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FAULT DETECTION FOR TIME-DELAY SYSTEMS WITHIN ROBUST RESIDUAL BASED ON FINITE MEMORY OBSERVERS.

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Abstract: Fault detection is usually achieved within residuals generation (based on analytical redundancy) and evaluation. Such methods need the knowledge of a system model and therefore have to be robust to disturbances such as model uncertainties and measurement errors. In order to free oneself from the drawbacks of classic methods, as the observer performance degradation, the concept of finite memory observer has been introduced. This paper gives a short summary of the observer structure and some of its properties to further focus on the choice of the residual. In general the estimation error is taken as the residual but another residual is defined here. This residual is the difference of two state estimations at the same instant but based on two overlapping observation horizons. The relevance of this paper is to prove that this special residual is 'more' robust with respect to some model uncertainties than the estimation error if the fault detection problem is tackled. Finally, the proposed method is applied to detect sensor faults for time-delay systems. The relevance of this paper is that the observer is based on a simplified model of the time-delay system that does not take into account the delayed states. The example given is a simulated one.

Key words: Time-delay systems, finite memory observer, robustness, fault detection, sliding window.

1. INTRODUCTION

This paper deals with a problem of huge interest for process control: fault detection and isolation (FDI) (Willsky, 1976) (Frank, 1990) (Patton, 1994). The diagnostic procedure has to be robust with respect to system uncertainties such as measurement noise and parameter variations. This paper only considers robust residuals generation based on observers. As all the process history is taken into account to compute the estimation (i.e. infinite memory), the model errors can be accumulated in the estimation error (the residual) that might then diverges. The finite memory observer (Medvedev and Toivonen, 1991a, 1991b) is designed to prevent such effect on the estimation error thanks to an estimation based only on a minimal number of measured data (Darouach, *et al.,* 1994). The residual proposed in this paper is defined as the difference between two estimations at the same instant but based on two overlapping horizons so that a detection window is defined (Kratz, *et al.*, 1997) (Nuninger, 1997). The relevance of this paper is to demonstrate that this special residual is better for fault detection as it is more robust with respect to model uncertainties as parameter variations for instance than the estimation error. In addition a criterion is given to chose the optimal sizes of the overlapping horizons. Therefore, no further hypothesis is required to prove the existence of the detection window. It is stressed that in this paper, time-delay systems are considered but the finite memory observers are based on simplified models (not taking into account the delayed states). As a consequence, these states stand for the model uncertainties to which ones the generated residual has to be robust.

This paper is organised as follows. First, the problem formulation and the finite memory observer structure are given. Second, the observer properties are summarised. Then, the special residual is defined and the residual sensitivity to additive sensor faults and to model uncertainties are studied. Finally, the residual generator is applied to a simulated time-delay system when only a simplified model is used to design the observer. A conclusion is also presented.

2. PROBLEM FORMULATION

2.1. System models

This paper considers the following state-space model of time-delay systems in continuous-time representation:

$$\dot{x}(t) = \sum_{s=0}^{S} A_s x(t - t_s) + \sum_{j=0}^{J} B_j u(t - t_j) \qquad (1.a)$$

$$y(t) = Cx(t) + v(t) \qquad (1.b)$$

where x is the state vector of dimension n, y the output measurement vector of dimension p, u the control input vector of dimension m and v the measurement noise vector (zero-mean with known covariance matrix V). Matrices A_s $(s = 0,\ldots,S)$, B_j $(j = 0,\ldots,J)$ and C are constant with appropriate dimensions. Integers s and j denote the number of time-delays in the state and control vectors respectively. Further, the design of the finite memory observer is based on the following simplified model of system (1) that does not take into account the influence of the delayed state and control vectors:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (2.a)$$

$$y(t) = Cx(t) + v(t) \qquad (2.b)$$

where A = A_0 and B = B_0. The sensibility of the proposed residual with respect to additive sensor faults and to model uncertainties will be studied further.

