = z(t) + My(t). \quad (4)$$

where $z \in I\!R^n$ and $\hat{x} \in I\!R^n$ is the estimated state.

The matrices $H(\rho)$, $L(\rho)$ and $J(\rho)$ are obtained by interpolation between extremal values. These extremal values correspond to the observers estimating the state of the system when $\rho \in \mathcal{V}$. The interpolation procedure is linear according to each parameter.

The problem of the observer design is reduced to find this extremal observers such that the reconstruction error between the scheduled observer (3)-(4) and the affine LPV system (1)-(2) is affinely quadratically stable irrespective of the initialisations $x(0)$ and $z(0)$, the control input $u(t)$ and the law of variation of $\rho(t)$.

By affine quadratic stability we mean the existence of an affine parameter dependent Lyapunov function according to the following definition (see [6]).

Definition 2.1. The linear system $\dot{x}(t) = A(\rho(t))x(t)$ is affinely quadratically stable (AQS) if there exist $K+1$ symmetric matrices P_0, P_1,..., P_K such that the following inequalities

$$P(\rho) = P_0 + \rho_1 P_1 + \ldots + \rho_K P_K > 0$$
$$\mathcal{F}(\rho, \dot{\rho}) = A(\rho)^T P(\rho) + P(\rho)A(\rho) + P(\dot{\rho}) - P_0 < 0$$

hold for all admissible trajectories of the parameter vector $\rho = \begin{bmatrix} \rho_1 & \rho_2 & \ldots & \rho_K \end{bmatrix}^T$.

Using the AQS concept, one looks for a parameter dependent Lyapunov function for testing stability of the error dynamic. Hence, the obtained conditions are less conservative than those based on quadratic stability where a single Lyapunov function is used for testing stability over the whole parameter variation domain [2].

3 Interpolation method

In this section we describe the interpolation method according to parameter vector. Note that the interpolation procedure is linear according to each parameter but, we will see bellow that globally, according to the parameter vector, the interpolation procedure is a product of these linear interpolations and thus it is nonlinear.

The observer state matrix $H(\rho)$ and observer gain matrix $L(\rho)$ are obtained by linear interpolation of each parameter. Since the parameter vector ρ belongs to a parameter box defined by the vertices set $\mathcal{V}$, then $H(\rho)$ and $L(\rho)$ are delimited by a polytope of $I\!R^{n\times n}$ and respectively $I\!R^{n\times p}$:

$$\mathcal{H} = \{\overline{H}_0, \overline{H}_1, \ldots, \overline{H}_{2^K-1}\} \quad (5)$$
$$\mathcal{L} = \{\overline{L}_0, \overline{L}_1, \ldots, \overline{L}_{2^K-1}\} \quad (6)$$
$$\mathcal{J} = \{\overline{J}_0, \overline{J}_1, \ldots, \overline{J}_{2^K-1}\}. \quad (7)$$

Each corner $\overline{H}_i$ of $\mathcal{H}$, $\overline{L}_i$ of $\mathcal{L}$ and respectively $\overline{J}_i$ of $\mathcal{J}$, corresponds to a certain corner of $\mathcal{V}$. The link between these corners are given by the following:

> Let $(b_0\, b_1\, b_2\, \ldots b_{K-1})$ be the binary representation of the index i. Then the parameter box corner corresponding to $\overline{H}_i$, $\overline{L}_i$ and respectively $\overline{J}_i$ is $(\widetilde{\rho}_1, \widetilde{\rho}_2, \ldots, \widetilde{\rho}_K)$
>
> $$\left.\begin{array}{l}\overline{H}_i\\ \overline{L}_i\\ \overline{J}_i\end{array}\right\} \Longleftrightarrow (\widetilde{\rho}_1, \widetilde{\rho}_2, \ldots, \widetilde{\rho}_K) \text{ where}$$
>
> $$\widetilde{\rho}_j = \begin{cases} \underline{\rho_j} & \text{when } b_j = 0 \\ \overline{\rho_j} & \text{when } b_j = 1 \end{cases}.$$

