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Bounding approach to the diagnosis of a class of uncertain static systems

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Abstract. A new approach to the diagnosis of a class of uncertain static systems is described. Using this approach, a diagnostic procedure can take into account some uncertainties at the time of a decision, instead of dismissing them, as is often the case. After first itemizing the basic principles and then formulating the diagnostic problem in an uncertain context, we will describe a direct resolution method and highlight its lacunae. We will then set forth an indirect method based on an analysis of parallelotopes representative of the uncertainties in question. The presentation of this approach is the major theoretical part of this work. We will subsequently show how to determine an indicator representing the difference between the real behavior of the system and the limit of its normal behavior. We will compare this approach with a traditional parity space approach in order to highlight the fact that the bounding approach offers guarantees which parity space approaches cannot offer.

Key-words: *fault diagnosis, uncertain static linear models, bounding approach*

1. Introduction

Many works in the international literature have dealt with diagnosis for the past 15 years or so. De Kleer and Williams (1987) proposed a particularly interesting diagnostic methodology. Their works, needless to add, only apply to deterministic static systems, but their considerable interest resides in the fact that they formulate the diagnostic problem in a very interesting way, in particular by introducing the concept of consistency. Elsewhere, Chow and Willsky (1984), followed by Massoumnia and Van der Velde (1988) introduced parity space techniques. These techniques, which are especially well-suited to sensor fault detection, usually constitute the hypothesis, during fault isolation, that, on the one hand, the physical system is behaving normally and, on the other, that the system model is certain. When the model is uncertain, these techniques attempt to dismiss the uncertainties without taking them into consideration during a diagnostic operation. In tandem, Patton and Frank (1989) developed approaches based on state observers, but just like the previous ones, these approaches attempt to dismiss uncertainties instead of taking them into consideration. Needless to say, the Kalman filter (Keller, 1997) is an exception here, because it includes vectors of stochastic variables and helps to determine, automatically, the thresholds indexed on standard differences. The uncertainties appearing in the models used by Kalman filters have an additive structure, which can only lead to regular thresholds around variables reconstructed by the filter. One or two researchers have shown an interest on techniques taking uncertainties into consideration (Horak and Allison, 1990; Chang et al, 1995).

About ten years ago, we saw the publication of works formulating identification problems in a membership context (Walter and Piet-Lahanier, 1987; Norton, 1987). Instead of representing the uncertainties by Gaussian stochastic variables, these approaches--also known as bounding approaches--represent the uncertainties by a set of possible values, where only the bounds were known. Based on this idea, many works have seen the light of day; they have been collected in a collective volume that presents the main results (Milanese et al, 1996). But these works have barely got beyond the estimation theory context.

In this article, we describe a technique akin to the bounding approach, even if the formalism may differ in a good many points, in order to adept to the problems inherent to the diagnosis of uncertain systems. In this article we will limit ourselves to the example of a class of uncertain static systems.

2. Problem formulation

It is easy to show that any diagnostic operation is based on a consistency test between observations and a behavioral model (Ploix, 1998). The model may represent a normal pattern of behavior or an abnormal one, when it is dedicated to a specific fault (Travé-Massuyès et al, 1997). In the first instance, we talk in terms of normal-operation-oriented procedures; in the second, of abnormal-operation-oriented procedures.

Independently of the type of procedures involved, the diagnosis resides in the conclusions drawn from consistency tests: the behavior in question is akin to the behavior represented or otherwise. In theory, an inconsistency inevitably reveals that the behavior observed does not match the behavior represented. Conversely, in the event of consistency, the only possible conclusion is a pattern of behavior that apparently complies with the behavior represented, although it is not possible to be categorical about this. We will return to this aspect in part five. An inconsistency must be understood in the strict sense "the behavior does not match the behavior represented" whereas a consistency must be interpreted with certain reservations "the behavior is akin to the behavior represented". It is nevertheless clear that a consistency is all the more probable when it is maintained during different operating modes.

Having thus reminded readers of the general principles of diagnosis, it now remains for us to take a look at what a consistency test becomes when the behavioral model involves uncertainties. In the case of a model that includes no uncertainty, it is enough to check whether the observations in turn check the models' deterministic equations. What happens in an uncertain context? First and foremost, it is obvious that a model containing uncertainties represents a set of possible types of behavior. This set may be represented by the laws of distribution specific to the stochastic context. It nevertheless seems preferable to work in a membership context, which describes a set solely by its bounds. If, in fact, the laws of Gaussian distribution are stable for addition, they no longer are for multiplication; as far as the laws of uniform distribution are concerned, these are already not stable for addition. Otherwise put, if the Gaussian stochastic variables are particularly suitable for the representation of additive structures of uncertainties, the other situations are particularly delicate in this context.

In order to tackle models containing uncertainties by a bounding approach, we will introduce the concept of abstract space, given as $\mathcal{A}(\ )$, by analogy with the stochastic variables. If X is a bounded variable, in other words, if it is only known by the space to which it may belong, then this space will be given as $\mathcal{A}(X)$. The notation x should designate a particular realization of X; nevertheless, in order to simplify the signs, we will merge the notation of a realization x with that of the bounded variable X itself. Henceforth, x will designate, alternately, the bounded variable and one of its realization, and $\mathcal{A}(x)$ will designate the abstract space of the bounded variable.