2.2. Static form of the system model (2)

At the instant t, a sliding observation window is considered defined by k+1 time-delay output measurements: $y_i = y(t - \tau_i)$, $i = 0,\ldots,k$. First x(t) is deduced from $x(t-\tau_i)$ by integration of equation (2.a) on the interval $[t-\tau_i, t]$. This expression is further replaced in the expression of y_i deduced from (2.b) so that the following relation is obtained:

$$y(t - \tau_i) = w_i x(t) - \bar{u}(t - \tau_i, t) + v(t - \tau_i) \qquad (3.a)$$

with:

$$w_i = Ce^{-A\tau_i} \qquad (3.b)$$

$$\bar{u}(t - \tau_i, t) = \int_{t-\tau_i}^{t} Ce^{A(t-\tau_i-\delta)} Bu(\delta)d\delta \qquad (3.c)$$

Note that the expression (3.c) requires the knowledge of an analytical form of the input but in practice, this expression is computed numerically within the assumption that the input is constant over the considered time interval. Thanks to equation (3) which is true for each data of the sliding observation window, system (2) can be described by the following « static » form already used by Chow and Willsky (1984):

$$Z_k(t) = W_k x(t) + N_k(t) \qquad (4.a)$$

where Z_k of dimension p(k+1) and the matrix W_k of dimension p(k+1)xn are defined by:

$$Z_k(t) = Y_k(t) + U_k(t) \qquad (4.b)$$

$W_k = [w_i]$ with i from 1 to k

$$Y_k(t) = \begin{bmatrix} y_0 \\ \vdots \\ y_k \end{bmatrix},\ U_k(t) = \begin{bmatrix} \bar{u}(t-\tau_0, t) \\ \vdots \\ \bar{u}(t-\tau_k, t) \end{bmatrix},\ N_k(t) = \begin{bmatrix} v(t-\tau_0) \\ \vdots \\ v(t-\tau_k) \end{bmatrix}$$

Note that N_k is the extended vector of the noise which variance is defined by:

$$R_k(t) = \mathrm{diag}\big(V(t-\tau_0),\ldots,V(t-\tau_k)\big) = \big(Q_k^T Q_k\big)^{-1} \qquad (5)$$

2.3. Finite memory observer

Within the use of the "static" form (4) of system (2), the state estimation is given in term of maximum likelihood by minimisation of the weighted square output estimation error other the defined observation window, i.e. the minimisation of the following criterion:

$$\phi_k = \frac{1}{2}\left\| Z_k(t) - \hat{Z}_k(t) \right\|^2_{R_k^{-1}} \qquad (6.a)$$

under the constraint:

$$\hat{Z}_k(t) = \hat{Y}_k(t) + U_k(t) = W_k \hat{x}_k(t) \qquad (6.b)$$

The classical solution in the least square sense is obtained:

$$\hat{x}_k(t) = \Omega_k^{-1} W_k^T R_k^{-1} Z_k(t) \qquad (7)$$

where the square matrix of dimension nxn:

$$\Omega_k(t) = W_k^T R_k^{-1}(t) W_k \qquad (8)$$

is assumed to be invertible. Note that the designed estimation is the product of a gain with a linear combination of the measurement z_i. It is therefore intrinsically of finite memory which size is equal to the number of data taken into account, i.e. k+1 and relies on the horizon definition $[t-\tau_k, t-\tau_0]$. The extended output estimation $\hat{Y}_k$ is deduced from (6.b), whose p first components stand for the estimation of the output at the instant t, i.e.:

$$\hat{y}_k(t) = S_{p,p(k+1)} \hat{Y}_k(t) = C\hat{x}_k(t) \qquad (9.a)$$

with:

$$S_{p,p(k+1)} = \begin{bmatrix} I_p & 0_{p,pk} \end{bmatrix} \quad (9.b)$$

Only the properties of the state estimation error (10.a) are given below as the ones of the output estimation error (10.b) can be easily deduced thanks to (9).

$$e_k(t) = \hat{x}_k(t) - x(t) \quad (10.a)$$

$$\varepsilon_k(t) = \hat{y}_k(t) - y(t) = S_{p,p(k+1)}\left(\hat{Y}_k(t) - Y_k(t)\right) \quad (10.b)$$