Thus, the interpolated observer matrices are:

$$H(\rho) = \mu_0(\rho)\overline{H}_0 + \ldots + \mu_{2^K-1}(\rho)\overline{H}_{2^K-1} \quad (8)$$
$$L(\rho) = \mu_0(\rho)\overline{L}_0 + \ldots + \mu_{2^K-1}(\rho)\overline{L}_{2^K-1} \quad (9)$$
$$J(\rho) = \mu_0(\rho)\overline{J}_0 + \ldots + \mu_{2^K-1}(\rho)\overline{J}_{2^K-1} \quad (10)$$

where $\mu_0(\rho)$, $\mu_1(\rho)$, ..., $\mu_{2^K-1}(\rho)$ are nonlinear interpolation functions. This functions are featured by:

- $\mu_0(\rho) + \mu_1(\rho) + \ldots + \mu_{2^K-1}(\rho) = 1$, $0 \le \mu_i(\rho) \le 1$ for $i = 0, \ldots, 2^K - 1$
- each interpolation function is given by

$$\mu_i(\rho) = \prod_{\substack{j=0 \\ i\equiv(b_0\, b_1\, \ldots b_{K-1})}}^{K-1} \frac{\alpha_j \rho_j + \beta_j}{\underline{\rho}_j - \overline{\rho}_j}$$

$$\text{where } \alpha_j = \begin{cases} 1 & \text{when } b_j = 0 \\ -1 & \text{when } b_j = 1 \end{cases} \quad (11)$$

and $\beta_j = \begin{cases} -\overline{\rho_j} & \text{when } b_j = 0 \\ \underline{\rho_j} & \text{when } b_j = 1 \end{cases}$ with $(b_0\, b_1\, \ldots b_{K-1})$ the binary representation of the index i .

From the equations (8), (9), (10), (11) we find for the observer matrices the following structure:

$$H(\rho) = \sum_{i=0}^{2^K-1} \Big(\prod_{\substack{j=0 \\ i\equiv(b_0\, b_1 \ldots b_{K-1})}}^{K-1} \rho_j^{b_j} \Big) H_i \tag{12}$$

$$= H_0 + \underbrace{\sum_i \rho_i H_i}_{C_K^1 \text{ terms}} + \underbrace{\sum_{i\neq j} \rho_i \rho_j H_{i+j+1}}_{C_K^2 \text{ terms}} + \tag{13}$$

$$\ldots + \underbrace{\prod_i \rho_i H_{2^K-1}}_{C_K^K \text{ terms}} \tag{14}$$

$$L(\rho) = \sum_{i=0}^{2^K-1} \Big(\prod_{\substack{j=0 \\ i\equiv(b_0\, b_1 \ldots b_{K-1})}}^{K-1} \rho_j^{b_j} \Big) L_i. \tag{15}$$

$$J(\rho) = \sum_{i=0}^{2^K-1} \Big(\prod_{\substack{j=0 \\ i\equiv(b_0\, b_1 \ldots b_{K-1})}}^{K-1} \rho_j^{b_j} \Big) J_i. \tag{16}$$

With this structure of the interpolated observer, the problem of the observer design is reduced to find the matrices H_0, H_1,..., H_{2^K-1}, L_0, L_1,..., L_{2^K-1} and J_0, J_1,..., J_{2^K-1} such that the reconstruction error dynamic is affinely quadratically stable.

4 Gain-scheduled observer design

In this section, the proposed results are based on the AQS of the observation error dynamics, by the means of a Lyapunov function with affine dependence on the parameters. We give sufficient conditions for the existence of a full-order state observer, independently of the initial conditions x_0 and z_0, the input $u(t)$ and the evolution of the parameter vector $\rho(t)$.

Define the state reconstruction error as: $e = \hat{x} - x$. By introducing a matrix T of appropriate dimension, such that $T = I_n - MC$, the estimation error is given by: $e = z - Tx$.

The dynamic of the estimation error is then expressed as :

$$\dot{e} = H(\rho)e + (H(\rho)T + L(\rho)C - TA(\rho))x + (J(\rho) - TB(\rho))u. \tag{17}$$

Using the equation (17), we can state the following proposition.