We will deal with the affine uncertain static models in the abstract variables. It should be noted that by uncertain we mean that certain parameters correspond to bounded variables. So we will only be considering those physical systems which can be represented by a static model including a term $N(\tilde{y}_k)$, representing the certain part of the model and a linear uncertain term in relation to the uncertainties $M(\tilde{y}_k)$, of the type:

$$M(\tilde{y}_k)\upsilon_k + N(\tilde{y}_k) = 0 \qquad (1)$$

$N(\tilde{y}_k)$ is a vector of $\mathbb{R}^n$ reliant upon observations $\tilde{y}_k$ made at the moment k. The vector υ_k contains m standardized bounded variables, in other words, υ_k verifies $\mathcal{A}(\upsilon_k)=\mathcal{B}_\infty^m$ where $\mathcal{B}_\infty^m$ is a unitary ball brought about by an infinite norm $\mathcal{B}_\infty^m=\{\vartheta\in\mathbb{R}^m/\|\vartheta\|_\infty\le 1\}$. The matrix $M(\tilde{y}_k)$ thus measures nxm and is dependent upon the observations $\tilde{y}_k$. Where the matrix $M(\tilde{y}_k)$ is concerned, we will presume that it is full line rank.

Let us note that this hypothesis is in no way restrictive. In effect, if the matrix $M(\tilde{y}_k)$ is not full line rank, it can always be broken down into $M(\tilde{y}_k)=M_1(\tilde{y}_k)^{\mathrm{T}}M_2(\tilde{y}_k)$ where the matrices $M_1(\tilde{y}_k)$ and $M_2(\tilde{y}_k)$ are both full line rank. In this case, we deduce that the consistency test is identical to the following:

$$\begin{cases} M_2(\tilde{y}_k)\upsilon_k + \left(M_1(\tilde{y}_k)M_1(\tilde{y}_k)^{\mathrm{T}}\right)^{-1} M_1(\tilde{y}_k)N(\tilde{y}_k) = 0 \\ \left(I_{rank(M_1)} - M_1(\tilde{y}_k)^{\mathrm{T}}\left(M_1(\tilde{y}_k)M_1(\tilde{y}_k)^{\mathrm{T}}\right)^{-1} M_1(\tilde{y}_k)\right)N(\tilde{y}_k) = 0 \end{cases}$$

The second test corresponds to a deterministic model and is carried out in a straightforward way, while the first test may be put in form (1). Consequently, even if $M(\tilde{y}_k)$ is not full line rank, it is still possible to revert to an equivalent form where $M(\tilde{y}_k)$ becomes it. Do the hypothesis is not restrictive; it is aimed at packaging the problem in a standard form.

In order to go ahead with a consistency test between the observations $\tilde{y}_k$ and the model (1), two different approaches may be adopted. The direct approach tests whether

$$\exists\upsilon_k \in \mathcal{B}_\infty^m \,/\, M(\tilde{y}_k)\upsilon_k + N(\tilde{y}_k) = 0 \qquad (2a)$$

while the indirect approach verifies whether

$$\{0\} \in \mathcal{A}\left(M(\tilde{y}_k)\upsilon_k + N(\tilde{y}_k)\right) \qquad (2b)$$

with

$$\mathcal{A}\left(M(\tilde{y}_k)\upsilon_k + N(\tilde{y}_k)\right) = \left\{M(\tilde{y}_k)\upsilon_k + N(\tilde{y}_k); \|\upsilon_k\|_\infty \le 1\right\}$$

We will start by taking a close look at the direct approach.

3. Direct approach of consistency tests

The observations $\tilde{y}_k$ will be consistent with the model (1) if there is a vector υ_k verifying (2a). By not taking the restriction $\|\upsilon_k\|_\infty\le 1$ into consideration, we deduct that the set of solutions υ_k verifying (2a) is as follows

$$\left\{\begin{aligned} \upsilon_k / \upsilon_k &= -M(\tilde{y}_k)^+ N(\tilde{y}_k) + \cdots \\ &\cdots\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi ; \chi \in \mathbb{R}^{m\times 1} \end{aligned}\right\} \qquad (3)$$

where $M(\tilde{y}_k)^+ = M(\tilde{y}_k)^{\mathrm{T}}\left(M(\tilde{y}_k)M(\tilde{y}_k)^{\mathrm{T}}\right)^{-1}$.