3. PROPRIETIES OF THE FINITE MEMORY ESTIMATION

3.1. Necessary and sufficient condition

The finite memory observer exists if and only if the matrix Ω_k is invertible which is equivalent to matrix Ω_k of full column rank. As it is a symmetric matrix depending on W_k and the regular matrix R_k, a sufficient condition is that W_k is full column rank. As W_k is defined by the static form (4) of the system, it can easily be proved that this condition is equivalent to the observability condition of the model (4). Therefore, the minimal number of measurements to take into account so that the observer exists is given by the observability gramian (11) condition taking into account the k+1 data of the observation horizon (static form).

$$\Gamma_k(t) = W_k^T(t) Q_k^T(t) Q_k(t) W_k(t) \quad (11)$$

3.2. Optimal number of measurements

The optimal number is given by the evolution of the norm of the state estimation error variance. Basing the proof on a sequential form (12) of the estimation (7) of x(t) expressed at a frozen instant t by adding one more data (z_{k+1}), it can be shown that matrix Ω_k is the solution of an algebraic Riccati equation depending on parameter k (Nuninger, 1997). Thanks to the proprieties of such equation, there exists an integer k_{opt} so that the inverse of matrix Ω_k does not change significantly for $k > k_{opt}$. This means that the new estimation of x(t), $\hat{x}_{k+1}$, based on one more data (z_{k+1}) does not differ significantly from the previous estimation $\hat{x}_k$; the adjunction of more data is therefore useless (no disturbances assumed).

$$\hat{x}_{k+1}(t) = \hat{x}_k(t) + G_{k+1}(t)\left\{z_{k+1}(t) - w_{k+1}\hat{x}_k(t)\right\} \quad (12.a)$$

$$G_{k+1}(t) = \Omega_k^{-1} w_k^T \left\{r_{k+1} + w_{k+1}\Omega_k^{-1} w_{k+1}^T\right\} \quad (12.b)$$

As a consequence, in practice, the optimal number of data to take into account is given by the evolution of the Euclidean matrix norm (for instance) of the estimation error variance Ω_k^{-1} or of the observability gramian Γ_k when k increases. Note that the optimal number of measurement does not rely on the τ_i that should nevertheless represent a proper virtual sampling of the data (Shannon criterion).

3.3. Estimation error proprieties

Consider additive deterministic faults d(t) on the output measurement in spite of the noise (i.e. $D_k(t) = \left[d(t - \tau_i)\right]$ for the considered horizon). Then, the output measurement of the static model is rewritten as (13) and the estimation error is (14).

$$\bar{Z}_k(t) = Z_k(t) + D_k(t) \quad (13)$$

$$\bar{e}_k(t) = e_k(t) + \Delta e_k(t) \quad (14.a)$$

where:

$$e_k(t) = \Omega_k^{-1} W_k^T R_k^{-1} N_k(t) \quad (14.b)$$

$$\Delta e_k(t) = \Delta\hat{x}_k(t) = \Omega_k^{-1} W_k^T R_k^{-1} D_k(t) \quad (14.c)$$

Note that $e_k(t)$ stands for the error when no fault occurs whereas the error differs from this value by the quantity $\Delta e_k(t)$ while faults appear. As a consequence, the estimation error is unbiased when there is no disturbances as it is proved within the estimation error mean and variance:

$$\mathrm{Esp}\left\{\bar{e}_k(t)\right\} = 0 + \mathrm{Esp}\left\{\Delta e_k\right\} = \Omega_k^{-1} W_k^T R_k^{-1} D_k \quad (15.a)$$

$$\mathrm{Var}\left\{\bar{e}_k(t)\right\} = \mathrm{Var}\left\{e_k(t)\right\} = \Omega_k^{-1} \quad (15.b)$$

Unfortunately, because of model uncertainties the estimation error is not a good residual to detect incipient faults. Indeed, as the real system is described by (1), the real output measurement is in fact (16.a) where ΔZ_k stands for the influence of the model uncertainties (16.b) not taken into account in the static model (3) of the system.

$$\bar{Z}_k(t) = Z_k(t) + D_k(t) + \Delta Z_k(f) \quad (16.a)$$

$$f(x,u,t) = \sum_{s=1}^{S} A_s x(t - t_s) + \sum_{j=1}^{J} B_i u(t - t_j) \quad (16.b)$$