Proposition 4.1. *The system* (3)-(4) *is an affine quadratically stable observer for the LPV system* (1)-(2) *if there exist matrices T, M, H_0,..., H_{2^K-1}, L_0,..., L_{2^K-1}, J_0,..., J_{2^K-1} of appropriate dimensions such that*

$$H(\rho) \text{ is } AQS \tag{18}$$
$$H(\rho)T + L(\rho)C - TA(\rho) = 0 \tag{19}$$
$$T = I_n - MC \tag{20}$$
$$J(\rho) = TB(\rho) \tag{21}$$

where $H(\rho)$, $L(\rho)$ and $J(\rho)$ are given by the relations (14), (15) *and* (16).

Proof: In the presence of the equations (19), (21) the estimation error is :

$$\dot{e} = H(\rho)e. \tag{22}$$

When the condition (18) is satisfied, the observer is AQS. □

The equation (21) gives the solutions:

$$\begin{cases} J_i = TB_i & \text{for } i = 1, \ldots, K \\ J_i = 0 & \text{for } i = K+1, \ldots, 2^K - 1 \end{cases}. \tag{23}$$

The equation (19) is a Sylvester equation with nonconstant matrices. The resolution of this equation gives the solutions:

$$H(\rho) = -K(\rho)C + TA(\rho) \tag{24}$$
$$L(\rho) = K(\rho) + H(\rho)M \tag{25}$$

where $K(\rho)$ is an arbitrary matrix according to the parameters. We choose for this matrix the same structure as the matrices $H(\rho)$ and $L(\rho)$:

$$K(\rho) = \sum_{i=0}^{2^K-1} \Big(\prod_{\substack{j=0 \\ i\equiv(b_0\, b_1 \ldots b_{K-1})}}^{K-1} \rho_j^{b_j} \Big) K_i. \tag{26}$$

wich is affine according to each parameter but it is nonlinear according to the parameter vector ρ. It is this choice of the structure of the matrix $K(\rho)$ which allows us to formulate the AQS condition (18) of the matrix $H(\rho)$ as an LMI feasibility condition with a rank constraint. From the relation (26) we find the equivalences:

$$\begin{cases} H_i = -K_iC + TA_i & \text{for } i = 0, ..., K \\ H_i = -K_iC & \text{for } i = K+1, ..., 2^K - 1 \\ L_i = K_i + H_iM & \text{for } i = 0, ..., 2^K - 1 \end{cases}. \tag{27}$$

Thereafter, we give the sufficient conditions for the existence of the observer in terms of LMI feasibility conditions with a rank-n constraint. This conditions are deduced by using the Gahinet *et al.*'s approach wich gives sufficient LMI conditions for affine quadratic stability (see [6]). Gahinet's approach relies on the concept of multiconvexity, that is, convexity along each direction of the parameter space. The solution of the state observer design problem is given by the following theorem.

Theorem 4.2. *The system* (3)-(4) *is an AQS observer for the LPV system* (1)-(2) *if there exist matrices* T, M, $H_0,\dots, H_{2^K-1}$, $L_0,\dots, L_{2^K-1}$, $K_0,\dots, K_{2^K-1}$, $J_0,\dots, J_{2^K-1}$ *and the* $K+1$ *symmetric matrices* P_0, $P_1,\dots, P_K$ *such that the equations* (20), (21) *are satisfied and the following LMI conditions with a rank-n constraint admit a solution*

$$P_0 > 0 \tag{28}$$

$$\mathcal{F}(\omega,\tau) = \Gamma(\omega)\mathcal{R}\Phi(\omega) + \Phi^T(\omega)\mathcal{R}^T\Gamma^T(\omega) + \Lambda(\tau)\mathcal{P} < 0,\ \text{for all } (\omega,\tau) \in \mathcal{V}\times\mathcal{S} \tag{29}$$