By introducing the restriction $\|\upsilon_k\|_\infty \le 1$, we obtain the fact that the consistency testing in a direct way is the same as looking for a value χ, like:

$$\left\|-M(\tilde{y}_k)^+ N(\tilde{y}_k)+\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi\right\|_\infty \le 1$$

This problem may be reformulated in the following way:

$$\inf_{\chi\in\mathbb{R}^m}\left(\left\|-M(\tilde{y}_k)^+ N(\tilde{y}_k)+\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi\right\|_\infty\right) \le 1 \quad (4)$$

The minimum appearing in (4) may be assessed numerically using the simplex method, for example. Nevertheless, this kind of approach fails to guarantee that the result is an global minimum and, what is more, the number of iterations necessary for the calculation can be considerable. When the condition (4) is met, the conclusion must be a positive consistency test, if not, it is awkward to conclude categorically with inconsistency. An analytical approach would be preferable even though it is hard to apply because the infinite norm cannot be derived in 0. To put this problem right, we will look for the minimum appearing in (4) by substituting, for example, the Euclidean norm for the infinite norm. Because the Euclidean norm is more restrictive than the infinite norm in the sense that $\{x/\|x\|_2 \le 1\} \subset \{x/\|x\|_\infty \le 1\}$, we deduce a sufficient condition of consistency:

$$\inf_{\chi\in\mathbb{R}^m}\left(\left\|\begin{array}{l}-M(\tilde{y}_k)^+ N(\tilde{y}_k)+\cdots\\ \cdots\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi\end{array}\right\|_2\right) \le 1$$

The problem of minimization corresponds to a classic problem of least squares. The criterion to be minimized is the following;

$$\begin{array}{l}J(\chi) = \tfrac{1}{2}\left(-M(\tilde{y}_k)^+ N(\tilde{y}_k)+\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi\right)^{\mathrm{T}}\cdots\\ \qquad\cdots\left(-M(\tilde{y}_k)^+ N(\tilde{y}_k)+\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi\right)\end{array}$$

The minimum is obtained when the derivative of this criterion in relation to χ is cancelled out. It is easy to show that this is obtained when:

$$\left(I_m - M(\tilde{y}_k)^+ M(\tilde{y}_k)\right)\chi = 0$$

By inserting this result in (4), we obtain the following sufficient condition of consistency:

$$\left\|M(\tilde{y}_k)^+ N(\tilde{y}_k)\right\|_\infty \le 1 \qquad (5)$$

This condition, needless to say, is less conservative that (4) but it has the advantage of being quicker to assess.

The consistency test can thus be broken down into two levels. The first corresponds to the sufficient condition of consistency (5) while the second consists in minimization (4). Figure 1 represents the sequence of these two test levels. The second hatched one is difficult to achieve not only because it calls for a considerable computation time which cannot be bounded, but also because it may lead to a dubious conclusion: the categorical aspect of an inconsistency ends up altered. Polytope construction techniques correct these problems.

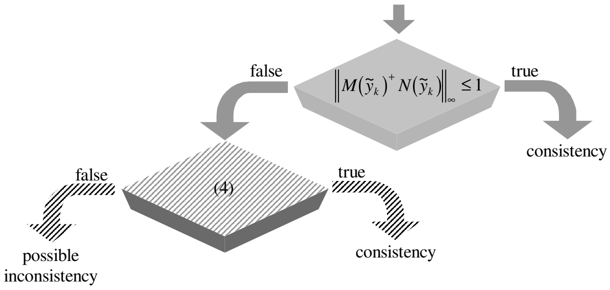

Figure I – 2 levels of direct consistency test

4. Indirect approach of consistency tests

The general principle behind this approach is to construct the polytope defined by the field of vectors appearing in (2b) then to check whether the origin of the coordinate axes belongs to this domain. Before assessing this domain (part 4.2), we will begin by assessing the aligned orthotope which circumscribes it. In part 4.3, we will see how the consistency between the measurements and the model may be appreciated.

To simplify the symbols, in this part, we will omit references to the fact that the matrices of the field of vectors depend on the measurements $\tilde{y}_k$ and we will note the abstract variables υ instead of υ_k.

4.1. Design of a circumscribed aligned orthotope

Before solving (2b), let us briefly turn our attention to the concept of circumscribed aligned orthotope. Let us posit $z = M\upsilon + N$. We have seen that the notation $\mathcal{A}(z)$ designates the abstract space of z or the set of its possible values. We note $\square\mathcal{A}(z)$ the orthotope aligned with the coordinate axes and circumscribed to $\mathcal{A}(z)$. This domain has the advantage of being more easily assessable than $\mathcal{A}(z)$. It is enough to look separately for the bounds which each variable z_i of z achieved, in other words $\mathcal{A}(z_i)$. ψ_i will designate a dimension line vector n, nil with the exception of the first element equal to 1. The domain $\mathcal{A}(z_i)$ will then be written $\mathcal{A}(\psi_i M\upsilon + \psi_i N)$ and the formulas of the interval arithmetic (Moore, 1979), enable us to conclude that:

$$z \in \square\mathcal{A}(z) \Leftrightarrow \forall i \in \{1,\cdots,n\}, \left|z_i - \psi_i N\right| \le \left\|\psi_i M\right\|_1 \qquad (6)$$

However, with the exception of the case where z is scalar, substituting $\square\mathcal{A}(z)$ in the consistency test (2b) leads to marked inaccuracy (see part 5).

4.2. Design of the exact uncertain domain

The abstract domain $\mathcal{A}(z)$ is a parallelotope (Vicino and Zappa, 1996; Ziegler, 1998), centered in N, in other words a convex domain delimited by two by two hyperplans which are parallel with each other. To simplify the problem, we will posit $z'=z-N$ and calculate $\mathcal{A}(z')$, centered on the origin, instead of $\mathcal{A}(z)$.