As a consequence, the estimation error is rewritten as (14.a) where $e_k(t)$ is still given by (14.b) but with the variation from this value, $\Delta e_k(t)$, now defined by:

$$\Delta e_k(t) = \Omega_k^{-1} W_k^T R_k^{-1}\left\{D_k(t) + \Delta Z_k(f)\right\} \quad (17)$$

It is obvious that both influence of faults and model uncertainties can not be distinguished as the corresponding sensitivities of the error, defined by the Jacobean matrices, are equal:

$$\frac{\partial \bar{e}_k}{\partial D_k^T} = \left[\frac{\partial \bar{e}_k(i)}{\partial D_k^T(j)}\right]_{n,(k+1)p} = \Omega_k^{-1} W_k^T R_k^{-1} = \frac{\partial \bar{e}_k}{\partial \Delta Z_k^T} \quad (18)$$

4. NEW RESIDUAL

4.1. Definition

The finite memory estimation can be computed on-line on a fixed horizon within the recursive formulation (12). This formula (similar to the Kalman filter innovation equation) brings up the expression of the new residual. Indeed, assume a fault on the more recent data (z_{k+1}) then, from (12) it is obvious that only $\hat{x}_{k+1}$ will be wrong as it depends on the faulty z_{k+1} which is not the case of $\hat{x}_k$. Therefore, the difference between $\hat{x}_{k+1}$ and $\hat{x}_k$ is not equal to zero and the fault can be detected. On the contrary, model uncertainties influence any estimation quite the same way; making the difference can eliminate some of them so that it is less sensitive to model uncertainties. So, in order to detect incipient faults despite of model uncertainties, a robust residual (19) is defined by extension as the difference of two estimations $\hat{x}_{0,k}(t)$ and $\hat{x}_{r,k}(t)$ of the same state x(t).

$$e_{k/r}(t) = \hat{x}_{0/k}(t) - \hat{x}_{r/k}(t) \qquad (19)$$

The estimations $\hat{x}_{0,k}$ and $\hat{x}_{r,k}$ are computed on two overlapping data-sets $\{y(t-\tau_k),\ldots,y(t-\tau_0)\}$ and $\{y(t-\tau_k),\ldots,y(t-\tau_r)\}$ as shown in Figure 1.

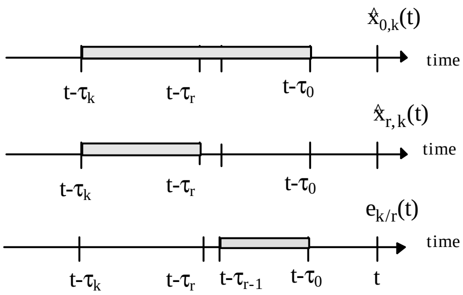

Fig. 1. Definition of the residual

It can be proved that the data of the detection window, i.e. $\{y(t-\tau_{r-1}),\ldots,y(t-\tau_0)\}$, have not the same influence than the data of the smallest horizon which is included in the largest one (these data are used for the computation of the two estimations ; this is not the case for the data of the detection window). Then, thanks to a special choice of (k,r), it is possible to give a greater sensitivity of the residual to the data of the detection window (i.e. the ones containing the information about the incipient fault) than the other data. Therefore, it is possible to design a residual which is less sensitive to some model uncertainties.

4.2. Residual formulation and statistical proprieties

Consider each estimation $\hat{x}_{0,k}(t)$ and $\hat{x}_{r,k}(t)$ deduced from (7) with the proper definitions of vectors and matrices on the given horizons denoted by the sub-indices '0,k' and 'r,k' respectively (see Figure 1). $\hat{x}_{0,k}(t)$ is given by (20) and $\hat{x}_{r,k}(t)$ satisfies the same equation with the proper notations.

$$\bar{\hat{x}}_{0,k}(t) = \hat{x}_{0,k}(t) + \Delta\hat{x}_{0,k}(t) \qquad (20.a)$$

$$\hat{x}_{0,k}(t) = x(t) + \Omega_{0,k}^{-1} W_{0,k}^{T} R_{0,k}^{-1} N_{0,k}(t) \qquad (20.b)$$

$$\Delta\hat{x}_{0,k}(t) = \Omega_{0,k}^{-1} W_{0,k}^{T} R_{0,k}^{-1} \{D_{0,k} + \Delta Z_{0,k}\} \qquad (20.c)$$