$$\mathcal{G}(\omega) = \Delta_i\mathcal{R}\Psi_i(\omega) + \Psi_i^T(\omega)\mathcal{R}^T\Delta_i^T \geq 0,\ \text{for all } i = 1,\dots,K \text{ and } \omega \in \mathcal{V} \tag{30}$$

$$\text{rank}\begin{bmatrix}\mathcal{P} & \mathcal{R}\end{bmatrix} = n \tag{31}$$

where $\mathcal{R} = \mathcal{P}\mathcal{K}$ *and*

$$\Gamma(\rho) = \begin{bmatrix}I_n & \rho_1 I_n & \dots & \rho_K I_n\end{bmatrix}$$
$$\Phi(\rho) = \begin{bmatrix}C & \rho_1 C & \dots & (\prod_{i=1}^K \rho_i)C & A(\rho)\end{bmatrix}^T$$
$$\mathcal{P} = \begin{bmatrix}P_0 & P_1 & \dots & P_K\end{bmatrix}^T$$
$$\mathcal{K} = \begin{bmatrix}-K_0 & -K_1 & \dots & -K_{2^K-1} & T\end{bmatrix}$$
$$\Lambda(\dot\rho) = \begin{bmatrix}0_n & \dot\rho_1 I_n & \dots & \dot\rho_K I_n\end{bmatrix}\ \text{and}$$

$$\Delta_i = \frac{\partial\Gamma(\rho)}{\partial\rho_i}\ ,\ \Psi_i(\rho) = \frac{\partial\Phi(\rho)}{\partial\rho_i}.$$

Proof: If the constraints (19), (20) and (21) are satisfied then the estimation error is given by (22). We must ensure the AQS of this equation. With the affine Lyapunov function $V(e,\rho) = \dot{e}P(\rho)e$ where $P(\rho) = P_0 + \rho_1 P_1 + \dots + \rho_K P_K$, we obtain the equation:

$$\frac{d\,P(\rho)}{dt} = \mathcal{F}(\rho,\dot\rho) = H(\rho)^T P(\rho) + P(\rho)H(\rho) + P(\dot\rho) - P_0.$$

By replacing $H(\rho)$ with the expression (24) we find: $\mathcal{F}(\rho,\dot\rho) = \Gamma(\rho)\mathcal{R}\Phi(\rho) + \Phi^T(\rho)\mathcal{R}^T\Gamma^T(\rho) + \Lambda(\dot\rho)\mathcal{P}$. The constraint (29) expresses the condition that $\mathcal{F}(\rho,\dot\rho)$ be negative definite to the corners of $\mathcal{V}\times\mathcal{S}$. To garanty that $\mathcal{F}(\rho,\dot\rho)$ is negative definite for all the polytope and implicitly the AQS, we impose the condition of multiconvexity:

$$\frac{\partial^2\mathcal{F}(\rho,\tau)}{\partial\rho_i^2} = 2\frac{\partial\Gamma(\rho)}{\partial\rho_i}\mathcal{R}\frac{\partial\Phi(\rho)}{\partial\rho_i} + 2\left(\frac{\partial\Phi(\rho)}{\partial\rho_i}\right)^T\mathcal{R}^T\left(\frac{\partial\Gamma(\rho)}{\partial\rho_i}\right)^T = 2\mathcal{G}(\rho) \geq 0. \tag{32}$$

The inequality (32) is depending on the parameter vector ρ and to satisfy this inequality we impose positive definition at the corners, that gives the conditions (30). The conditions (30) are sufficient to ensure the inequality (32) because it has the property of multiconvexity:

$$\frac{\partial^2\mathcal{G}}{\partial\rho_i^2} = \frac{\partial^2\Delta_i\mathcal{R}\Psi_i(\omega)}{\partial\rho_i^2} + \frac{\partial^2(\Delta_i\mathcal{R}\Psi_i(\omega))^T}{\partial\rho_i^2} = 0.$$

The condition that the equality $\mathcal{R} = \mathcal{P}\mathcal{K}$ has a solution, is assured by the constraint (31).
As the point $(0,0,\dots,0)$ belongs to the parameter box then $P(0) = P_0$. Condition (28) and continuity of the eigenvalues of $P(\rho)$ ensure that $P(\rho)$ is positive definite in the entire parameter box. □