By its very nature, a parallelotope is the intersection of strip constraints $\mathcal{S}_i$, written in a general manner:

$$S_i = \{ z' / |H_i z'| \le K_i \} \text{ with } K_i \in \mathbb{R} \tag{7a}$$

because $\mathcal{A}(z')$ is centered on the origin. This strip constraint can be referred to $\mathcal{A}(z)$:

$$S_i = \{ z / -K_i + H_i N \le H_i z \le K_i + H_i N \} \tag{7b}$$

The purpose of this part is to look for the set of strip constraints such as

$$\mathcal{A}(z) = \bigcap_i \mathcal{S}_i \tag{8}$$

with the aim of carrying out the consistency tests (2b). In fact, by disposing of all the strip constraints, we can end up with a consistency if $\{0\} \in \mathcal{A}(z)$, in other words, if

$$\forall i, -K_i + H_i N \le 0 \le K_i + H_i N \tag{9}$$

In the event that one of these conditions were not satisfied, it would be necessary to conclude with an inconsistency.

In order to assess the expressions of the hyperplans $\mathcal{B}_i^+$ and $\mathcal{B}_i^-$, let us look for what must verify z' in order to belong to the facet of $\mathcal{A}(z')$ included in $\mathcal{B}_i^+$, for example. To get to this point, we will argue on the hyperplans delimiting the strip constraints. Each strip may be associated with two hyperplans delimiting it:

$$\mathcal{B}_i^+ = \{ z' / H_i z' \le K_i \} \text{ and } \mathcal{B}_i^- = \{ z' / -H_i z' \le K_i \} \tag{10}$$

Let us place ourselves in a point $z' \in \mathcal{A}(z')$, in other words, z' is such that there exists $\upsilon \in \mathcal{B}_\infty^m$ for which $z'=M\upsilon$. Let us note that if υ varies slightly from $\partial\lambda$ ($\partial\lambda \in \mathbb{R}$) in the direction v, then, by noting $L_v(z')=Mv$ the Lie derivative following v, we deduce that z' will vary from $\partial z'=L_v(z')\partial\lambda$. If z' belongs to $\mathcal{A}(z') \cap \mathcal{B}_i^+$, then the two following conditions must be verified.

Firstly, the hyperplan $\mathcal{B}_i^+$ must be spanned by n-1 independent vectors $\{L_{v_1}(z'),\dots,L_{v_{n-1}}(z')\}$ associated with n-1 independent vectors of $\mathbb{R}^m$ $\{v_1,\dots,v_{n-1}\}$:

$$rank\left(M \begin{bmatrix} v_1 & \cdots & v_{n-1} \end{bmatrix} \right) = n-1 \tag{11a}$$

Whatever the nature of v not belonging to the subspace $\mathcal{E}_{\{v_1,\dots,v_{n-1}\}}$ spanned by the vectors $\{v_1,\dots,v_{n-1}\}$, a variation in a direction $v\partial v$ must lead outside the domain $\mathcal{A}(z')$, whereas a variation in the other direction leads inside $\mathcal{A}(z')$. By noting $\mathcal{E}_{\{v_n,\dots,v_m\}}$ the complement, spanned by the vectors $\{v_n,\dots,v_m\}$, of $\mathcal{E}_{\{v_1,\dots,v_{n-1}\}}$ in $\mathbb{R}^m$, we will convey this condition by the following equation:

$$\forall v \in \mathcal{E}_{\{v_n,\cdots,v_m\}}, \exists \upsilon \in \mathcal{B}_\infty^m, \partial\lambda \in \mathbb{R}^* / \\ \begin{cases} \|\upsilon + v\partial\lambda\|_\infty \le 1 \\ \|\upsilon - v\partial\lambda\|_\infty > 1 \end{cases} \tag{11b}$$

Because the dimension of the kernel of M is a priori not nil, the conditions (10) are necessary but not sufficient.

The condition (10b) is not particularly easy to manipulate in this form. To reformulate, let us break down υ into $\upsilon=[v_1 \dots v_{n-1}]\mu+[v_n \dots v_m]\zeta$ and v into $v=[v_n \dots v_m]\chi$, the condition (10b) then becomes: $\forall\chi \in \mathbb{R}^{m-n+1}$:

$$\exists\left(\begin{bmatrix} v_1 & \cdots & v_{n-1} \end{bmatrix}\mu, \begin{bmatrix} v_n & \cdots & v_m \end{bmatrix}\zeta \right) \in \mathcal{B}_\infty^m \times \mathcal{B}_\infty^m, \partial\lambda \in \mathbb{R}^* / \\ \begin{cases} \left\| \begin{bmatrix} v_1 & \cdots & v_{n-1} \end{bmatrix}\mu + \begin{bmatrix} v_n & \cdots & v_m \end{bmatrix}(\zeta + \chi\partial\lambda) \right\|_\infty \le 1 \\ \left\| \begin{bmatrix} v_1 & \cdots & v_{n-1} \end{bmatrix}\mu - \begin{bmatrix} v_n & \cdots & v_m \end{bmatrix}(\zeta + \chi\partial\lambda) \right\|_\infty > 1 \end{cases}$$