Note that $\hat{x}_{0,k}$ (20.a) stands for the estimation while there is no faults and no model uncertainties, otherwise the estimation differs from this value by the quantity $\Delta\hat{x}_{0,k}$ (20.c). Thanks to the vectors and matrices bloc-proprieties, the estimation $\hat{x}_{0,k}$ on the bigger horizon can be expressed with respect to the vectors and matrices defining the estimation $\hat{x}_{r,k}$ based on the smaller horizon. Indeed, the following relations are satisfied (the sub-indices '0,r-1' denotes the data defined inside the detection window):

$$Y_{0,k} = \begin{bmatrix} Y_{0,r-1} \\ Y_{r,k} \end{bmatrix}$$, and so on for $U_{0,k}$, $Z_{0,k}$, $N_{0,k}$ and $D_{0,k}$

$$R_{0,k}^{-1} = \begin{bmatrix} R_{0,r-1}^{-1} & 0_{r,k-r+1} \\ 0_{k-r+1,r} & R_{r,k}^{-1} \end{bmatrix}, W_{0,k} = \begin{bmatrix} W_{0,r-1} \\ W_{r,k} \end{bmatrix}$$

$$\Omega_{0,k} = \Omega_{r,k} + \Omega_{0,r-1}$$

First assume that the model uncertainties are additive deterministic and constant parameter variations (ΔA, ΔB and ΔC) on the matrices defining the real system which is therefore given by ($A+\Delta A$, $B+\Delta B$, $C+\Delta C$). The not exact model is kept as (A,B,C). As a consequence, the output measurement Z_k can be rewritten with ΔZ_k now defined by (21) where the matrix ΔW_k and the vector ΔU_k stand for the additive and deterministic parameter variations of the matrices and vectors defining the static form of the real system. Basing the demonstration on the way W_k and U_k are designed when the indicated parameter variations occur, it is easily proved that $\Delta Z_{0,k}$ can also be rewritten with respect to $\Delta Z_{o,r-1}$ and $\Delta Z_{r,k}$ the same way $Z_{0,k}$ is expressed.

$$\Delta Z_k(t) = \Delta W_k x(t) - \Delta U_k(t) \qquad (21)$$

Because of the limited number of pages required for this paper, only the final expression of the residual (22) is given (Nuninger, 1997) that leads to the statistical proprieties (23). From (22) and the further sensitivity study, it is obvious that both influence of faults and parameter variations (additive and deterministic) can be split apart thanks to the choice of the pair (k, r).

$$\bar{e}_{k/r}(t) = e_{k/r}(t) + \Delta e_{k/r}(t) \qquad (22.a)$$

$$\begin{aligned} e_{k/r}(t) = {} & \Omega_{0,k}^{-1} W_{0,r-1}^{T} R_{0,r-1}^{-1} N_{0,r-1}(t) + \\ & \left(\Omega_{0,k}^{-1} - \Omega_{r,k}^{-1}\right) W_{r,k}^{T} R_{r,k}^{-1} N_{r,k}(t) \end{aligned} \qquad (22.b)$$

$$\begin{aligned} \Delta e_{k/r}(t) = {} & \Omega_{0,k}^{-1} W_{0,r-1}^{T} R_{0,r-1}^{-1} \left[\Delta Z_{0,r-1}(t) + D_{0,r-1}(t)\right] + \\ & \left(\Omega_{0,k}^{-1} - \Omega_{r,k}^{-1}\right) W_{r,k}^{T} R_{r,k}^{-1} \left[\Delta Z_{r,k}(t) + D_{r,k}(t)\right] \end{aligned} \qquad (22.c)$$

$$Esp\{\bar{e}_{k/r}(t)\} = Esp\{\Delta e_{k/r}(t)\} = \Delta e_{k/r}(t) \qquad (23.a)$$

$$Var\{\bar{e}_{k/r}(t)\} = \Omega_{r,k}^{-1} - \Omega_{0,k}^{-1} \qquad (23.b)$$

Second, the model uncertainties are due to not modelled time-delay in the real system. Therefore, $\Delta Z_{0,k}$ must be rewritten like (24) that leads to the new expression (25) of $\Delta e_{k/r}(t)$.