Remark 4.1.
1. To design the observer we must solve the LMI conditions (28), (29) and (30) with the rank-n constraint (31). Solving an LMI with rank constraint is a difficult non-convex problem. In [9], Henrion *et al.* formulate the co-stabilisation of a family of SISO linear systems as a rank one constrained LMI problem. They solve this rank one constrained LMI problem by an LMI relaxation procedure and they propose a heuristic design algorithm based on potential reduction method. As an extension, it is possible to solve the rank-n constrained problem via a heuristic based on convex relaxations (implementation of a cone-complementarity algorithm). For more details on the cone-complementarity algorithm and SDP[2] relaxations see [5], [4], [10]. The heuristic is not guaranteed to provide a solution, even when one exists.
2. The rank constraint can be avoided by replacing $\mathcal{R}$ by $\mathcal{P}\mathcal{K}$ and resolving the BMI (29) with the LMI (28) and (30). The BMI problems are not convex and can have multiple local solutions. To solve them locally one can use iterative schemes which allow to reduce computational complexity of BMI to the computational complexity of LMI. In [8], Hassibi *et al.* propose a path-following (homotopy) method for locally solving BMI. See [7], [3] for global methods for solving BMI problems.

In the next section we give an example wich illustrates the results of Theorem 4.2.

5 Numerical example

In this section we present a numerical example for an affine LPV system dependent on two parameters. The parameter vector ranges in a hyper-rectangle wich is presented in Figure 1.

The system is described by the matrices:

[2] semidefinite programming

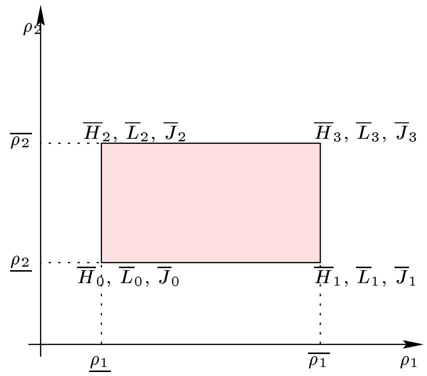

Figure 1: The parameter box.

$$A(\rho) = \begin{bmatrix} 1+0.4\rho_1+1.5\rho_2 & 0.4+1.5\rho_1+0.1\rho_2 \\ 3.7+1.1\rho_1+0.2\rho_2 & 1.4+0.9\rho_1+0.44\rho_2 \end{bmatrix},$$

and $C = \begin{bmatrix} 1 & 0.5 \end{bmatrix}$.

The observer matrices are:

$$H(\rho) = H_0 + \rho_1 H_1 + \rho_2 H_2 + \rho_1\rho_2 H_3$$
$$L(\rho) = L_0 + \rho_1 L_1 + \rho_2 L_2 + \rho_1\rho_2 L_3$$
$$J(\rho) = J_0 + \rho_1 J_1 + \rho_2 J_2 + \rho_1\rho_2 J_3$$

and the matrix $K(\rho)$ has the same structure.

For the parameters, we choose the extremal values: $\rho_1(t) \in [-0.2, 0.2]$, $\rho_2(t) \in [-0.1, 0.1]$,
$\dot{\rho}_1(t) \in [-0.4, 0.4]$ and $\dot{\rho}_2 \in [-0.1571, 0.1571]$.

With the Lyapunov function
$V(e,\rho) = \dot{e}(P_0 + \rho_1 P_1 + \rho_2 P_2)e$, the observer design algorithm are given by the feasibility conditions of the Theorem 4.2.