This condition is easy to interpret in so much as the vectors $\{v_1,\dots,v_m\}$ define an orthonormal base of $\mathbb{R}^m$. In this instance, the condition in fact becomes:

$$\|\zeta\|_\infty = 1 \tag{12}$$

By introducing this result into the expression of z', we deduce that the set of the values z' belonging to the border $\mathcal{B}_i^+$, is written thus:

$$z' = M\begin{bmatrix} v_1 & \cdots & v_{n-1} \end{bmatrix}\mu + M\begin{bmatrix} v_n & \cdots & v_m \end{bmatrix}\zeta ; \|\zeta\|_\infty = 1$$

where the vectors v_i are the vectors of an orthonormal base.

In order to refer back to a Cartesian equation, it is necessary to get rid of the term in μ, in other words, it is necessary to find a matrix H_i such as:

$$H_i M[v_1 \dots v_{n-1}]=0 \tag{13}$$

Now, as a result of the condition (11a), the set of solutions is given by a rank 1 line matrix. Therefore, all the combinations of n-1 vectors defining an orthonormal base of $\mathbb{R}^m$ lead, when the condition (11a) is verified to a matrix H_i defining a Cartesian equation of a border $\mathcal{B}_i^+$ of strip constraint:

$$H_i z' = H_i M\begin{bmatrix} v_n & \cdots & v_m \end{bmatrix}\zeta ; \|\zeta\|_\infty = 1$$

Nevertheless, ζ is not yet determined. This results come from the fact that the conditions (11) were necessary on their own. If the matrix H_i necessarily verifies (13) then $H_i z'=H_i M\upsilon$ and the formulae of interval arithmetic allow the equation:

$$-\|H_i M\|_1 \le H_i z' \le \|H_i M\|_1$$

The constraint K_i of (9) associated with M_i is unique: it is valid for $K_i=\|H_i M\|_1$.

We may conclude from this that each one of the strip constraints (7) corresponds to an n-1 uplets of vectors of an orthonormal base of $\mathbb{R}^m$ verifying $rank\left(M\left[v_1 \quad \cdots \quad v_{n-1}\right]\right)=$ n-1. At most, then, there are C_m^{n-1} strip constraints. In other words $2\,C_m^{n-1}$ facets. Each of the strip constraints (7) is defined by the matrices H_i and K_i defined by:

$$H_iM[v_1 \ldots v_{n-1}]=0 \text{ et } K_i=\|H_iM\|_1 \qquad (14)$$

Let us take the example of a domain $z=M\upsilon$ ($z=z'$ because the matrix N has been chosen nil) defined by the following matrix M:

$$M=\begin{bmatrix}2 & -1 & -1 & 1 & 0\\ 1 & 1 & 0 & 0 & -1\\ 1 & -1 & 2 & 0 & 0\end{bmatrix}$$

In this case, n is equal to 3 and m to 5. So it is a question of finding all the doublets of vectors of an orthonormal base of $\mathbb{R}^5$. Let us take a look, for example, at the following doublets presented in the form of a two-column matrix:

$$\Omega_{12}=\begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\end{bmatrix}\;\Omega_{13}=\begin{bmatrix}1&0\\0&0\\0&1\\0&0\\0&0\end{bmatrix}\;\Omega_{23}=\begin{bmatrix}0&0\\1&0\\0&1\\0&0\\0&0\end{bmatrix}\;\ldots$$

We should note that each doublet of vectors corresponds to the elimination of a doublet of abstract variables of υ in the Cartesian equation of the strip constraint. The strip constraints obtained are written thus:

$$\mathcal{S}_{12}: \left|\begin{bmatrix}-2 & 1 & 3\end{bmatrix}z\right| \le 11$$
$$\mathcal{S}_{13}: \left|\begin{bmatrix}2 & -5 & 1\end{bmatrix}z\right| \le 15$$
$$\mathcal{S}_{23}: \left|\begin{bmatrix}2 & 3 & 1\end{bmatrix}z\right| \le 13$$

By proceeding in the same way for C_5^2 possible doublets, we deduce the facets of the domain $\mathcal{A}(z)$ appearing in figure II:

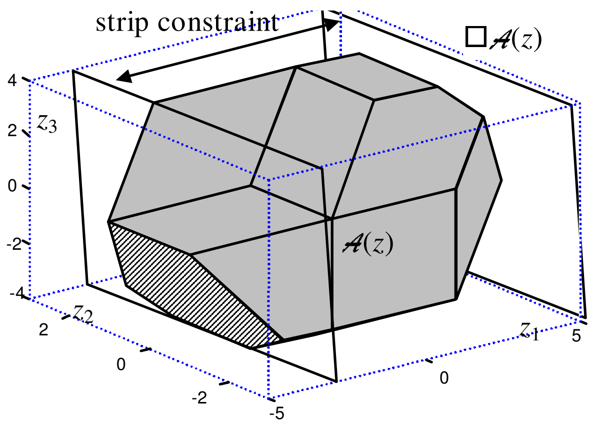

Figure II – Domain $\mathcal{A}(z)$ and strip constraint

4.3. Computation of a distance to border

The indirect approach turns out to be clearly more effective than the direct approach in so much as it executes precisely the consistency test being sought. Nevertheless, the dichotomous results of the test (2b) may appear somewhat poor when we consider all the information at our disposal. The aim of this part is no longer to determine whether the element {0} belongs or not to the domain $\mathcal{A}(Mz+N)$, but to assess the distance separating the origin of the coordinate axes to the border closest to the parallelotope $\mathcal{A}(z)$. In this way, a decision of consistency will be combined with a distance in relation to the decision of inconsistency or vice versa.