$$\Delta Z_{0,k}(t) = \begin{bmatrix} \Delta Z_{0,k}[0, r-1] \\ \Delta Z_{0,k}[r, k] \end{bmatrix} \qquad (24)$$

$$\Delta e_{k/r}(t) = -\Omega_{r,k}^{-1} W_{r,k}^{T} R_{r,k}^{-1} \left\{ \Delta Z_{r,k} + D_{r,k} \right\} + \Omega_{0,k}^{-1} W_{0,r-1}^{T} R_{0,r-1}^{-1} \left\{ \Delta Z_{0,k}[0, r-1] + D_{0,r-1} \right\} + \Omega_{0,k}^{-1} W_{r,k}^{T} R_{r,k}^{-1} \left\{ \Delta Z_{0,k}[r,k] + D_{r,k} \right\} \quad (25)$$

However, the proof is omitted in this paper, it can be shown that $\Delta Z_{0,k}[r,k]$ rely on $\Delta Z_{r,k}$ because of the way the static form of the real system is designed. This is quite clear for discrete time representation. Therefore, a slightly similar result as (22) can be written but in a more complex way because of coupling terms but a close conclusion could be brought out anyway. The extension for the statistical proprieties is not direct.

4.4. Residual sensitivity to parameter variations

Because of the previous remark, only the residual sensitivity to additive deterministic parameter variations is studied and compared to the residual sensitivity to additive faults. Within the previous definition of sensitivity, the following results can be easily proved from the expression (22) of the residual:

$$\frac{\partial \bar{e}_{k/r}(t)}{\partial D_{0,r-1}^{T}(t)} = \frac{\partial \bar{e}_{k/r}(t)}{\partial \Delta Y_{0,r-1}^{*T}(t)} = \Omega_{0,k}^{-1} W_{0,r-1}^{T} R_{0,r-1}^{-1} \quad (25.a)$$

$$\frac{\partial \bar{e}_{k/r}(t)}{\partial D_{r,k}^{T}(t)} = \frac{\partial \bar{e}_{k/r}(t)}{\partial \Delta Y_{r,k}^{*T}(t)} = \left(\Omega_{0,k}^{-1} - \Omega_{r,k}^{-1} \right) W_{r,k}^{T} R_{r,k}^{-1} \quad (25.b)$$

From theses results it is concluded that for a given horizon, the influence of faults and uncertainties can not be distinguished. Note that the same result applied for the state estimation error $\bar{e}_k(t)$ based on the biggest horizon. The residual sensitivity to the faults $D_{0,r-1}$ appearing on the data of the detection window (25.a) is compared to the residual sensitivity to the fault $D_{r,k}$ appearing on the data from the non-common horizon (25.b). It is obvious that both sensitivities are different (the same result applied for $\Delta Z_{0,r-1}$ and $\Delta z_{r,k}$). This remark leads to a tool for the choice of the pair (k, r). Note that in comparison with the estimation error $\bar{e}_k(t)$, the proposed residual $\bar{e}_{r/k}(t)$ has not the same sensitivity with respect to incipient faults whereas the sensitivities are equal outside the detection window:

$$\frac{\partial \bar{e}_{k}}{\partial D_{r,k}^{T}} = \Omega_{0,k}^{-1} W_{r,k}^{T} R_{r,k}^{-1} = \frac{\partial \bar{e}_{k/r}}{\partial D_{r,k}^{T}} + \Omega_{r,k}^{-1} W_{r,k}^{T} R_{r,k}^{-1} \quad (26.a)$$

$$\frac{\partial \bar{e}_{k}(t)}{\partial D_{0,r-1}^{T}(t)} = \frac{\partial \bar{e}_{k}(t)}{\partial \Delta Y_{0,r-1}^{*T}(t)} = \Omega_{0,k}^{-1} W_{0,r-1}^{T} R_{0,r-1}^{-1} = \frac{\partial \bar{e}_{k/r}(t)}{\partial D_{0,r-1}^{T}(t)} \quad (26.b)$$