We have the folowing matrices:
$\Gamma(\rho) = \begin{bmatrix} I_n & \rho_1 I_n & \rho_2 I_n \end{bmatrix}$,
$\Phi(\rho) = \begin{bmatrix} C & \rho_1 C & \rho_2 C & \rho_1\rho_2 C & A(\rho) \end{bmatrix}^T$,
$\Lambda(\dot{\rho}) = \begin{bmatrix} 0_n & \dot{\rho}_1 I_n & \dot{\rho}_2 I_n \end{bmatrix}$,
$\Delta_1 = \begin{bmatrix} 0_n & I_n & 0_n \end{bmatrix}$, $\Delta_2 = \begin{bmatrix} 0_n & 0_n & I_n \end{bmatrix}$,
$\Psi_1(\rho) = \begin{bmatrix} 0_{p\times n} & C & 0_{p\times n} & \rho_2 C & A_1 \end{bmatrix}^T$
and $\Psi_2(\rho) = \begin{bmatrix} 0_{p\times n} & 0_{p\times n} & C & \rho_1 C & A_2 \end{bmatrix}^T$.

For solving the conditions of Theorem 4.2 we apply the method described in [1] and we obtain the following observer matrices:

$H(\rho) = H_0 + \rho_1 H_1 + \rho_2 H_2 + \rho_1\rho_2 H_3$ with
$H_0 = \begin{bmatrix} -5.1175 & -1.3431 \\ 0.6913 & -1.0631 \end{bmatrix}$, $H_1 = \begin{bmatrix} -3.0710 & -1.2516 \\ 3.1276 & -0.5155 \end{bmatrix}$,
$H_2 = \begin{bmatrix} -0.0369 & -1.6667 \\ -2.5895 & 1.6545 \end{bmatrix}$, $H_3 = \begin{bmatrix} -0.0373 & -0.0187 \\ 0.1074 & 0.0537 \end{bmatrix}$,

$$L(\rho) = \begin{bmatrix} -11.9986 - 4.5681\rho_1 - 0.7408\rho_2 - 0.0247\rho_1\rho_2 \\ 10.1780 + 3.0436\rho_1 - 1.9368\rho_2 + 0.0712\rho_1\rho_2 \end{bmatrix},$$

$M = \begin{bmatrix} 1.1653 \\ 0.9950 \end{bmatrix}$ and

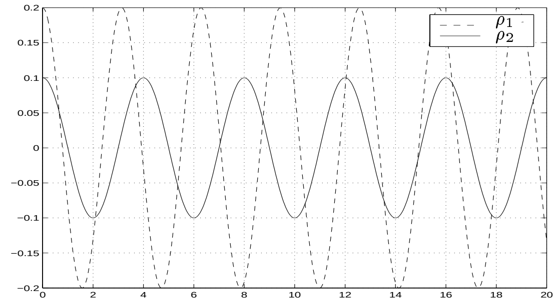

Figure 2: The parameters $\rho_1(t)$ and $\rho_2(t)$.

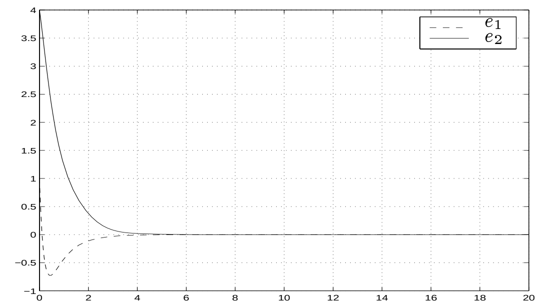

Figure 3: Estimation errors: e_1, e_2.

$$J(\rho) = TB(\rho) = \begin{bmatrix} 1.0058 & -2.9249 \\ -2.5978 & 3.7079 \end{bmatrix} B(\rho).$$

In the case of variation of the parameters $\rho_1(t)$ and $\rho_2(t)$ as in Figure 2 and with a nul command, this observer gives the estimation errors presented in Figure 3.

6 Conclusion

In this paper, a parameter varying observer design has been addressed. The method is based on gain-scheduling of a full-order observer for estimation of the state of an LPV system. Linear observers are designed for extremal values of the parameter vector and the scheduled observer is obtained by linear interpolation according to each parameter. The design of this linear observers are based on the existence of an affine parameter-dependent Lyapunov function and it is expressed as an LMI feasibility condition with rank-n constraint.

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