We have seen that these facets are hyperplans $\mathcal{B}_i^+$ or $\mathcal{B}_i^-$ given by:

$$\begin{cases}\mathcal{B}_i^+ : H_i z = H_i N + K_i\\ \mathcal{B}_i^- : H_i z = H_i N - K_i\end{cases}$$

Instead of assessing the distance of origin to the nearest border $\mathcal{B}^*$, it seemed to us more apposite to assess the quotient between this distance and the distance separating the center of the parallelotope and the border $\mathcal{B}^*$. The advantage of this quotient is that it normalizes the result: a distance of 1 in the event of consistency means that the origin O is merged with the center N of the parallelotope, whereas in the event of inconsistency, the value 1 means that we are as far from the border $\mathcal{B}^*$ as the center N is from this border. What is more, the more this quotient of distance, called d, tends towards zero, the closer we get to a border, in other words, the more we find ourselves at the limit between consistency and inconsistency.

We will note respectively J^* and Q , the points of the border $\mathcal{B}^*$ which are respectively the closest, in the sense of norm 2, to the origin O and the center N of the parallelotope (see figure III, where the circles appear as ellipses as a result of the scale). The quotient of distance d is expressed as follows:

$$d=\frac{\left\|\overrightarrow{OJ^*}\right\|_2}{\left\|\overrightarrow{NQ}\right\|_2} \qquad (15)$$

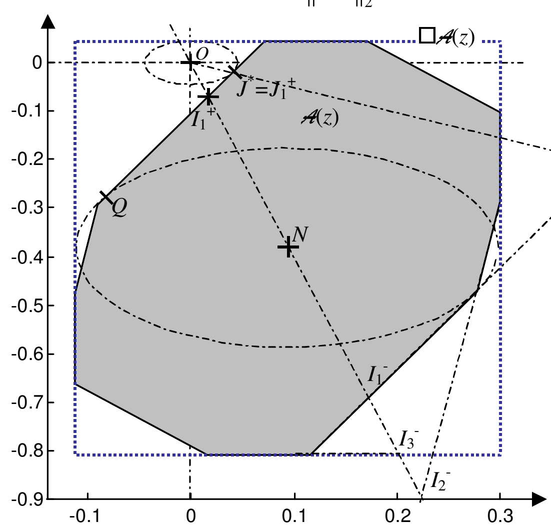

Figure III – Elements for computation of distance

To find $\mathcal{B}^*$, we will take advantage of the convexity of the domain $\mathcal{A}(z)$. First, assume the case $N=0$ is proceeded separately: it yields d=1. Otherwise, we will start by calculating the intersections between the straight line $\mathcal{D}$, connecting the origin O to the center N of the parallelotope, and each of the hyperplans $\mathcal{B}_i^+$ and $\mathcal{B}_i^-$. When it exists, the intersection between the straight line $\mathcal{D}$ and a hyperplan of the border is a point, it will be written I_i^+ when it is a matter of the hyperplan $\mathcal{B}_i^+$, and I_i^- for the hyperplan $\mathcal{B}_i^-$. The coordinates of these points are given by:

$$I_i^+ = \frac{H_i N + K_i}{H_i N} N \text{ and } I_i^- = \frac{H_i N - K_i}{H_i N} N$$

The search for the hyperplan $\mathcal{B}^*$ closest to the origin starts by a search for the hyperplan couple $\mathcal{B}_{i^*}^+$ and $\mathcal{B}_{i^*}^-$ to which $\mathcal{B}^*$ belongs. To do this, it is necessary to determine the hyperplan $\mathcal{B}_{i^*}^+$, for which the distance between N and the I_i^+ (or the I_i^-) is the smallest, in other words finding the index i^* as follows:

$$i^* = \arg\left(\min_i\left(\left|\frac{K_i}{H_i N}\right|\right)\right).$$

Because N is the center of the parallelotope, the two hyperplans $\mathcal{B}_{i^*}^+$ and $\mathcal{B}_{i^*}^-$ are as close to N. To go back to $\mathcal{B}^*$, we retain only the one which is the closest to the origin O. To do this, we orthogonally project O onto these two hyperplans (points J_1^+ and J_1^- in figure III), then we evaluate the distances:

$$\left\|\overrightarrow{OJ_{i^*}^+}\right\|_2 = \frac{|H_{i^*}N + K_{i^*}|}{\|H_{i^*}\|_2}, \left\|\overrightarrow{OJ_{i^*}^-}\right\|_2 = \frac{|H_{i^*}N - K_{i^*}|}{\|H_{i^*}\|_2}.$$

The smaller of these two distances determines the hyperplan $\mathcal{B}^*$. We will write J^* the orthogonal projection of O on $\mathcal{B}^*$.