4.3. Optimal choice of the detection window

Thanks to the previous results, it is proved that the residual $e_{r/k}(t)$ sensitivity can be improved with respect to incipient faults within a proper choice of the pair (k, r). This pair is chosen so that more influence is given to the data of the detection window with respect to the measurements outside this window. The aim is that the residual is more sensitive to incipient faults $D_{0,r-1}$ and less sensitive to some parameter variations $\Delta Z_{r,k}$ (note that the influence with respect to $\Delta Z_{0,r-1}$ can not be changed in comparison to the influence of $D_{0,r-1}$). First, the integer k is chosen so that the state estimation error is of minimal variance (the inverse of Ω_k is the solution of an algebraic Riccati equation). Second, for a fixed optimal k, integer $r < k$ is chosen so that the sensitivity of the residual with respect to the data from the detection window is greater than the sensitivity with respect to the data from the smallest horizon. In practice, such a consideration leads to the study of the following sensitivity performance index ratio $\rho D_{r,k}^{0,r-1}$ (27) when r increases. The result is a compromise between having enough data to compute the estimation and an optimal sensitivity to the incipient faults. The choice remains an heuristic one as the evolution is not simple to study in theory (especially if model uncertainties are considered instead of additive deterministic parameter variations). Anyway, the proposed residual (19) is more effective for fault detection than the classical estimation error.

$$\rho D_{r,k}^{0,r-1} = \left\| \frac{\partial \bar{e}_{k/r}}{\partial D_{0,r-1}^{T}} \right\|_2 \Bigg/ \left\| \frac{\partial \bar{e}_{k/r}}{\partial D_{r,k}^{T}} \right\|_2 \quad (27)$$

5. APPLICATION ON A SIMULATED EXAMPLE

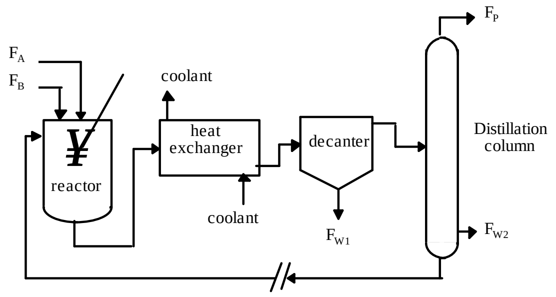

Fig. 2. Flowsheet of the chemical process (Williams et Otto, 1960).

The proposed example is a chemical process presented by (Williams and Otto, 1960). This system is a model of a refining plant (with separation and reaction involved) whose flow-sheet is shown in Figure 2 (Schoen, 1995). Sub-indices 'A' and 'B' denote the raw materials (feed rates F_A and F_B respectively), 'P' is the valuable product (F_P) whereas 'W1' stands for the by-product which is undesirable (F_{W1}) and 'W2' for the column purge lead off (F_{W2}). Note that the raw materials and by-product are recycled to the chemical reactor when reprocessed within the recycle loop that represents a significant transport lag. As a consequence, the equations of the process are non linear ones but a linearized model can be given anyway for a chosen operating point. In that case, for a recycling time of 10 minutes, the linearized model is the following (one time unit stands for 10 minutes) (Ross, 1971):

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-1) + Bu(t)$$
$$y(t) = Cx(t)$$

$$A_0 = \begin{bmatrix} -4.93 & -1.01 & 0 & 0 \\ -3.20 & -5.30 & -12.8 & 0 \\ 6.40 & 0.387 & -32.5 & -1.04 \\ 0 & 0.833 & 11.0 & -3.96 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1.92 & 0 & 0 & 0 \\ 0 & 1.92 & 0 & 0 \\ 0 & 0 & 1.87 & 0 \\ 0 & 0 & 0 & 0.724 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & -1 \end{bmatrix}$$

The four state components stand for the deviation in the weight composition of the materials (A, B, W1, W2 respectively). The control inputs are defined as $\partial F_A / 6V_R$ and $\partial F_B / 6V_R$ ($V_R = 2.628m^3$ is the volume of the reactor) and ∂F_A and ∂F_B are the deviations in the feed rates (in pounds per hour) from their nominal values. The proposed residual defined on the output $\varepsilon_{k/r}(t) = \hat{y}_{0,k} - \hat{y}_{r,k} = Ce_{k,r}(t)$ is designed from the evolution of the norm of the estimation error covariance matrix (k = 15) and from the evolution of the sensitivity performance index ratio $\rho D_{r,k}^{0,r-1}$ (r = 2 < k) plotted in Figure 3. In this figure the evolution of the variance norm of the residual is also plotted. In Figure 4 the residual is plotted when there is no faults (it is statistically close to zero). The output is also plotted. Two additive bias on y are assumed of magnitude 0.02, length 1.6s and 0.5s starting at instants 0.8s and 4s respectively. Two random values of magnitude 0.02 are also simulated at instants 7s and 8s. The measurement noise is of 5% magnitude.