Lastly, in compliance with (15), all that remains to do is to divide $\left\|\overrightarrow{OJ^*}\right\|_2$ by $\left\|\overrightarrow{NQ}\right\|_2$. We thus obtain:

$$d = \frac{\min\left(|H_{i^*}N - K_{i^*}|, |H_{i^*}N + K_i|\right)}{|K_{i^*}|} \quad (16)$$

with $i^* = \arg\left(\min_i\left(\left|\frac{K_i}{H_i N}\right|\right)\right)$

5. Example

To illustrate the previous developments, we describe a diagnostic procedure deriving from the classic static model of the direct current machine. We introduced three model uncertainties: on the resistance (υ_1(t)), on the coefficient of viscous friction (υ_2(t)) and on the electromagnetic constant (υ_3(t)). The model is made up of two static relations, the first represents the electrical part of the machine, the second the mechanical part.

$$\begin{cases} \frac{d[i]}{dt} = -(1+\rho_1\upsilon_1(t))\frac{[i]}{\tau_1} + (1+\rho_3\upsilon_3(t))\frac{\gamma-1}{\gamma\tau_1}[\omega] + \frac{[u]}{\gamma\tau_1} = 0 \\ \frac{d[\omega]}{dt} = (1+\rho_2\upsilon_2(t))\frac{[\omega]}{\tau_2} - (1+\rho_3\upsilon_3(t))\frac{[i]}{\tau_2} = 0 \end{cases} \quad (17a)$$

with $\tau_1 = L/R_n = 17ms$, $\tau_2 = J/f_n = 1s$, $\gamma = \frac{R_n f_n}{R_n f_n + K_n^2} = 0.118$.

R_n, f_n, K_n designate respectively the nominal values of the above mentioned uncertain parameters. Let us note that if the first uncertainty only has an influence on the electrical equation of the machine and the second only intervenes in the mechanical equation, the last one intervenes in both relations. The input corresponds to the voltage supply and the outputs are the current and the speed, with the brackets indicating that we are dealing with physical variables.

The equations of the measurements are then added. It is supposed that the power supply is a perfectly known quantity: $[u] = \tilde{u}$ where the symbol "~" denotes that the associated variable corresponds to the measurement. On the other hand, the measurements of current and speed are presumed to be imperfect:

$$\begin{cases} \tilde{i} = (1+\rho_4\upsilon_4(t))[i] + \rho_6\upsilon_6(t) \\ \tilde{\omega} = (1+\rho_5\upsilon_5(t))[\omega] + \rho_7\upsilon_7(t) \end{cases}. \quad (17b)$$

$\upsilon_4(t)$ and $\upsilon_5(t)$ are uncertainties of a multiplicative nature whereby it is possible to take into consideration the non-linearities of sensors. $\upsilon_6(t)$ and $\upsilon_7(t)$ are additive terms representing sensor noises. All the abstract variables, which are brought together in one vector $\upsilon(t)$, are standardized. The different coefficients ρ_i define the range of the different uncertainties. The scalar ρ_1 equals 0.5, which means that the resistance may vary by ±50% around its nominal value R_n. The value 0.2 is given to ρ_2 and ρ_3: while 0.05 represents the range of the four uncertainties ρ_4, ρ_5, ρ_6, ρ_7 of sensors.

By combining these relations (17) so that only measurements will appear from now on, and after linearization around low uncertainty values, we obtain, respectively, the following electrical and mechanical relations:

$$\begin{bmatrix} \tilde{y}(t)^T\theta_1 \\ \tilde{y}(t)^T\theta_2 \end{bmatrix} + \begin{bmatrix} \tilde{y}(t)^T T_1 & \gamma\rho_6 & (1-\gamma)\rho_7 \\ \tilde{y}(t)^T T_2 & \rho_6 & -\rho_7 \end{bmatrix}\upsilon(t) = 0 \quad \begin{matrix}(18a)\\(18b)\end{matrix}$$

where $\tilde{y}(t) = \begin{bmatrix}\tilde{i}(t) & \tilde{\omega}(t) & \tilde{u}(t)\end{bmatrix}$, the vector υ(t) contains in increasing order of indices the abstract variables υ_i(t). The expressions of the different matrices are as follows:

$$\theta_1 = \begin{bmatrix} -\gamma \\ \gamma-1 \\ 1 \end{bmatrix}, T_1 = \begin{bmatrix} -\gamma\rho_1 & 0 & 0 & \gamma\rho_4 & 0 \\ 0 & 0 & -(1-\gamma)\rho_3 & 0 & (1-\gamma)\rho_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, T_2 = \begin{bmatrix} 0 & 0 & -\rho_3 & \rho_4 & 0 \\ 0 & \rho_2 & 0 & 0 & -\rho_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We have simulated the system based on the linearized model by adding three parametric faults which will appear in sequence. Between moments 25 and 39, the resistance is kept at three times its nominal value. Between 40 and 54, the electromagnetic coefficient is fixed at 1.75 K_n, then between 55 and 69 it is the viscous friction that is fixed at 2.2 f_n. Under normal behavior, all the uncertain parameters develop in some sort of between their bounds.