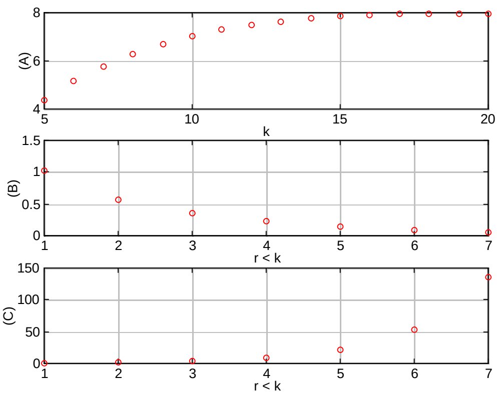

Fig. 3. Norms of the estimation error variance (A), ratio $\rho D_{r,k}^{0,r-1}$ (B) and of the residual variance (C).

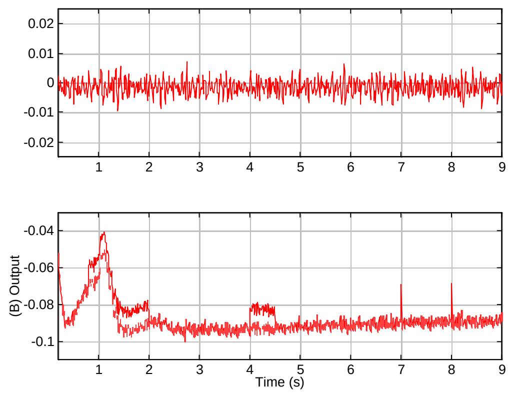

Fig. 4. (A) Residual when no faults. (B) Faulty and no-faulty output (dash) $y(t) = x_3(t) - x_2(t)$.

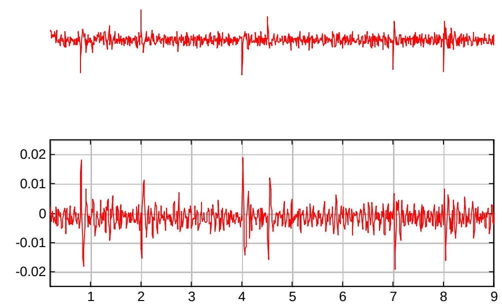

Fig. 5. Estimation error (A) and residual (B) (faults).

Finally, the residual and the estimation error are plotted in Figure 5. The error never converges to zero (even while no faults) and the evaluation procedure should be based on adaptive threshold whereas only a fixed threshold is required for the residual. It is therefore more efficient for fault detection as it is less sensitive to model uncertainties and the evaluation procedure is simpler.

6. CONCLUSION

Despite of the uncertain model (as time-delays are not taken into account to design the observer) the proposed residual is worthwhile for the detection of additive faults on the outputs. Indeed, the residual defined as the difference between two finite memory estimations, is robust with respect to some parameter uncertainties for a special choice of the parameters (k, r) that define the two overlapping horizons. Thanks to this choice, a detection window is defined and the residual is therefore more sensitive to incipient faults than to parameter uncertainties appearing in the measurements outside the detection window. This is true in comparison of the estimation error that accumulates uncertain parameters and could present a divergence phenomenon. Besides, both faults and parameter uncertainties can not be isolated. This is still true for the new residual but, it allows the isolation of incipient faults upon the detection window with respect to fault or disturbances that appeared before the detection window. Note that such a residual as also been developed and studied for discrete time representation (Kratz, *et al.*, 1997). In addition, in order to detect faults on the input, a generalised state finite memory observer is developed by Nuninger (1997). Future work will present this observer and study the residual proprieties.

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