In an initial phase, we construct, at each given moment, strip constraints associated with the two relations (18), then we assess the quotient of distance d representing the distance separating the origin from the polytope obtained (figure IV, uppermost graph). A minus sign indicates that the origin of the reference is situated within the parallelotope, and thus that the behavior of the system is akin to a normal pattern of behavior. Conversely, a plus sign indicates that the origin is outside the calculated polytope, thus revealing the presence of a fault. We note, in figure IV, that the first two faults are perfectly detected, whereas with the third, the test result only reveals the inconsistency at specific points. This could come as a surprise because the results of this approach are guaranteed: an inconsistency necessarily reveals a behavioral anomaly. The fact remains that this result can be explained by the fact that the excitation supply is practically nil in this area of operation--in other words, the machine is not being supplied with power any more. In these conditions, it is imminently understandable that the fault should not be diagnosed. Consistency does not necessarily imply the absence of faults, and this is independent of the method used.

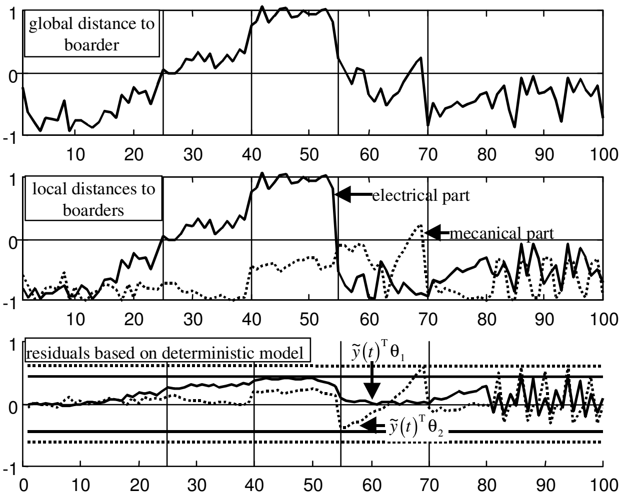

Figure IV – Results of consistency tests

In a second phase, we try to improve the diagnosis by making separate examinations of the electrical and mechanical parts of the machine. To this end, at any given moment, we separately determined the constraints associated with the relations (18a) representing the electrical part and (18b) representing the mechanical part, and then we assessed the associated distances. The two distances are represented in the central diagram of figure IV. The first two faults, one of which corresponds to a resistance fault and the other to a electromagnetic constant fault, lead to inconsistencies at the electrical level. This shows that these faults affect the electrical part of the machine. The distance associated with the mechanical part does not make it possible to assert categorically that the second fault is situated on K, because the distance has never become positive. It is nevertheless natural to envisage it in so far as in this simulation sector the distance has become considerably closer to zero. Lastly, it can be observed that the last fault only affects the mechanical part, which is natural for viscous friction.

We should note that the local tests are less precise than the overall tests. This can be explained by the fact that the local tests try to find out whether the origin is inside an aligned orthotope circumscribed by the abstract domain $\mathcal{A}(z)$. As a result of this, the detection becomes less accurate. This can be seen on figure IV at the moments between 55 and 60; the fault is detected by testing the abstract domain of the complete model, whereas the test relating to each of the relations reveals nothing. Figure III representing the abstract domain and the circumscribed orthotope associated with moment 55 visually confirms this fact.

The lower diagram shows two residuals obtained by not taking uncertainties into account in the model during the detection phase. The relations (18) are thus limited to:

$$\begin{bmatrix} \tilde{y}(t)^T\theta_1 \\ \tilde{y}(t)^T\theta_2 \end{bmatrix} = 0 \,. \tag{19}$$

The thresholds are obtained by studying the system in normal operation and by adjusting it in such a way as to avoid any false alarm. It should be noted that, with this example, the technique of classic parity space, which does not take uncertainties into account, does not detect any fault. In addition, this technique offers no guarantee. It is not because the residuals shift markedly away from zero that it is possible to conclude that we are necessarily in the presence of a behavioral anomaly.

6. Conclusion

We have proposed an alternative to the traditional diagnostic methods for uncertain static systems. Having rejected the direct solution of the problem, we have shown that the bounding approach makes it possible not only to take uncertainties into account during a diagnostic operation, but also to guarantee the results obtained. In addition to the guaranteed diagnosis in its dichotomous form, we have shown that it could be complemented by an index representing the distance separating the behavior observed from the limit of normal behavior. Needless to say, the results described only have a bearing on one class of uncertain static systems, but they may be extended to a broader class as well as dynamic systems (Ploix, 1998). Let us note, however, that in the case of vectorial fields which are not affine in the uncertainties, the calculation of distance in relation to the limit of normality becomes more problematic. In conclusion, and using one particular example, we have tried to highlight the new prospects opening up for this approach which is particularly well suited to the synthesis of complete diagnostic procedures.

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